General Motion in 2 or 3 Dimensions

Consider 2-dimensions first (easy to generalize to 3-D)

The vector $\mathbf{r}_1$ tells you where the object is with respect to the origin $(x=0,y=0)$ at time $t_1$.

As the object moves, the position changes from $\mathbf{r}_1$ to $\mathbf{r}_2$

$$\mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

Displacement:

or $\mathbf{\Delta r} = \Delta x \hat{i} + \Delta y \hat{j}$
General Motion in 2 or 3 Dimensions

Just as in 1-D, we can define an average velocity:

\[
\overrightarrow{v}_{\text{avg}} = \frac{\Delta \overrightarrow{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} = v_{\text{avg},x} \hat{i} + v_{\text{avg},y} \hat{j}
\]

And acceleration:

\[
\overrightarrow{a}_{\text{avg}} = \frac{\Delta \overrightarrow{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} = a_{\text{avg},x} \hat{i} + a_{\text{avg},y} \hat{j}
\]

an instantaneous velocity:

\[
\overrightarrow{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{r}}{\Delta t} = v_x(t) \hat{i} + v_y(t) \hat{j}
\]

\[
\overrightarrow{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{v}}{\Delta t} = a_x(t) \hat{i} + a_y(t) \hat{j}
\]
Example: The position of an object with time is given by:

\[ \vec{r}(t) = (45t) \hat{i} + (1.5 - 4.9t^2) \hat{j} \]

Note: The units of both quantities in ( ) are in m. This means 45 = 45 m/s and 4.9 = 4.9 m/s².

a) What is \( \vec{v}(t) \)?

\[ \vec{v}(t) = \frac{dx(t)}{dt} \hat{i} + \frac{dx(t)}{dt} \hat{j} \]

\[ \vec{v}(t) = 45 \, \hat{i} - 9.8t \, \hat{j} \]
Example: Continued

b) What is $a(t)$?

\[
\vec{a}(t) = \frac{dv_x(t)}{dt} \hat{i} + \frac{dv_y(t)}{dt} \hat{j}
\]

\[
\vec{a}(t) = (0) \hat{i} - 9.8 \hat{j}
\]

\[
\vec{a}(t) = -9.8 \hat{j}
\]

c) Draw a picture of the y vs x motion from $t = 0$ s to $t = 0.5$ s.

\[ \text{Table:} \]

<table>
<thead>
<tr>
<th>$t$(s)</th>
<th>$r_x$(m)</th>
<th>$r_y$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.50</td>
</tr>
<tr>
<td>0.1</td>
<td>4.5</td>
<td>1.45</td>
</tr>
<tr>
<td>0.3</td>
<td>13.5</td>
<td>1.06</td>
</tr>
<tr>
<td>0.5</td>
<td>22.5</td>
<td>0.28</td>
</tr>
</tbody>
</table>

$V_0 = 45$ m/s

1.5 m