General Motion in 2 or 3 Dimensions

Consider 2-dimensions first (easy to generalize to 3-D)

Path of particle as a function of time = \( \mathbf{r}(t) \)

The vector \( \mathbf{r}_1 \) tells you where the object is with respect to the origin \((x=0, y=0)\) at time \( t_1 \).

As the object moves, the position changes from \( \mathbf{r}_1 \) to \( \mathbf{r}_2 \)

Displacement:

\[
\begin{align*}
\Delta \mathbf{r} &= \mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} \\
&= \Delta x \hat{i} + \Delta y \hat{j}
\end{align*}
\]

General Motion in 2 or 3 Dimensions

Just as in 1-D, we can define an average velocity:

\[
\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} = v_{\text{avg},x} \hat{i} + v_{\text{avg},y} \hat{j}
\]

And acceleration:

\[
\mathbf{a}_{\text{avg}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} = a_{\text{avg},x} \hat{i} + a_{\text{avg},y} \hat{j}
\]

an instantaneous velocity:

\[
\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = v_x(t) \hat{i} + v_y(t) \hat{j}
\]

And acceleration:

\[
\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = a_x(t) \hat{i} + a_y(t) \hat{j}
\]
Example: The position of an object with time is given by:

\[ \mathbf{r}(t) = (45t) \mathbf{i} + (1.5 - 4.9t^2) \mathbf{j} \]

Note: The units of both quantities in ( ) are in m. This means \(45 = 45\text{m/s}\) and \(4.9 = 4.9\text{m/s}^2\).

a) What is \(v(t)\)?

\[ \mathbf{v}(t) = \frac{dx(t)}{dt} \mathbf{i} + \frac{dy(t)}{dt} \mathbf{j} \]

\[ \mathbf{v}(t) = 45 \mathbf{i} - 9.8t \mathbf{j} \]

Example: Continued

b) What is \(a(t)\)?

\[ \mathbf{a}(t) = \frac{dv_x(t)}{dt} \mathbf{i} + \frac{dv_y(t)}{dt} \mathbf{j} \]

\[ \mathbf{a}(t) = (0) \mathbf{i} - 9.8 \mathbf{j} \]

\[ \mathbf{a}(t) = -9.8 \mathbf{j} \]

c) Draw a picture of the y vs x motion from \(t = 0\text{s}\) to \(t = 0.5\text{s}\).

<table>
<thead>
<tr>
<th>t(s)</th>
<th>(r_x(\text{m}))</th>
<th>(r_y(\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.50</td>
</tr>
<tr>
<td>0.1</td>
<td>4.5</td>
<td>1.45</td>
</tr>
<tr>
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<td>13.5</td>
<td>1.06</td>
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<tr>
<td>0.5</td>
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