

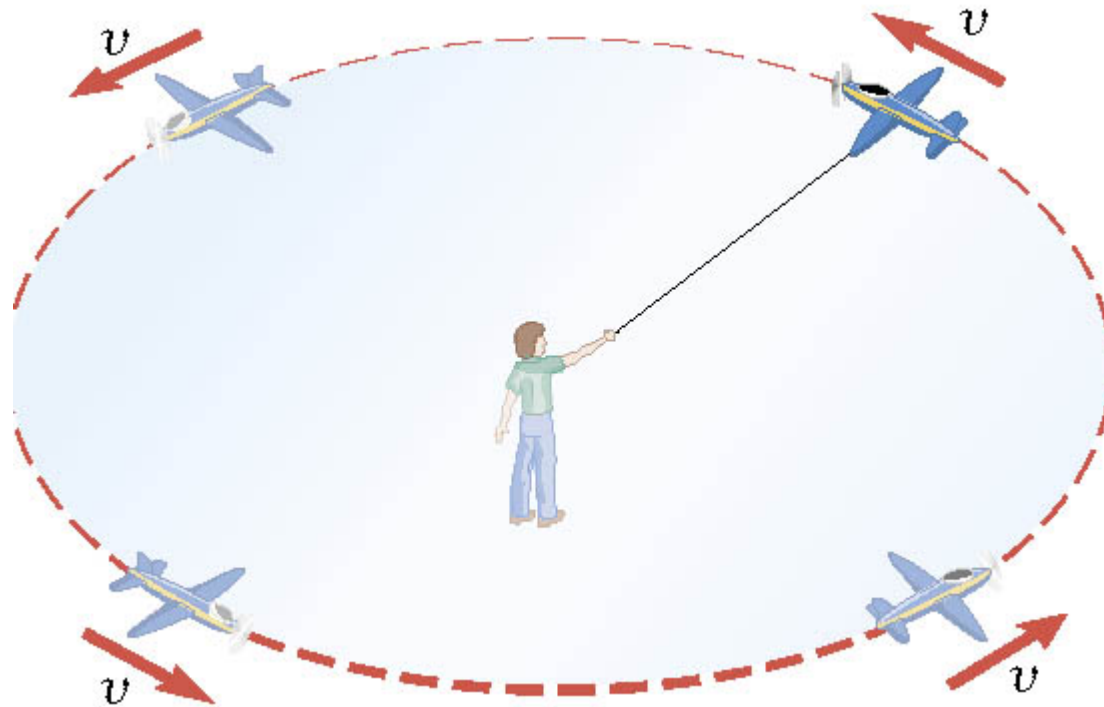
Chapter 5

Dynamics of Uniform Circular Motion

5.1 Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

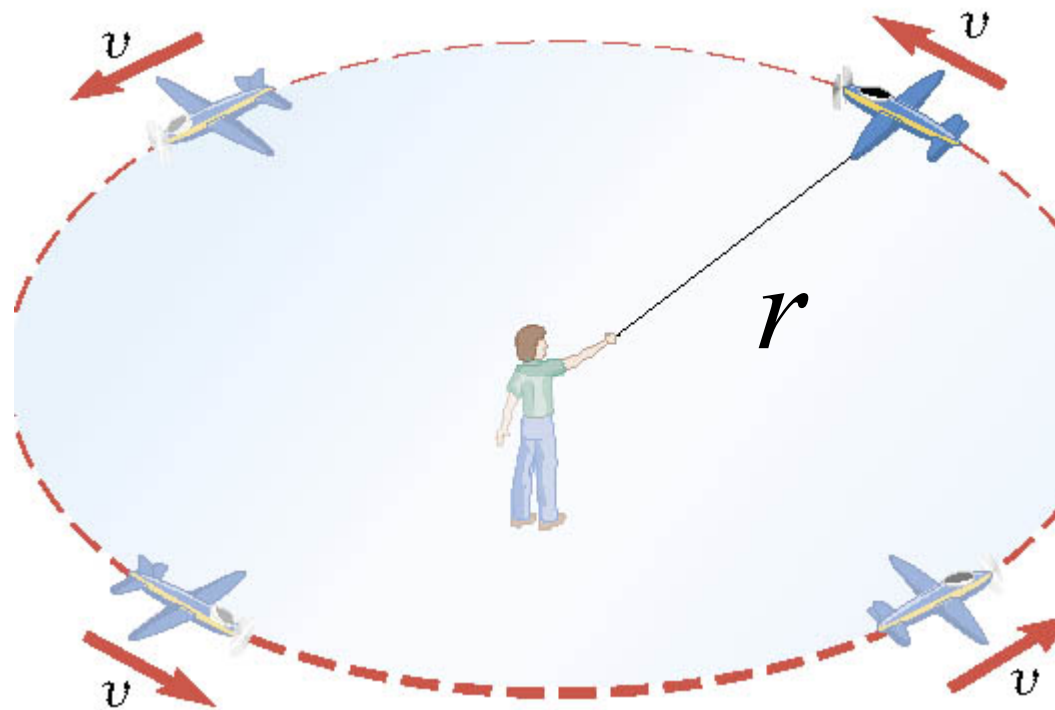


5.1 Uniform Circular Motion

Let T be the time it takes for the object to travel once around the circle.

$T \rightarrow$ **period** of the circular motion

$$v = \frac{2\pi r}{T}$$



5.1 Uniform Circular Motion

Example 1: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

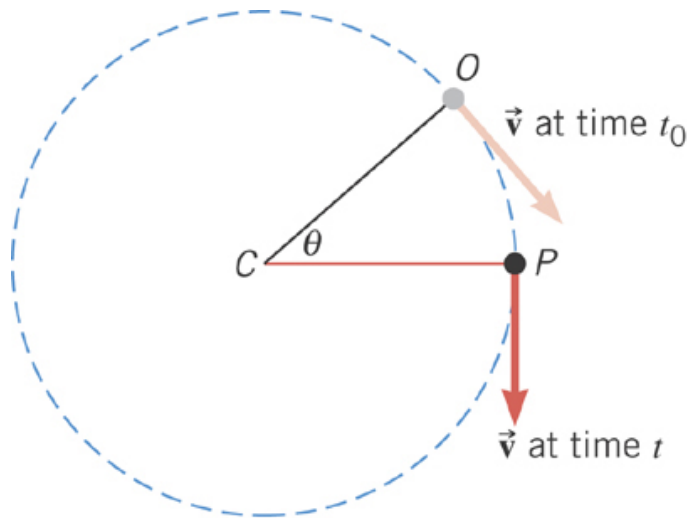
$$\frac{1}{830 \text{ revolutions/min}} = 1.2 \times 10^{-3} \text{ min/revolution}$$

$$T = 1.2 \times 10^{-3} \text{ min} = 0.072 \text{ s}$$

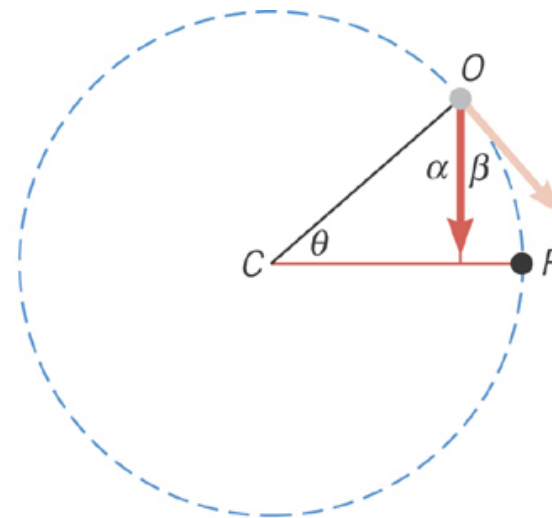
$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29 \text{ m})}{0.072 \text{ s}} = 25 \text{ m/s}$$

5.2 Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant*.



(a)



(b)

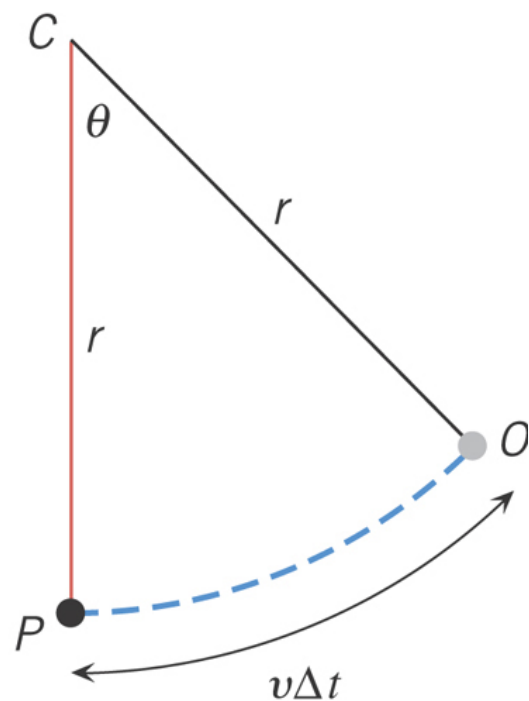
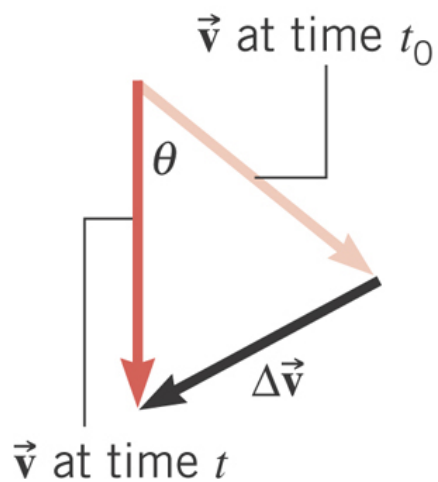
$$\alpha + \beta = 90^\circ$$

$$\alpha + \theta = 90^\circ$$



$$\beta = \theta$$

5.2 Centripetal Acceleration

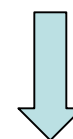


Centripetal acceleration

$$\frac{\Delta v}{v} = \frac{v \Delta t}{r}$$



$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

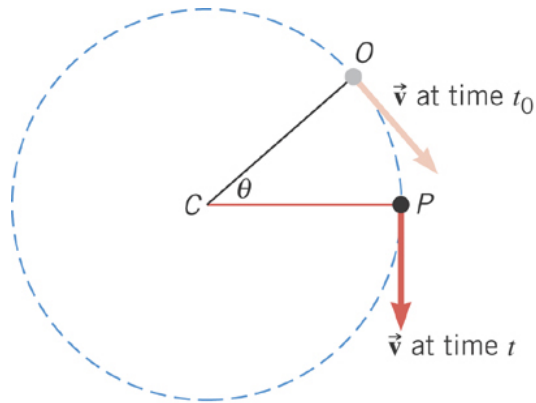


$$a_c = \frac{v^2}{r}$$

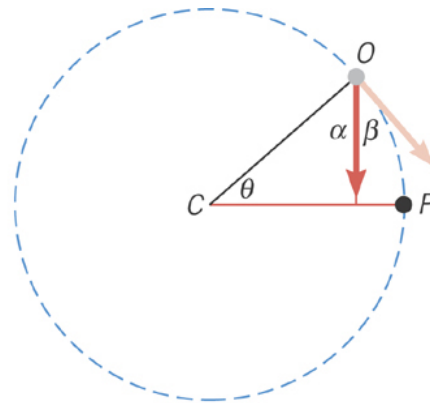
5.2 Centripetal Acceleration

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.

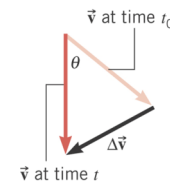
$$a_c = \frac{v^2}{r}$$



(a)



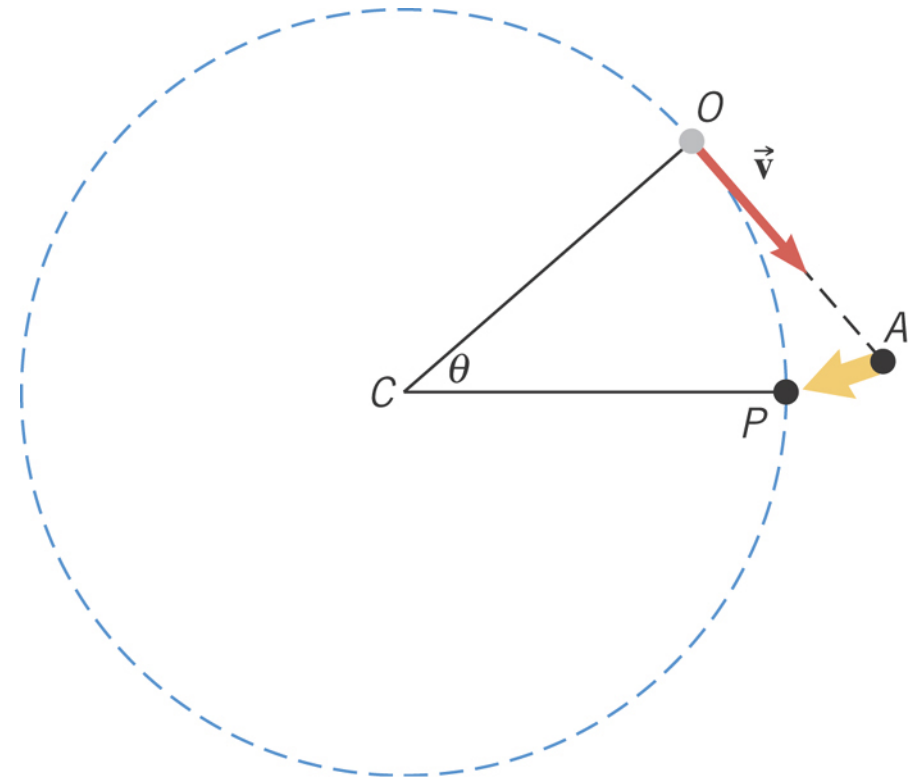
(b)



5.2 Centripetal Acceleration

Conceptual Example 2: Which Way Will the Object Go?

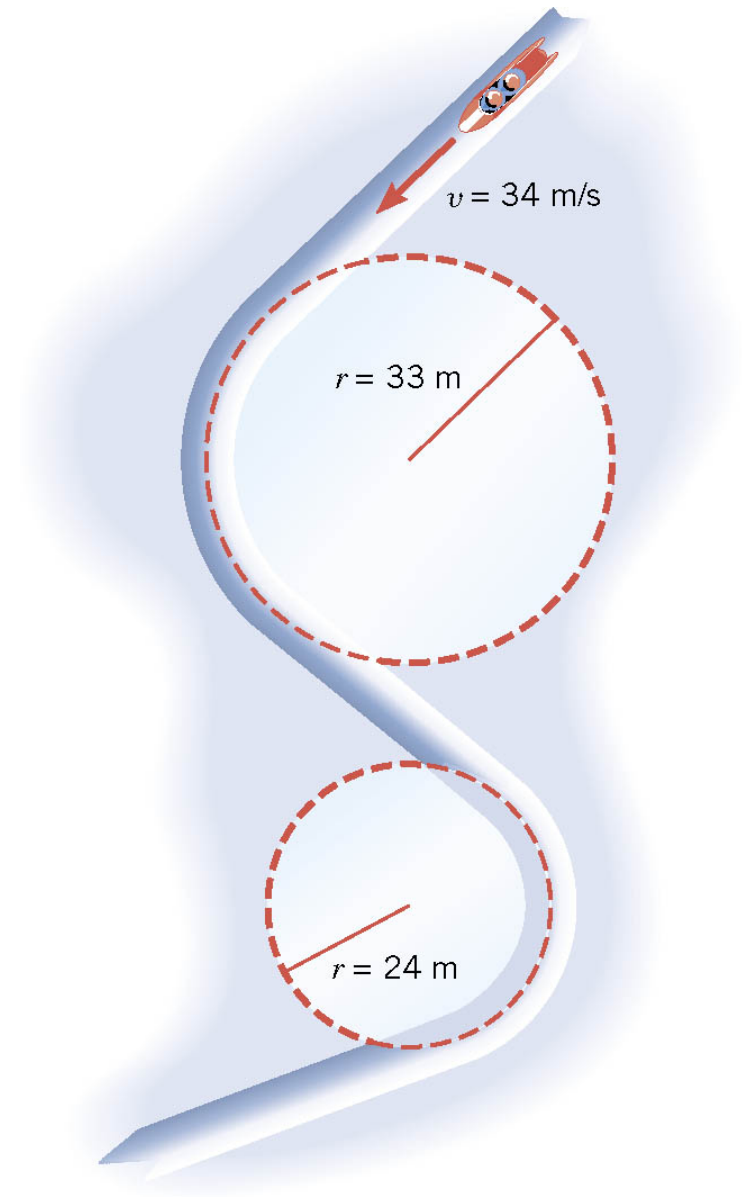
An object is in uniform circular motion. At point O it is released from its circular path. Does the object move along the straight path between O and A or along the circular arc between points O and P ?



5.2 Centripetal Acceleration

Example 3: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m. Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8 \text{ m/s}^2$.

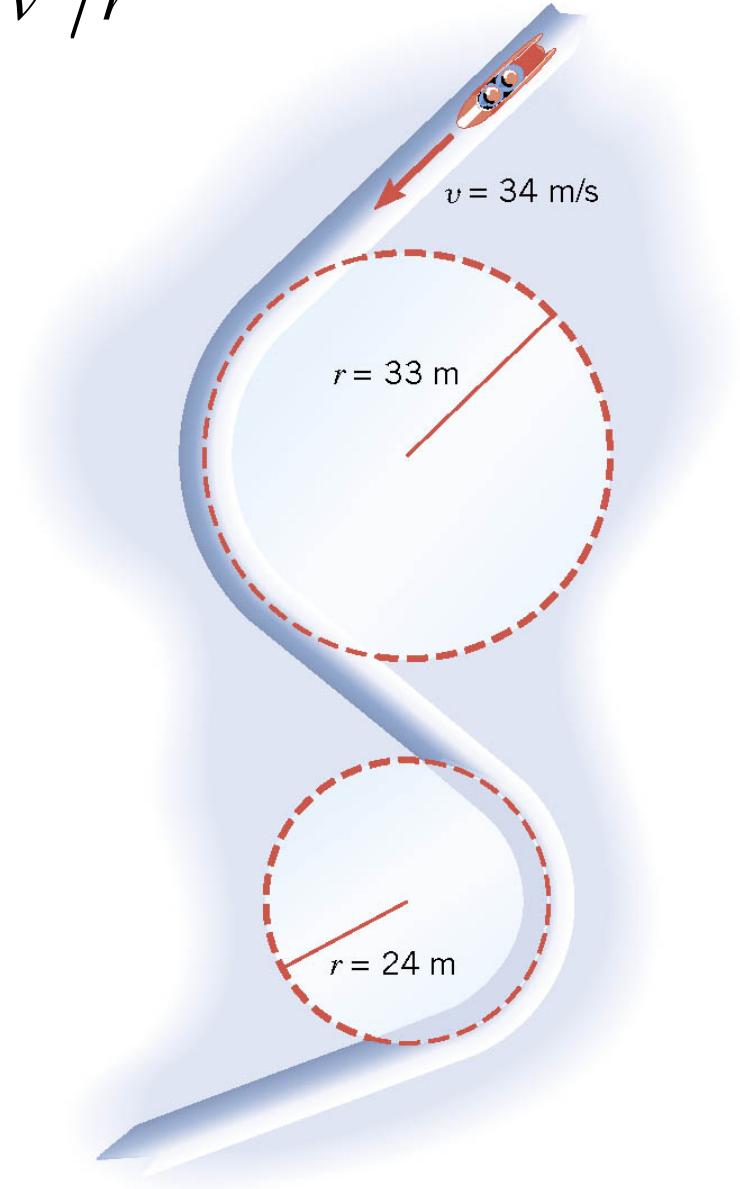


5.2 Centripetal Acceleration

$$a_c = v^2 / r$$

$$a_c = \frac{(34 \text{ m/s})^2}{33 \text{ m}} = 35 \text{ m/s}^2 = 3.6g$$

$$a_c = \frac{(34 \text{ m/s})^2}{24 \text{ m}} = 48 \text{ m/s}^2 = 4.9g$$



5.3 Centripetal Force

Recall Newton's Second Law

When a net external force acts on an object of mass m , the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

5.3 Centripetal Force

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

$$F_c = ma_c = m \frac{v^2}{r}$$

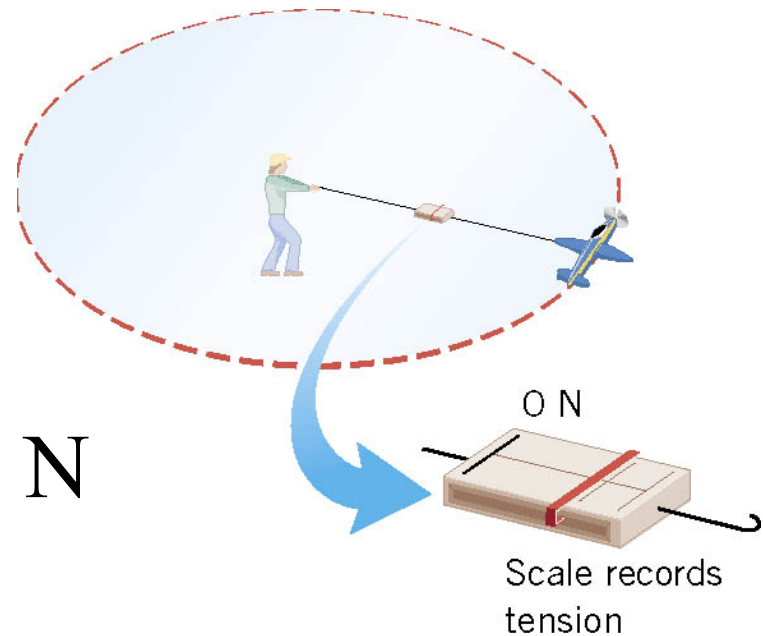
5.3 Centripetal Force

Example 5: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

$$F_c = T = m \frac{v^2}{r}$$

$$T = (0.90 \text{ kg}) \frac{(19 \text{ m/s})^2}{17 \text{ m}} = 19 \text{ N}$$



5.3 *Centripetal Force*

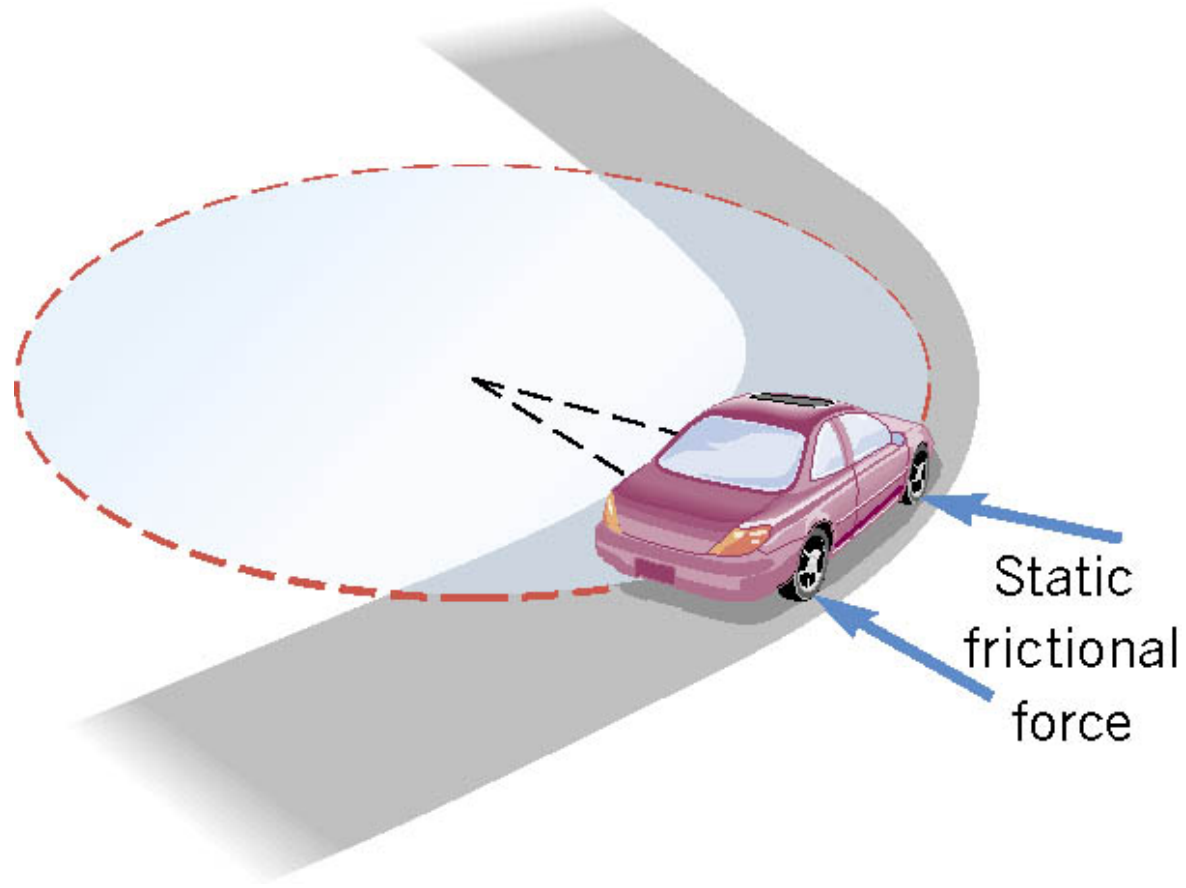
Conceptual Example 6: A Trapeze Act

In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight down and is stationary or when the partner is swinging through the straight-down position?



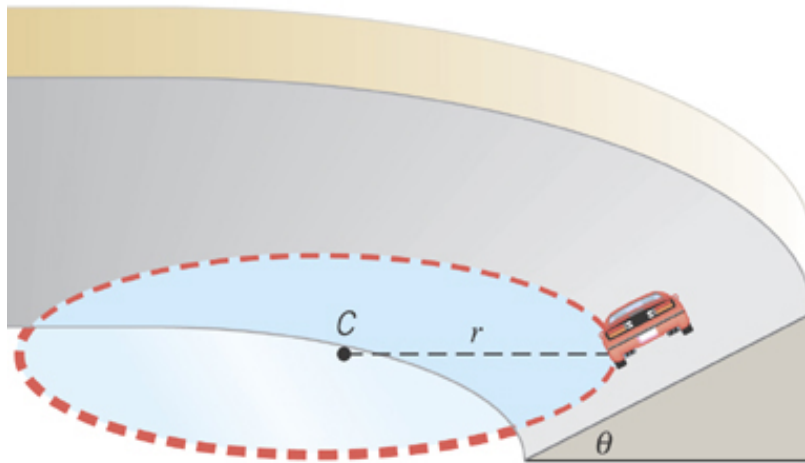
5.4 Banked Curves

On an unbanked curve, the static frictional force provides the centripetal force.

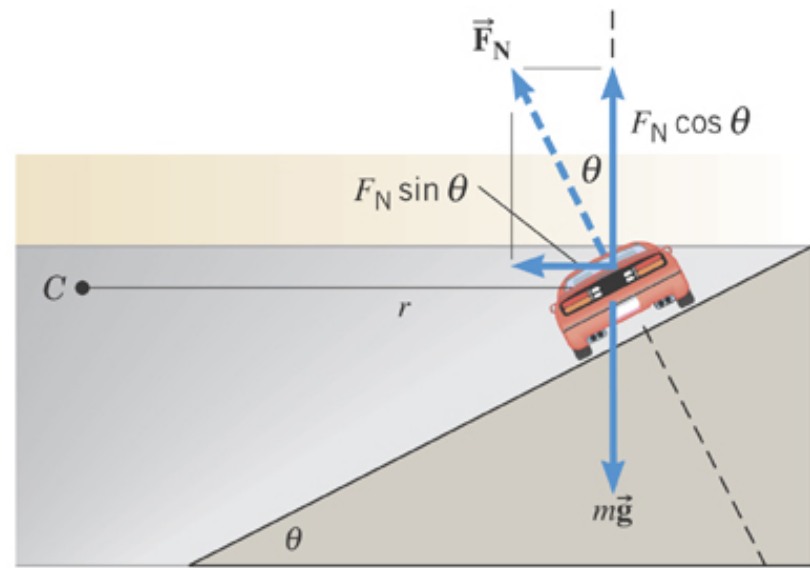


5.4 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.

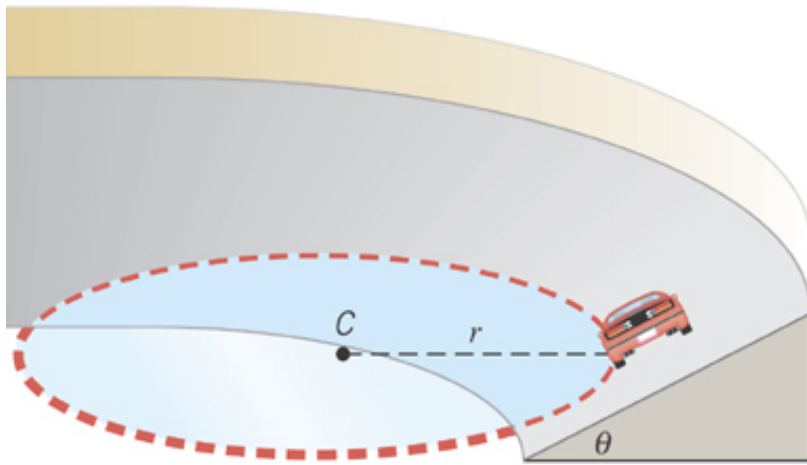


(a)

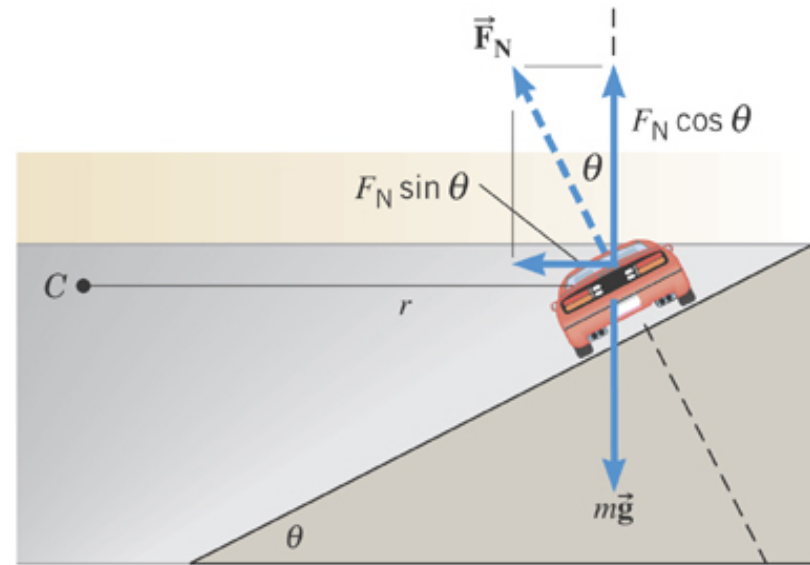


(b)

5.4 Banked Curves



(a)



(b)

$$F_c = F_N \sin \theta = m \frac{v^2}{r}$$

$$F_N \cos \theta = mg$$

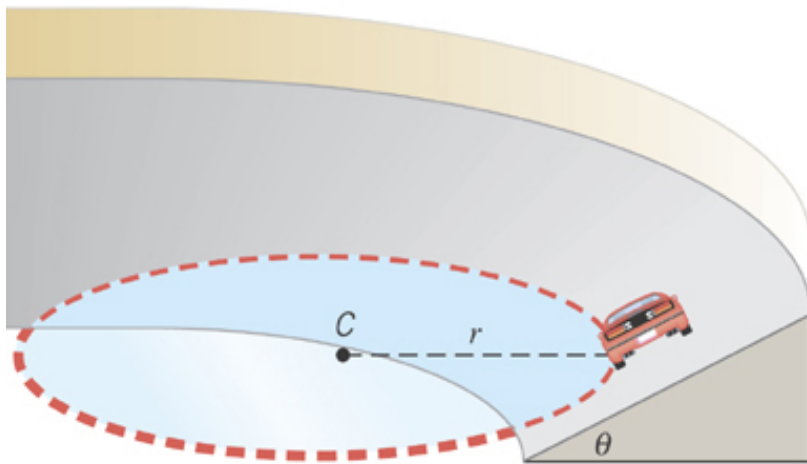
5.4 Banked Curves

$$F_N \sin \theta = m \frac{v^2}{r}$$

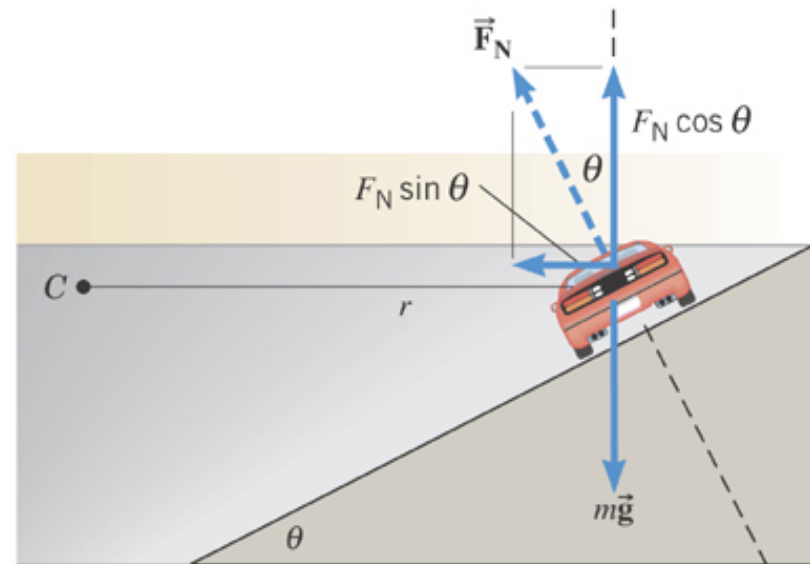


$$\tan \theta = \frac{v^2}{rg}$$

$$F_N \cos \theta = mg$$



(a)



(b)

5.4 Banked Curves

Example 8: The Daytona 500

The turns at the Daytona International Speedway have a maximum radius of 316 m and are steeply banked at 31 degrees. Suppose these turns were frictionless. As what speed would the cars have to travel around them?

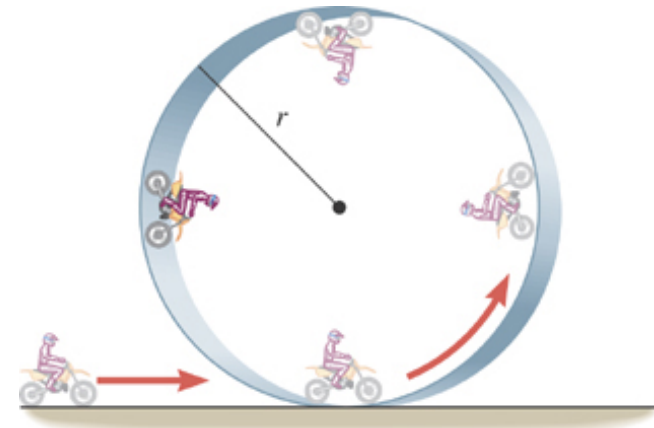
$$\tan \theta = \frac{v^2}{rg} \quad \longrightarrow \quad v = \sqrt{rg \tan \theta}$$

$$v = \sqrt{(316 \text{ m})(9.8 \text{ m/s}^2) \tan 31^\circ} = 43 \text{ m/s (96 mph)}$$

5.7 Vertical Circular Motion

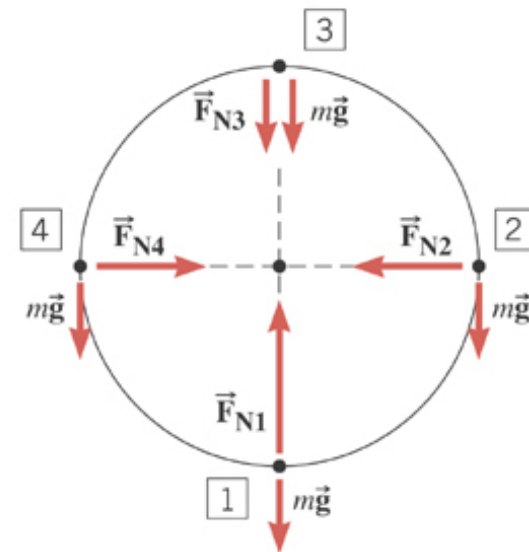
In vertical circular motion the gravitational force must also be considered.

An example of vertical circular motion is the vertical “loop-the-loop” motorcycle stunt. Normally, the motorcycle speed will vary around the loop.



(a)

The normal force, F_N , and the weight of the cycle and rider, mg , are shown at four locations around the loop.



(b)

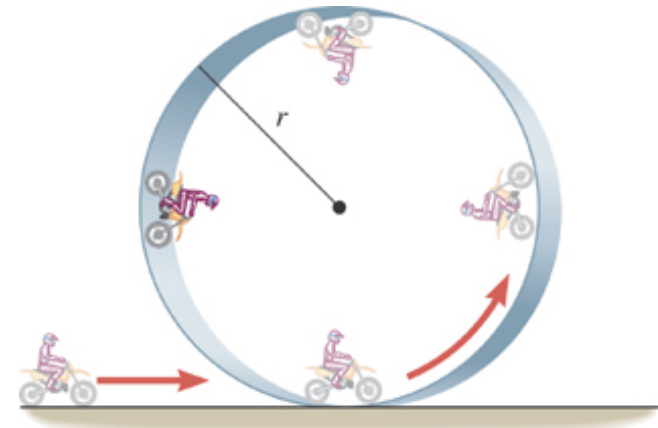
5.7 Vertical Circular Motion

$$1) \quad F_{N1} - mg = m \frac{v_1^2}{r}$$

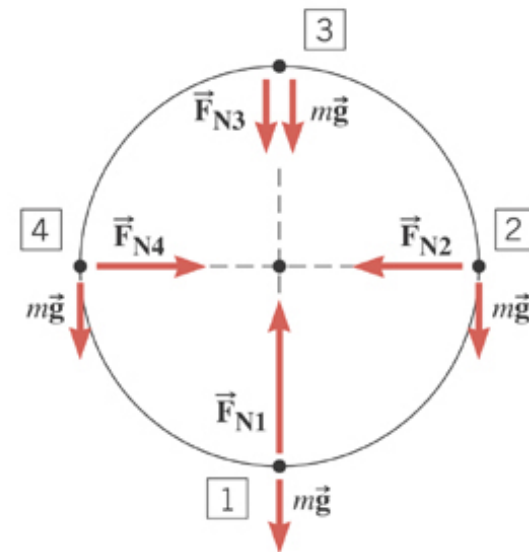
$$2) \quad F_{N2} = m \frac{v_2^2}{r}$$

$$3) \quad F_{N3} + mg = m \frac{v_3^2}{r}$$

$$4) \quad F_{N4} = m \frac{v_4^2}{r}$$



(a)



(b)

5.7 Vertical Circular Motion

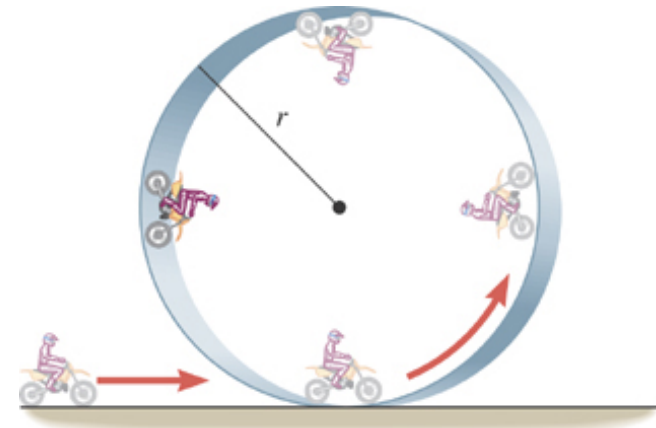
There is a minimum speed the rider must have at point 3 in order to stay on the loop.

This speed may be found by setting $F_{N3} = 0$ in the centripetal force equation for point 3, i.e. in $F_{N3} + mg = mv_3^2/r$

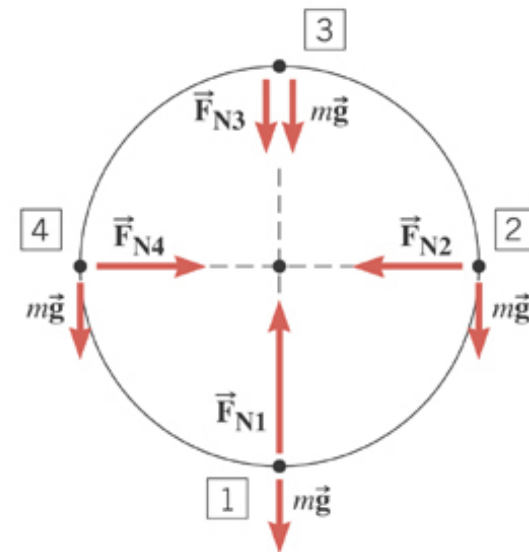
$$\rightarrow v_{3min} = (rg)^{1/2}$$

e.g. for a track with $r = 10$ m,

$$v_{3min} = ((10)(9.8))^{1/2} = 9.9 \text{ m/s} = 22 \text{ mph}$$



(a)



(b)