Chapter 6

Work and Energy
6.1 Work Done by a Constant Force

Work involves force and displacement.

\[ W = Fs \]

1 N \cdot m = 1 \text{joule (J)}
6.1 *Work Done by a Constant Force*

<table>
<thead>
<tr>
<th>System</th>
<th>Force</th>
<th>Distance</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>newton (N)</td>
<td>meter (m)</td>
<td>joule (J)</td>
</tr>
<tr>
<td>CGS</td>
<td>dyne (dyn)</td>
<td>centimeter (cm)</td>
<td>erg</td>
</tr>
<tr>
<td>BE</td>
<td>pound (lb)</td>
<td>foot (ft)</td>
<td>foot · pound (ft · lb)</td>
</tr>
</tbody>
</table>
6.1 *Work Done by a Constant Force*

More general definition of the work done on an object, $W$, by a constant force, $F$, through a displacement, $s$, with an angle, $\theta$, between $F$ and $s$:

$$W = (F \cos \theta)s$$

$\cos 0^\circ = 1$

$\cos 90^\circ = 0$

$\cos 180^\circ = -1$
Example 1 Pulling a Suitcase-on-Wheels

Find the work done if the force is 45.0 N, the angle is 50.0 degrees, and the displacement is 75.0 m.

\[ W = (F \cos \theta)s = (45.0 \text{ N})\cos 50.0^\circ \cdot 75.0 \text{ m} \]

\[ = 2170 \text{ J} \]
A weightlifter doing positive and negative work on the barbell:

Lifting phase --> positive work.
\[ W = (F \cos 0)s = Fs \]

Lowering phase --> negative work.
\[ W = (F \cos 180)s = -Fs \]
Example. Work done skateboarding. A skateboarder is coasting down a ramp and there are three forces acting on her: her weight \( W \) (675 N magnitude), a frictional force \( f \) (125 N magnitude) that opposes her motion, and a normal force \( F_N \) (612 N magnitude). Determine the net work done by the three forces when she coasts for a distance of 9.2 m.

\[
W = (F \cos \theta)s, \text{ work done by each force}
\]

1. Work done by weight
   \[
   W = (675 \cos 65^\circ)(9.2) = 2620 \text{ J}
   \]

2. Work done by frictional force
   \[
   W = (125 \cos 180^\circ)(9.2) = -1150 \text{ J}
   \]

3. Work done by normal force
   \[
   W = (612 \cos 90^\circ)(9.2) = 0 \text{ J}
   \]

The net work done by the three forces is:

\[
2620 \text{ J} - 1150 \text{ J} + 0 \text{ J} = 1470 \text{ J}
\]
6.1 *Work Done by a Constant Force*

**Example 3  Accelerating a Crate**

The truck is accelerating at a rate of +1.50 m/s\(^2\). The mass of the crate is 120 kg and it does not slip. The magnitude of the displacement is 65 m.

What is the total work done on the crate by all of the forces acting on it?
6.1 Work Done by a Constant Force

The angle between the displacement and the normal force is 90 degrees.

The angle between the displacement and the weight is also 90 degrees.

Thus, for $\vec{F}_N$ and $\vec{W}$,

$$W = (F \cos 90) s = 0$$
6.1 Work Done by a Constant Force

The angle between the displacement and the friction force is 0 degrees.

\[ f_s = ma = (120 \text{ kg})(1.5 \text{ m/s}^2) = 180 \text{ N} \]

\[ W = [(180 \text{ N}) \cos 0](65 \text{ m}) = 1.2 \times 10^4 \text{ J} \]
Consider a constant net external force acting on an object. The object is displaced a distance $s$, in the same direction as the net force.

The work is simply

$$ W = (\sum F)s = (ma)s $$
6.2 The Work-Energy Theorem and Kinetic Energy

\[ W = m(\Delta s) = m \frac{1}{2} \left( v_f^2 - v_o^2 \right) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]

\[ v_f^2 = v_o^2 + 2(ax) \]

constant acceleration kinematics formula

\[ (ax) = \frac{1}{2} \left( v_f^2 - v_o^2 \right) \]

DEFINITION OF KINETIC ENERGY

The kinetic energy KE of an object with mass \( m \) and speed \( v \) is given by

\[ KE = \frac{1}{2} m v^2 \]
6.2 *The Work-Energy Theorem and Kinetic Energy*

**THE WORK-ENERGY THEOREM**

When a net external force does work on an object, the kinetic energy of the object changes according to

\[ W = KE_f - KE_o = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]

(also valid for curved paths and non-constant accelerations)
6.2 The Work-Energy Theorem and Kinetic Energy

Example 4 Deep Space 1

The mass of the space probe is 474 kg and its initial velocity is 275 m/s. If the 56.0 mN force acts on the probe through a displacement of $2.42 \times 10^9$ m, what is its final speed?

![Diagram of space probe with force F and initial velocity \( \vec{v}_0 \) and final velocity \( \vec{v}_f \).]
6.2 *The Work-Energy Theorem and Kinetic Energy*

\[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]

\[ W = [(\Sigma F) \cos \theta]s \]
6.2 The Work-Energy Theorem and Kinetic Energy

\[
[(\Sigma F)\cos \theta]s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2
\]

\[
(5.60 \times 10^{-2}\, \text{N}) \cos 0^\circ (2.42 \times 10^9 \, \text{m}) = \frac{1}{2} (474\, \text{kg}) v_f^2 - \frac{1}{2} (474\, \text{kg})(275\, \text{m/s})^2
\]

\[
v_f = 805\, \text{m/s}
\]
Example. A 58 kg skier is coasting down a 25° slope as shown. Near the top of the slope, her speed is 3.6 m/s. She accelerates down the slope because of the gravitational force, even though a kinetic frictional force of magnitude 71 N opposes her motion. Ignoring air resistance, determine the speed at a point that is displaced 57 m downhill.
6.2 The Work-Energy Theorem and Kinetic Energy

\[ v_0 = 3.6 \text{ m/s} \quad s = 57 \text{ m} \quad 25^\circ \text{ slope} \quad m = 58 \text{ kg} \quad f_k = 71 \text{ N} \]

Find \( v_f \)
6.2 *The Work-Energy Theorem and Kinetic Energy*

\[
W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \quad \text{Work-Energy Theorem}
\]

\[
W = [(\Sigma F) \cos \theta] s
\]

\[
\sum F = mg \sin 25^\circ - f_k = (58)(9.8)(0.42) - 71 = 170 \text{ N}
\]

\[
W = [(170) \cos 0](57) = 9700 \text{ J}
\]

Solving the Work-Energy Theorem for \(v_f\):

\[
v_f = \left[\frac{2W + m v_o^2}{m}\right]^{1/2} = \left[\frac{(2(9700) + (58)(3.6)^2)/(58)}{58}\right]^{1/2}
\]

\[
= 19 \text{ m/s}
\]
6.2 The Work-Energy Theorem and Kinetic Energy

Conceptual Example 6  Work and Kinetic Energy

A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.
6.3 Gravitational Potential Energy

Find the work done by gravity in accelerating the basketball downward from $h_0$ to $h_f$.

$$W = (F \cos \theta)s$$

$$W_{\text{gravity}} = mg(h_o - h_f)$$
6.3 Gravitational Potential Energy

\[ W_{\text{gravity}} = mg(h_o - h_f) \]

We get the same result for \( W_{\text{gravity}} \) for any path taken between \( h_o \) and \( h_f \).
6.3 Gravitational Potential Energy

Example 7  A Gymnast on a Trampoline

The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?
6.3 Gravitational Potential Energy

\[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]

\[ W_{\text{gravity}} = mg(h_o - h_f) \]

\[ mg(h_o - h_f) = -\frac{1}{2} m v_o^2 \]

\[ v_o = \sqrt{-2g(h_o - h_f)} \]

\[ v_o = \sqrt{-2(9.80 \text{ m/s}^2)(1.20 \text{ m} - 4.80 \text{ m})} = 8.40 \text{ m/s} \]
DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy $PE$ is the energy that an object of mass $m$ has by virtue of its position relative to the surface of the earth. That position is measured by the height $h$ of the object relative to an arbitrary zero level:

$$PE = mgh$$

$1 \text{ N} \cdot \text{m} = 1 \text{ joule (J)}$