Chapter 8

Rotational Kinematics

8.3 The Equations of Rotational Kinematics

Table 8.2 Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	Linear Motion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

Table 8.1 The Equations of Kinematics for Rotational and Linear Motion

Rotational Motion (α = constant)		Linear Motion (a = constant)	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0{}^2 + 2ax$	(2.9)

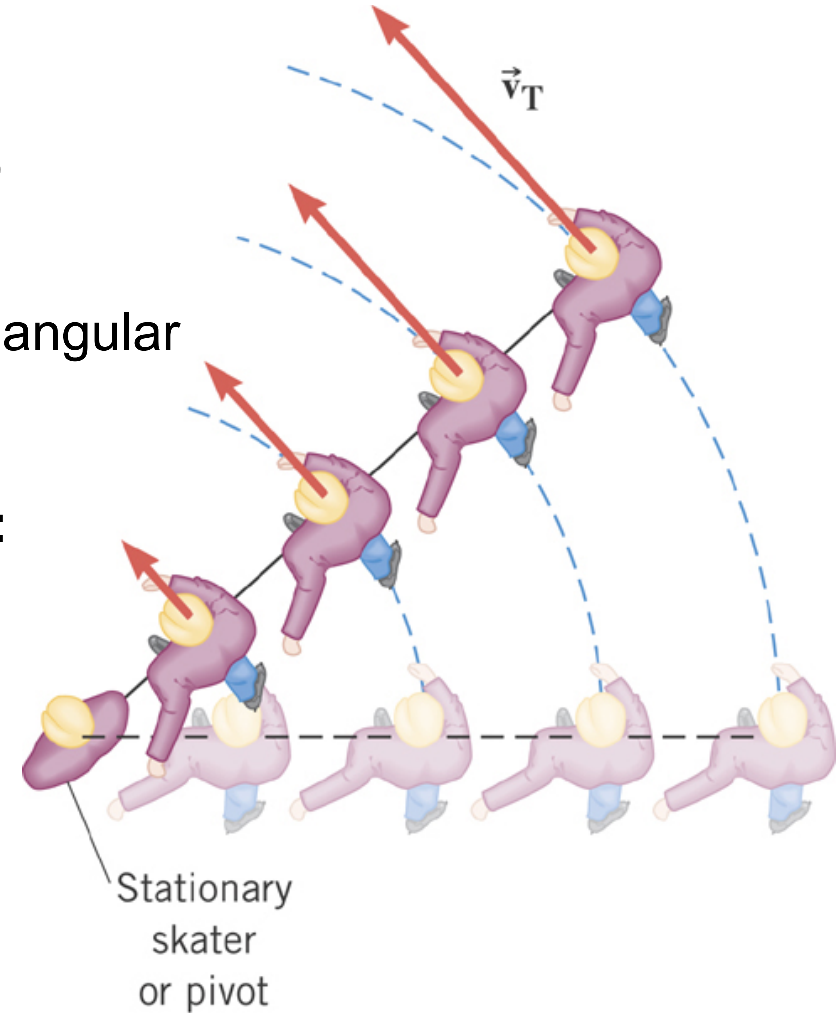
8.4 Angular Variables and Tangential Variables

The relationship between the (tangential) arc length, s, at some radius, r, and the angular displacement, θ, has been shown to be

$$s = r\theta \quad (\theta \text{ in radians})$$

Let's find other relationships between the angular and tangential variables.

Consider skaters "cracking-the-whip" :



$$\vec{\mathbf{v}}_{\mathbf{T}} = \text{tangential velocity}$$

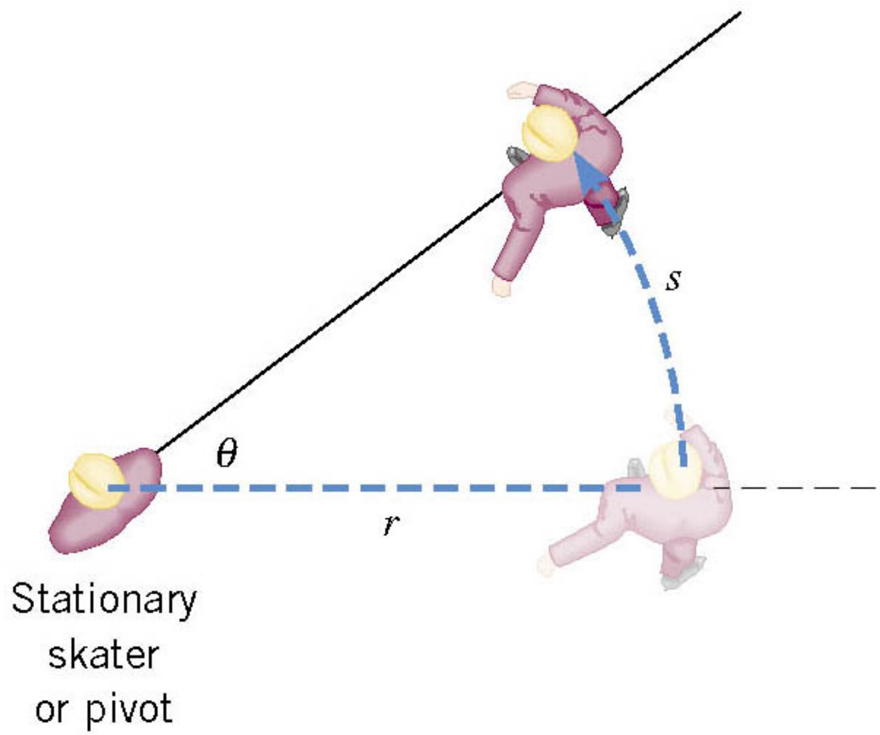
$$v_T = \text{tangential speed}$$

8.4 Angular Variables and Tangential Variables

$$v_T = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right)$$

$$\omega = \frac{\theta}{t}$$

$$v_T = r\omega \quad (\omega \text{ in rad/s})$$



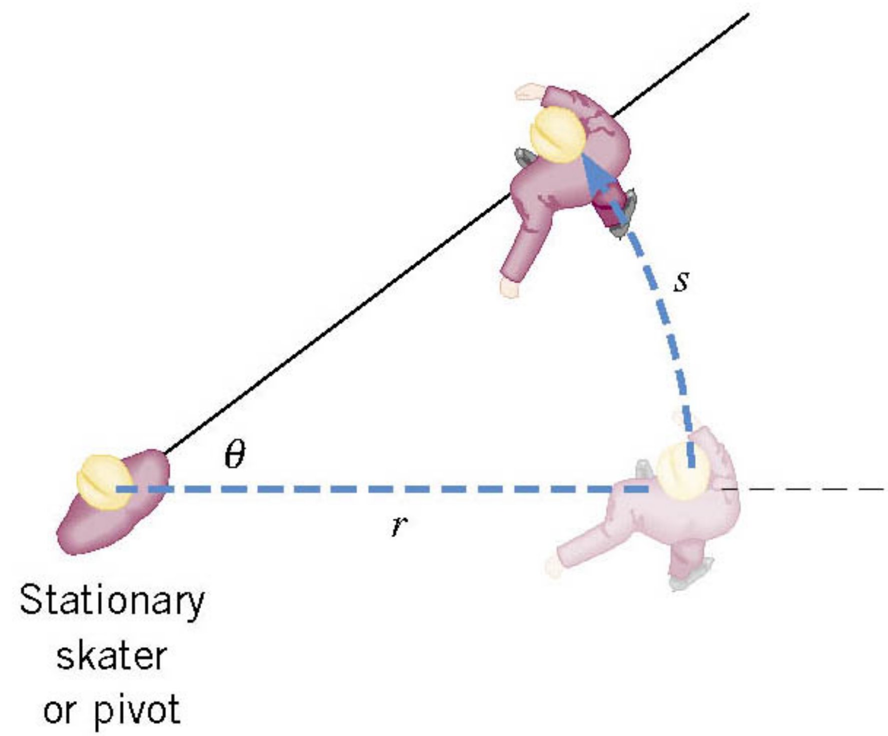
8.4 *Angular Variables and Tangential Variables*

The tangential acceleration, a_T, is the change of the tangential velocity per time:

$$\alpha = \frac{\omega - \omega_o}{t}$$

$$a_T = \frac{v_T - v_{To}}{t} = \frac{(r\omega) - (r\omega_o)}{t} = r\frac{\omega - \omega_o}{t}$$

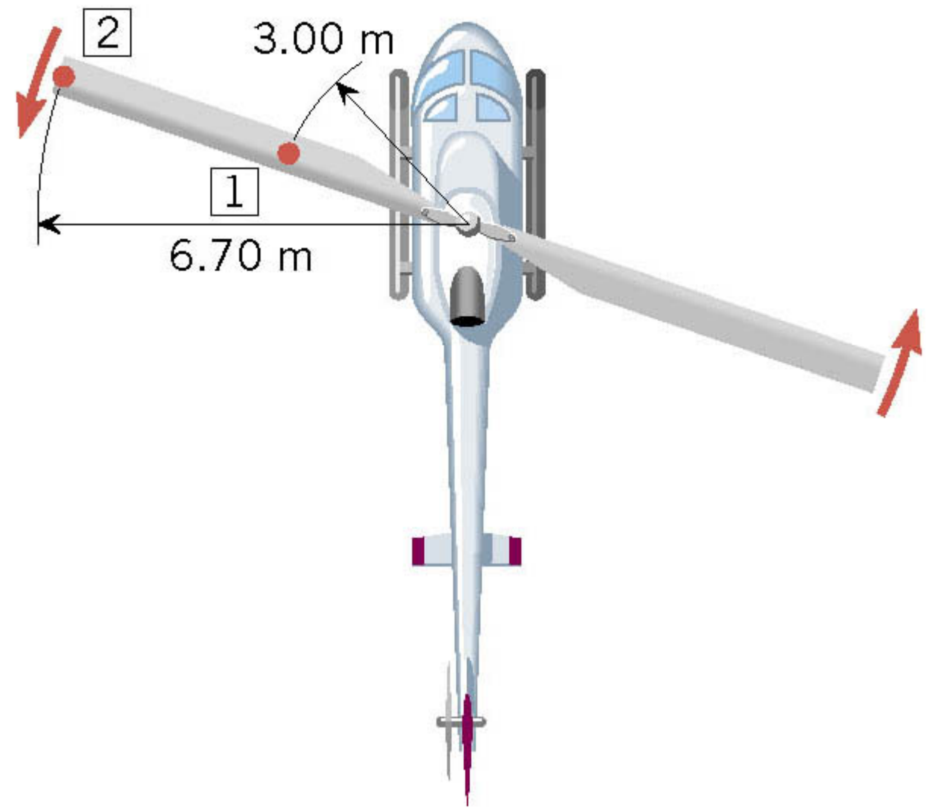
$$a_T = r\alpha \qquad (\alpha \text{ in rad/s}^2)$$



Example 6 A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.

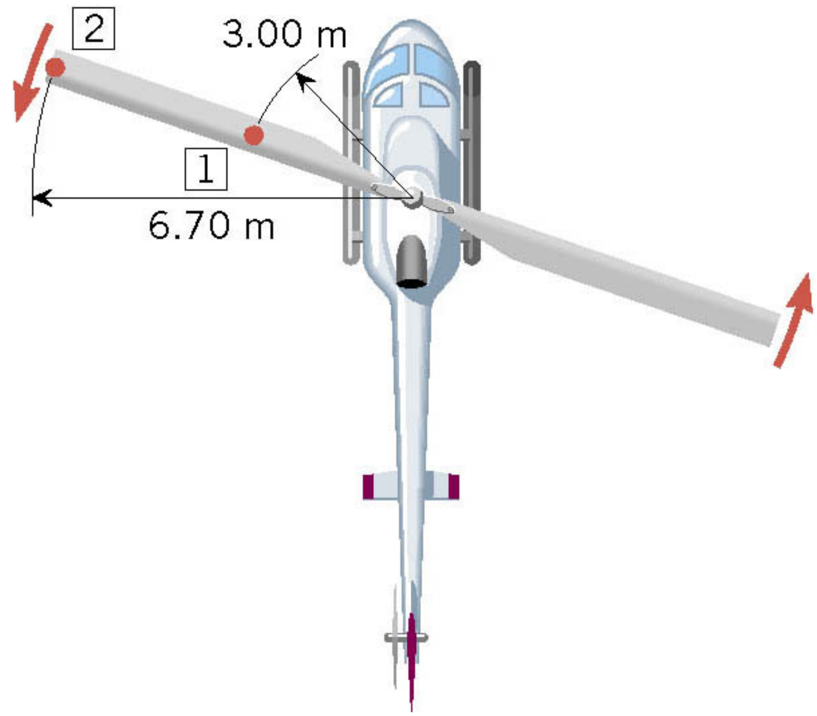
For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



8.4 Angular Variables and Tangential Variables

$$\omega = \left(6.50\,\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 40.8\,\text{rad/s}$$

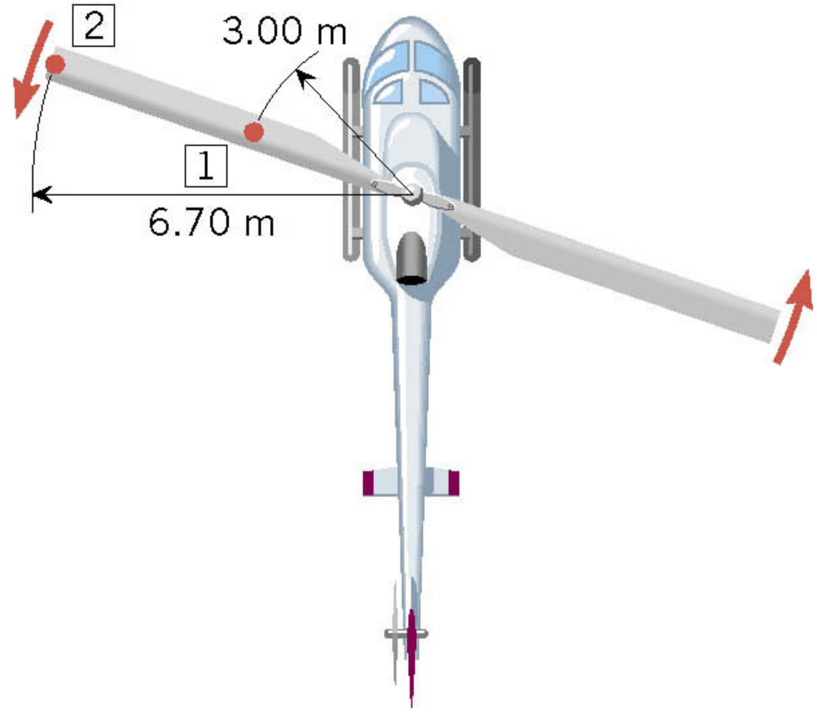
$$v_T = r\omega = (3.00\,\text{m})(40.8\,\text{rad/s}) = 122\,\text{m/s}$$



8.4 Angular Variables and Tangential Variables

$$\alpha = \left(1.30\,\frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 8.17\ \text{rad/s}^2$$

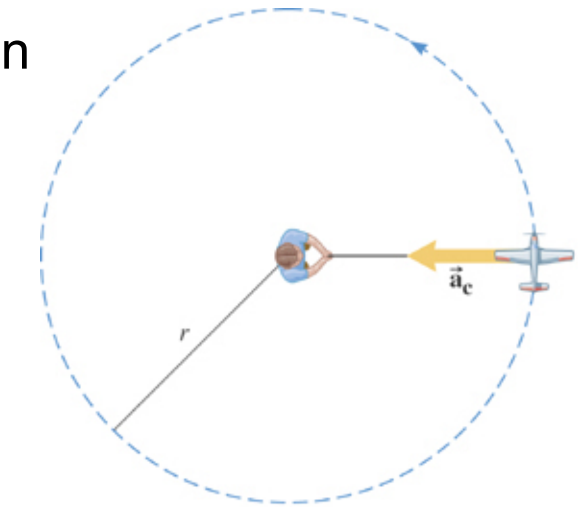
$$a_T = r\alpha = (3.00\ \text{m})\left(8.17\ \text{rad/s}^2\right) = 24.5\ \text{m/s}^2$$



8.5 Centripetal Acceleration and Tangential Acceleration

In uniform circular motion, the only acceleration present is the centripetal acceleration.

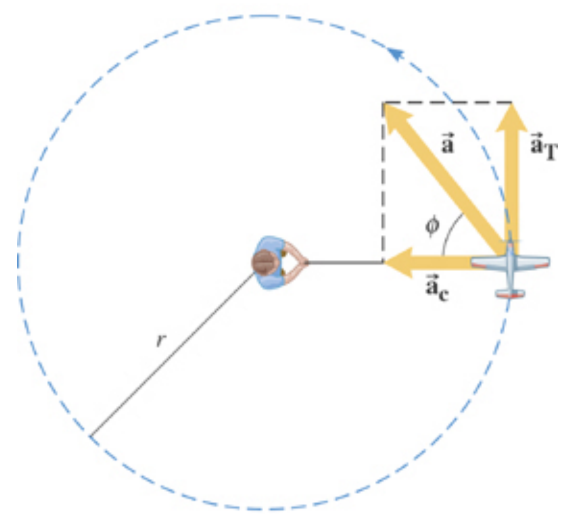
$$a_c = \frac{v_T^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \quad (\omega \text{ in rad/s})$$



(a) Uniform circular motion

In nonuniform circular motion, there are both a centripetal and a tangential acceleration.

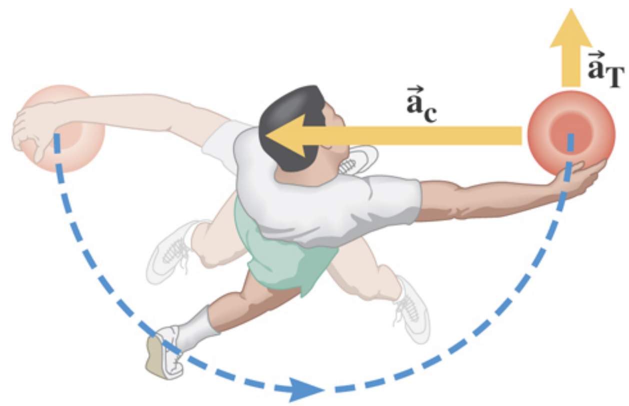
$$a_T = r\alpha \qquad (\alpha \text{ in rad/s}^2)$$



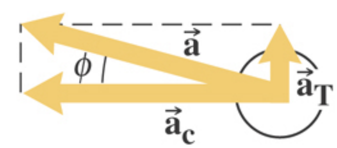
(b) Nonuniform circular motion

Example 7 A Discus Thrower

Starting from rest, the thrower accelerates the discus to a final angular speed of +15.0 rad/s in a time of 0.270 s before releasing it. During the acceleration, the discus moves in a circular arc of radius 0.810 m.



(*a*)

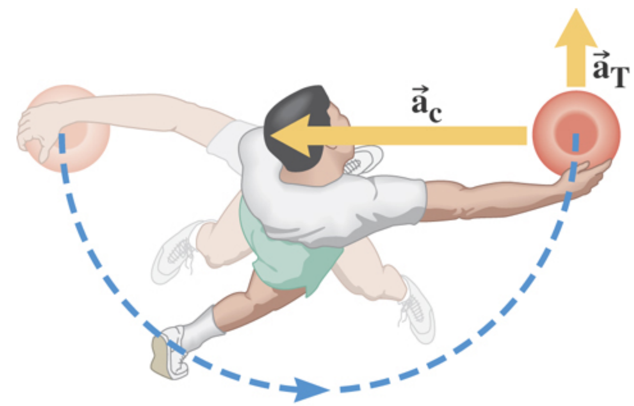


(*b*)

Find the magnitude of the total acceleration of the discus just before the discus is released.

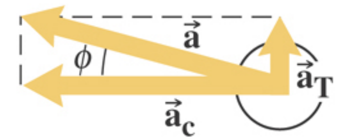
8.5 Centripetal Acceleration and Tangential Acceleration

$$a_c = r\omega^2 = (0.810\,\mathrm{m})(15.0\,\mathrm{rad/s})^2$$
$$= 182\,\mathrm{m/s^2}$$



(*a*)

$$a_T = r\alpha = r\frac{\omega - \omega_o}{t} = (0.810\,\mathrm{m})\left(\frac{15.0\,\mathrm{rad/s}}{0.270\,\mathrm{s}}\right)$$
$$= 45.0\,\mathrm{m/s^2}$$



(*b*)

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(182\,\mathrm{m/s^2})^2 + (45.0\,\mathrm{m/s^2})^2} = 187\,\mathrm{m/s^2}$$

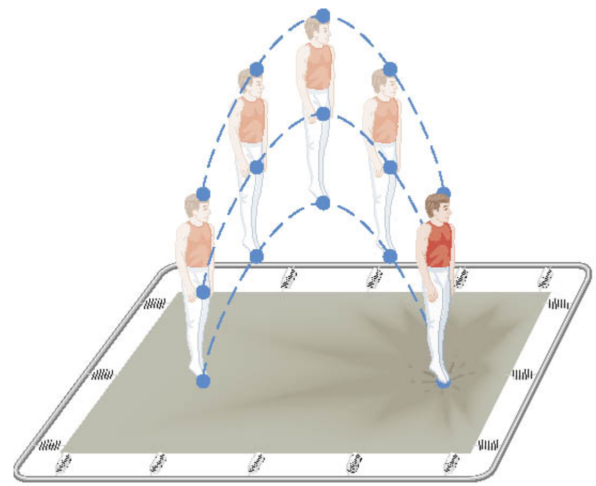
$$\phi = \tan^{-1} a_T/a_c = \tan^{-1}(45.0)/(182) = 13.9^\circ$$

Chapter 9

Rotational Dynamics

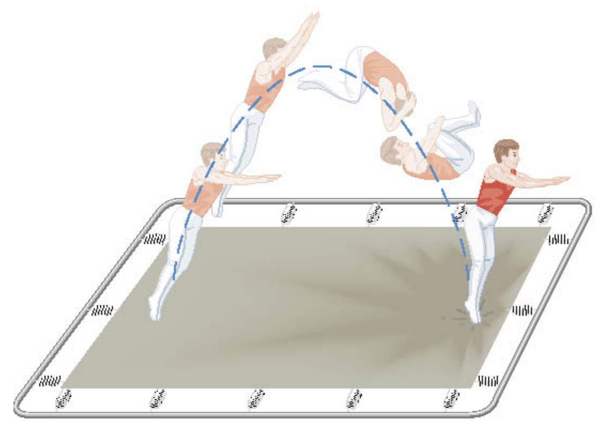
9.1 The Action of Forces and Torques on Rigid Objects

In pure translational motion, all points on an object travel on parallel paths.



(a) Translation

The most general motion is a combination of translation and rotation.



(b) Combined translation and rotation

9.1 The Action of Forces and Torques on Rigid Objects

According to Newton's second law, a net force causes an object to have an acceleration.

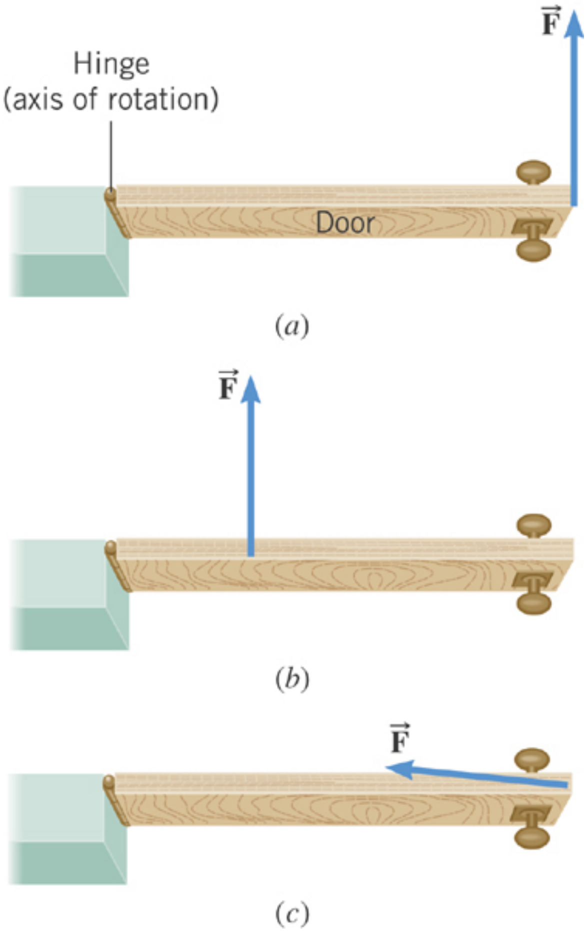
What causes an object to have an ***angular acceleration***?

↓

TORQUE

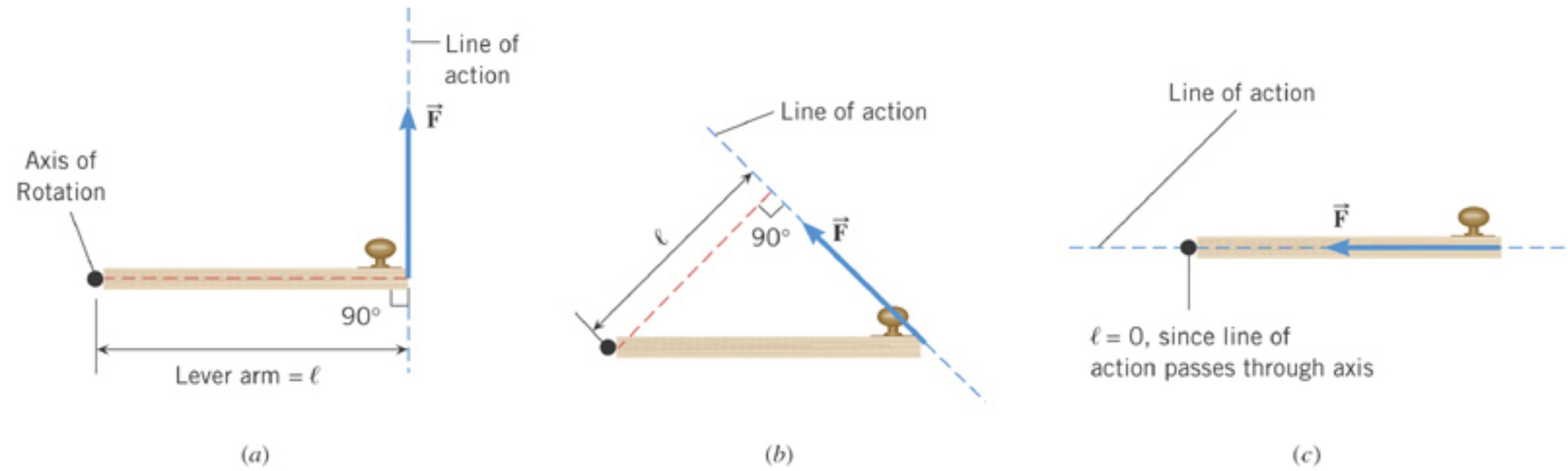
Torque tells you how effective a given force is at rotating something about some axis.

9.1 The Action of Forces and Torques on Rigid Objects



The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.

9.1 The Action of Forces and Torques on Rigid Objects



DEFINITION OF TORQUE

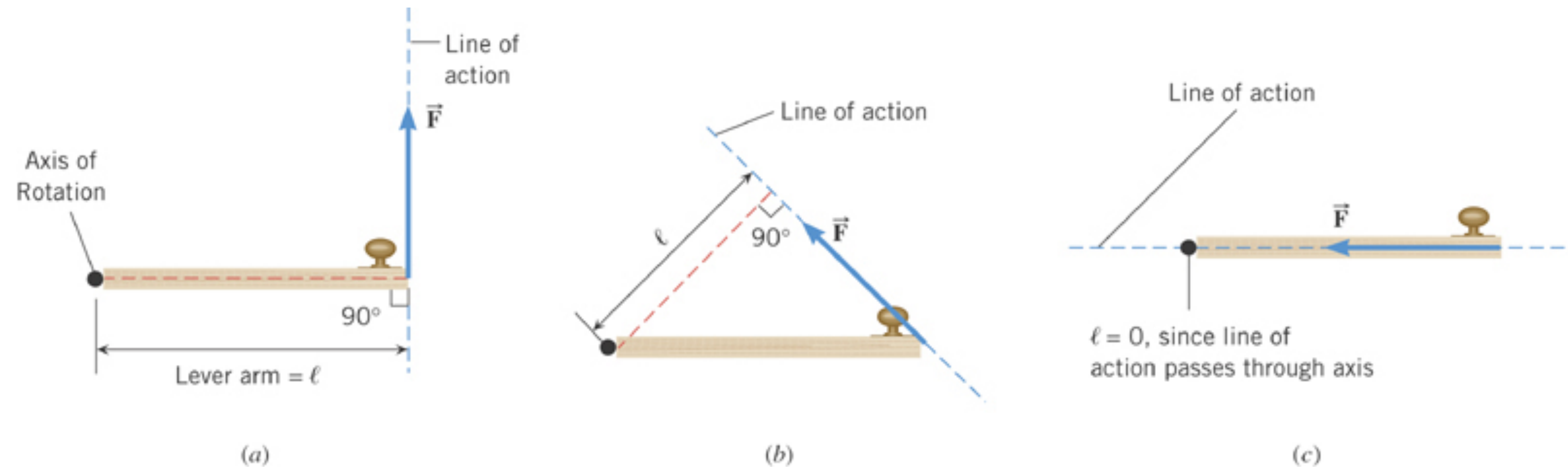
Magnitude of Torque = (Magnitude of the force) x (Lever arm)

$$\tau = F\ell$$

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.

SI Unit of Torque: newton x meter (N·m)

9.1 The Action of Forces and Torques on Rigid Objects

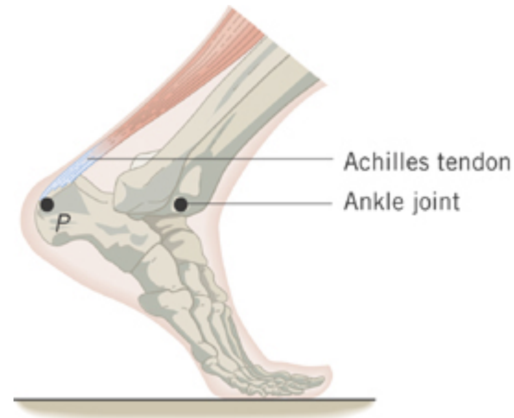


The lever arm, l, for some force acting about some rotation axis is defined as the distance between the rotation axis and a perpendicular to the force.

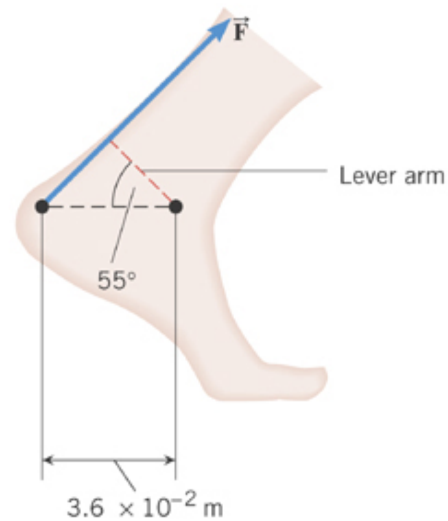
Example 2 The Achilles Tendon

The tendon exerts a force of magnitude 720 N with a geometry as shown in the figure.

Determine the torque (magnitude and direction) of this force about the ankle joint.



(a)

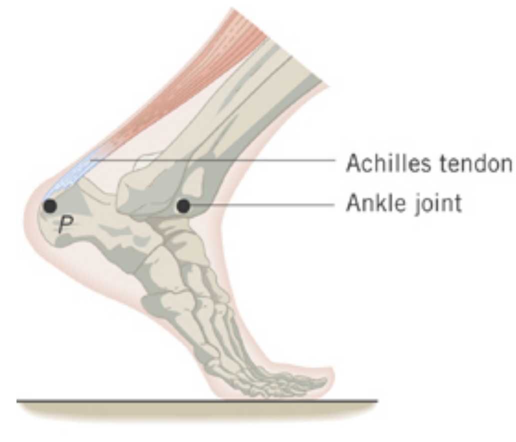


(b)

9.1 The Action of Forces and Torques on Rigid Objects

$$\tau = F\ell$$

$$\cos 55^\circ = \frac{\ell}{3.6\times10^{-2}\,\mathrm{m}}$$

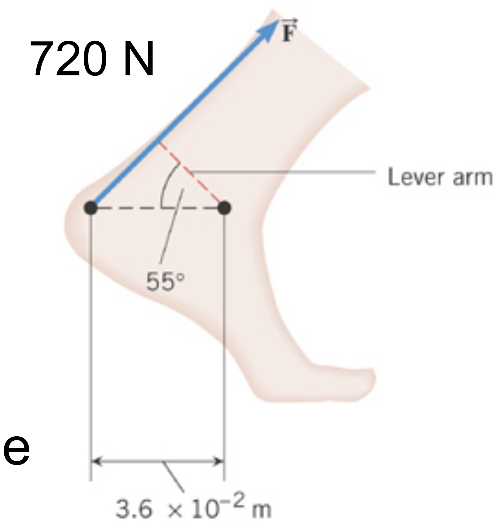


(a)

First, calculate the magnitude of τ

$$\tau = (720\ \mathrm{N})(3.6\times10^{-2}\,\mathrm{m})\cos 55^\circ$$

$$= 15\ \mathrm{N\cdot m}$$



(b)

Since the force rotates the foot about the ankle joint in a clockwise direction --> τ negative

τ **= - 15 N m** , if the direction is included

9.2 Rigid Objects in Equilibrium

If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.

$$a_x = a_y = 0 \qquad \alpha = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.

2. Draw a free-body diagram that shows all of the external forces acting on the object.

3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.

4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)

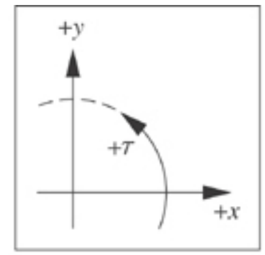
5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.

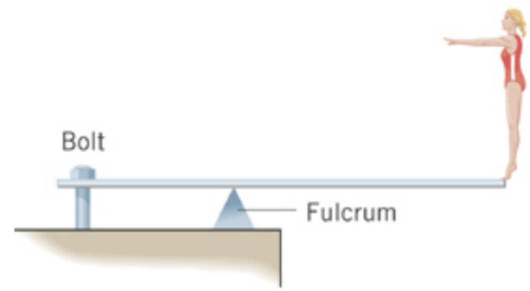
6. Solve the equations for the desired unknown quantities.

Example 3 A Diving Board

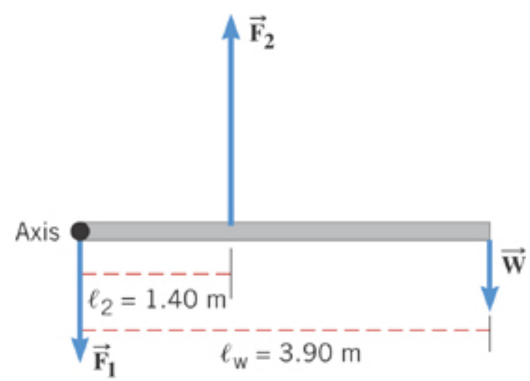
A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end.

Find the forces that the bolt and the fulcrum exert on the board.



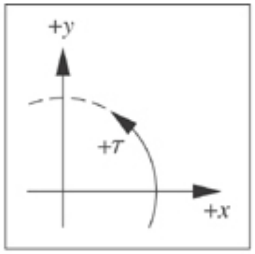


(*a*)

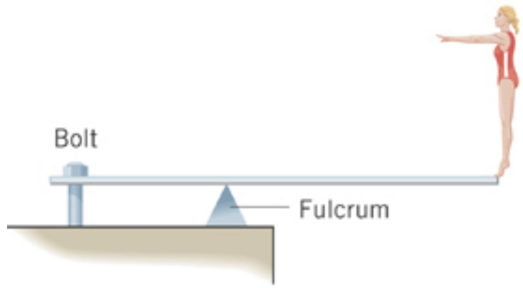


(*b*) Free-body diagram of the diving board

9.2 Rigid Objects in Equilibrium



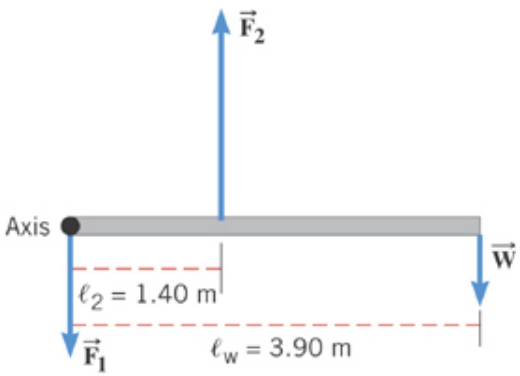
$$\sum \tau = F_2 \ell_2 - W \ell_W = 0$$



(a)

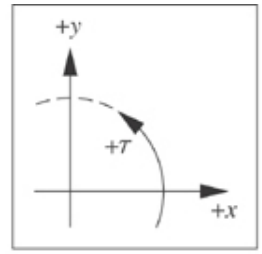
$$F_2 = \frac{W \ell_W}{\ell_2}$$

$$F_2 = \frac{(530\ \mathrm{N})(3.90\ \mathrm{m})}{1.40\ \mathrm{m}} = 1480\ \mathrm{N}$$

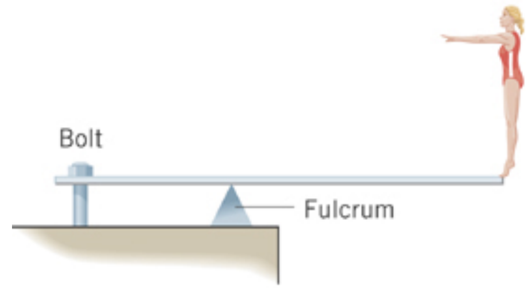


(b) Free-body diagram of the diving board

9.2 Rigid Objects in Equilibrium

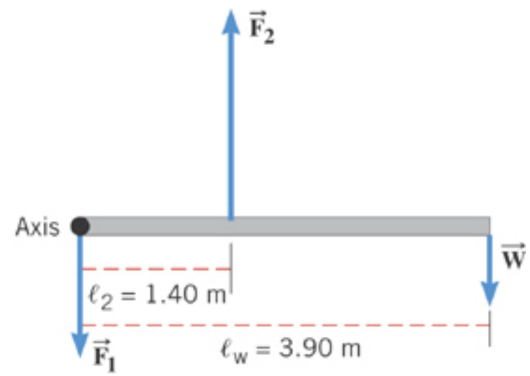


$$\sum F_y = -F_1 + F_2 - W = 0$$



(a)

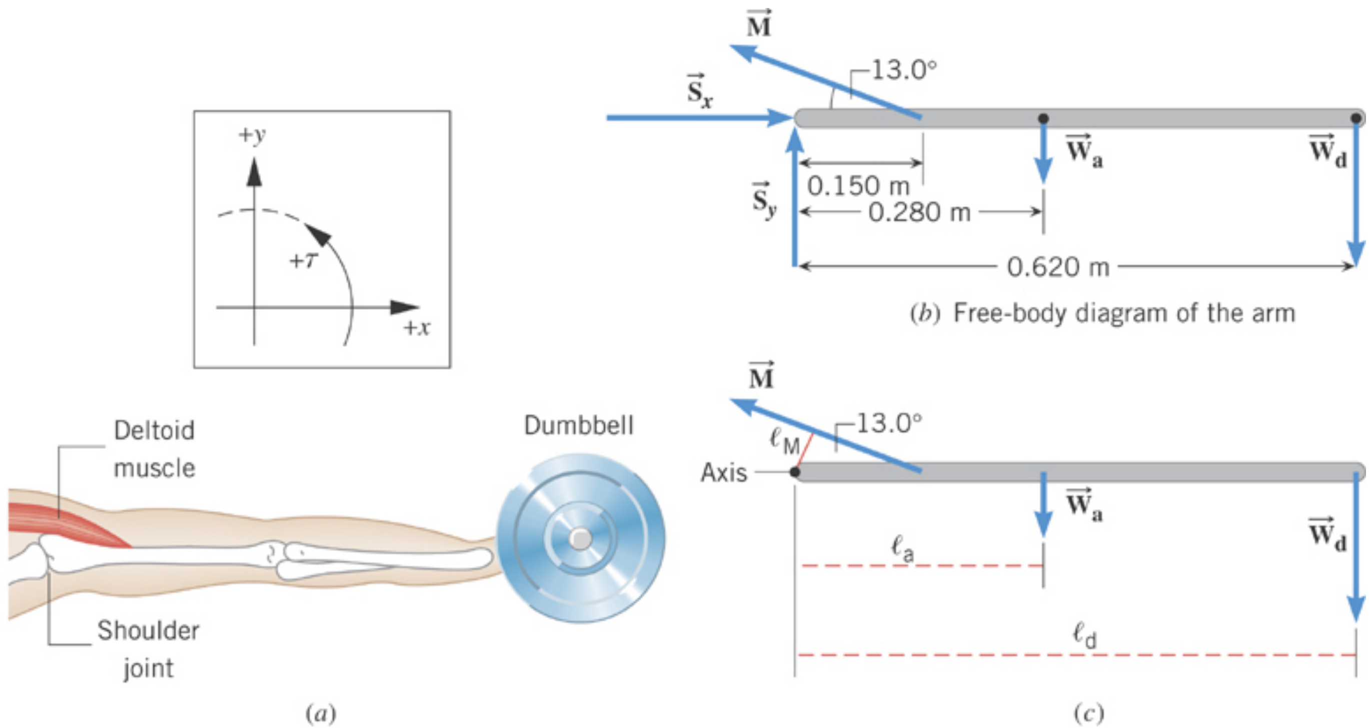
$$-F_1 + 1480\text{ N} - 530\text{ N} = 0$$



(b) Free-body diagram of the diving board

$$F_1 = 950\text{ N}$$

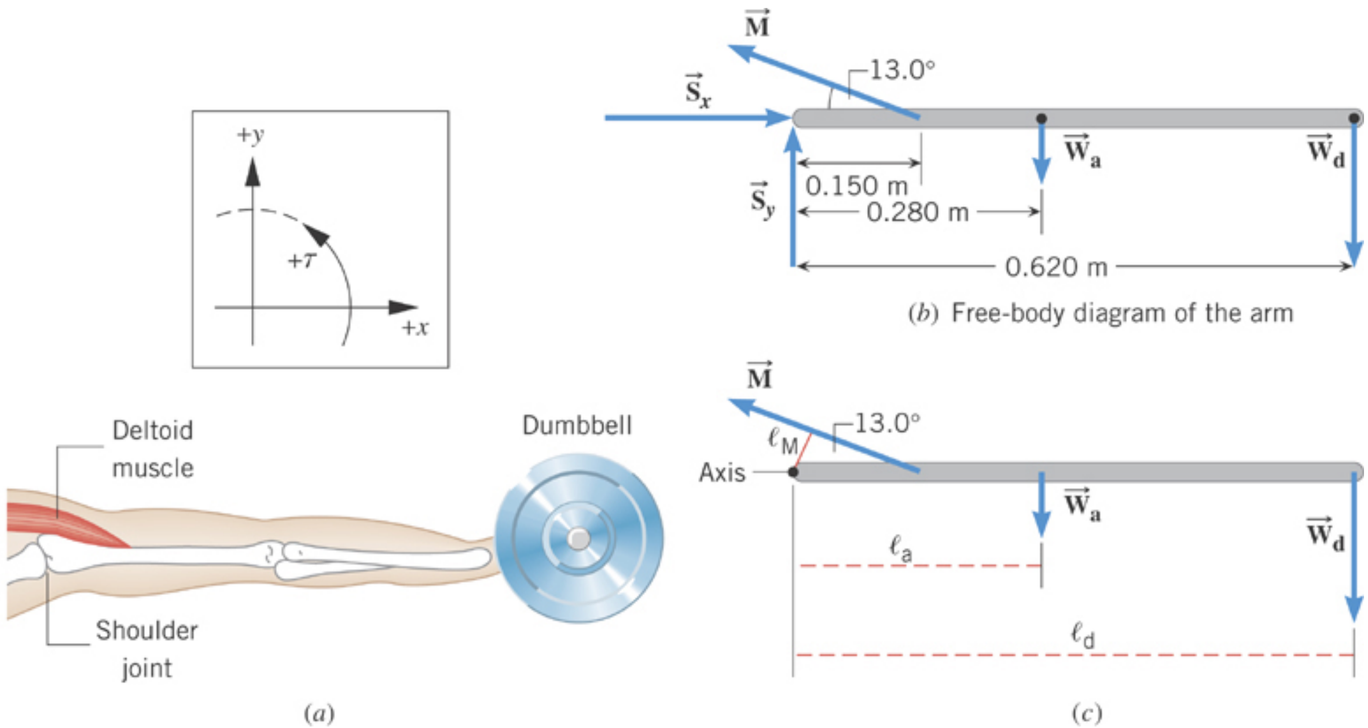
9.2 Rigid Objects in Equilibrium



Example 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbell he can hold?

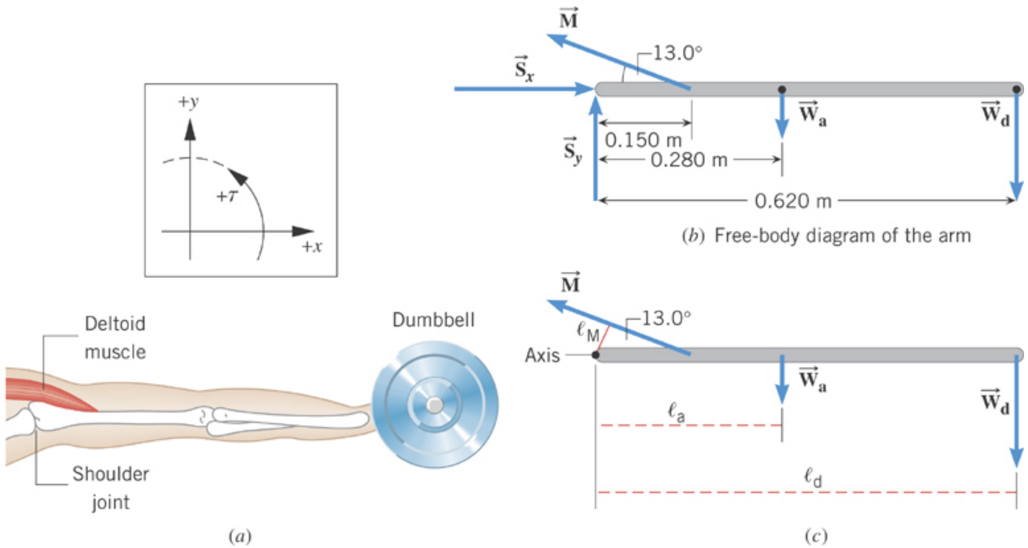
9.2 Rigid Objects in Equilibrium



$$\sum \tau = -W_a \ell_a - W_d \ell_d + M \ell_M = 0$$

$$\ell_M = (0.150\ \mathrm{m}) \sin 13.0^\circ$$

9.2 Rigid Objects in Equilibrium



$$W_d = \frac{-W_a \ell_a + M \ell_M}{\ell_d}$$

$$= \frac{-(31.0\ \text{N})(0.280\ \text{m}) + (1840\ \text{N})(0.150\ \text{m})\sin 13.0^\circ}{0.620\ \text{m}} = 86.1\ \text{N}$$