9.5 Rotational Work and Energy

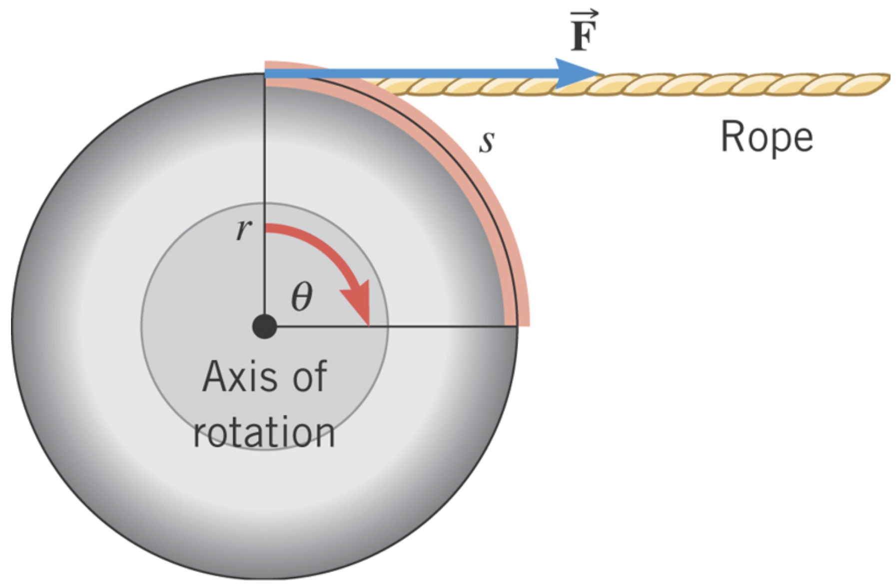
$$s = r\theta$$

$$W = Fs = Fr\theta$$

$$\tau = Fr$$

$$W = \tau\theta$$

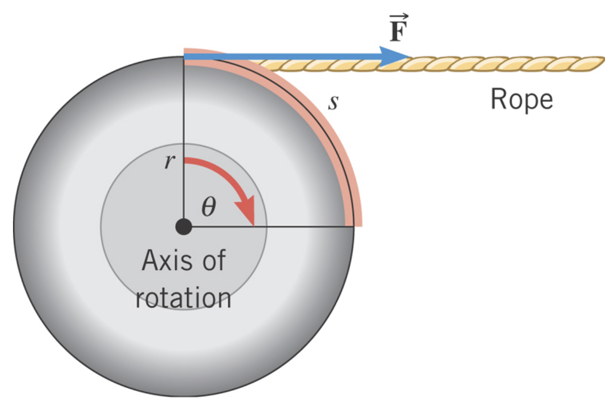
Consider the work done in rotating a wheel with a tangential force, F, by an angle θ.



DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

$$W_R = \tau\theta$$



Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

9.5 Rotational Work and Energy

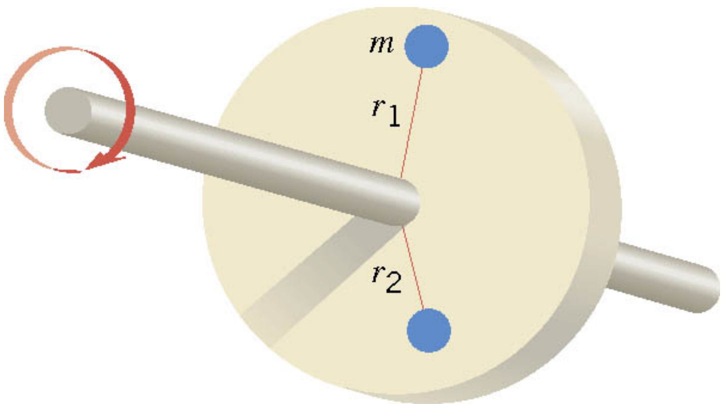
According to the Work-Energy theorem: $W = KE_f - KE_0$

So W_R should be able to produce rotational kinetic energy.

Calculate the kinetic energy of a mass m undergoing rotational motion at radius r and moving with tangential speed v_T

$$KE = \tfrac{1}{2} m v_T^2 = \tfrac{1}{2} m r^2 \omega^2$$

$$v_T = r\omega$$



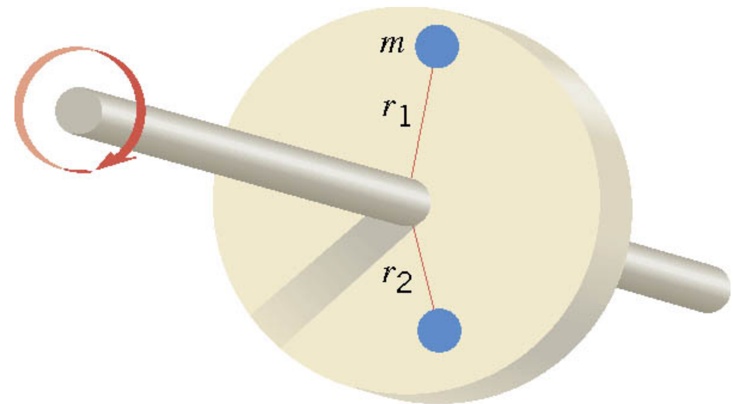
For a system of rotating masses, the total kinetic energy is the sum over the kinetic energies of the individual masses,

$$KE = \sum \left(\tfrac{1}{2} m r^2 \omega^2\right) = \tfrac{1}{2}\left(\sum m r^2\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$KE_R = \tfrac{1}{2} I\omega^2$$



Requirement: The angular speed must be expressed in rad/s.

SI Unit of Rotational Kinetic Energy: joule (J)

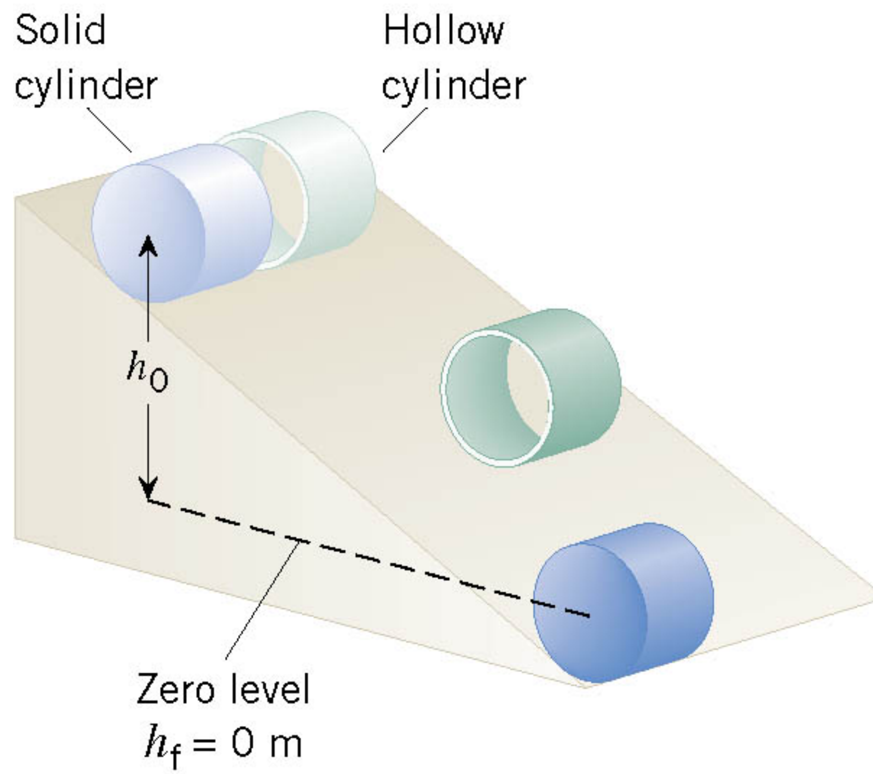
Thus, the **rotational version of the Work-Energy theorem** is:

$$W_R = KE_{Rf} - KE_{R0} \quad \text{where} \quad \begin{cases} W_R = \tau\theta \\ KE_R = \tfrac{1}{2} I\omega^2 \end{cases}$$

Example 13 Rolling Cylinders

A thin-walled hollow cylinder (mass = *m*, radius = *r*) and a solid cylinder (also, mass = *m*, radius = *r*) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



9.5 Rotational Work and Energy

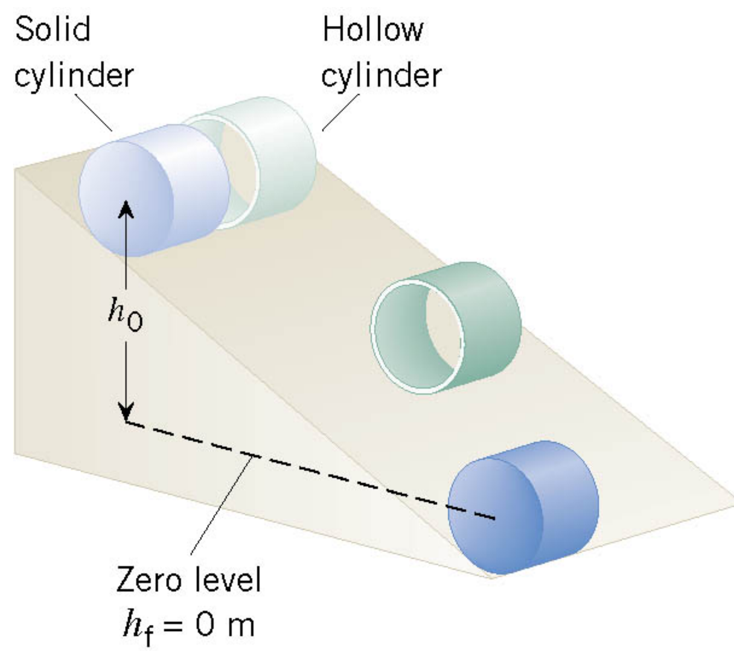
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgh$$

ENERGY CONSERVATION

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f = \tfrac{1}{2}mv_o^2 + \tfrac{1}{2}I\omega_o^2 + mgh_o$$

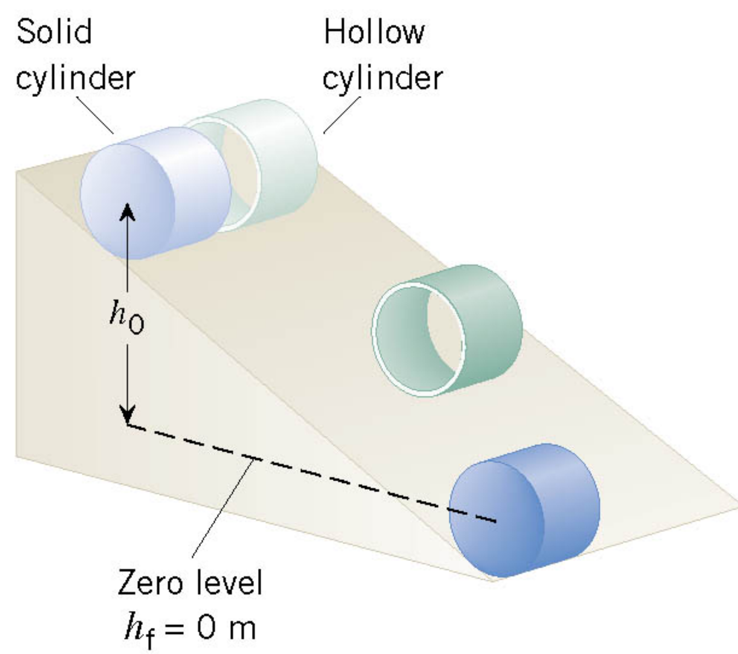
$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 = mgh_o$$

$$\omega_f = v_f / r$$



9.5 Rotational Work and Energy

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/r^2 = mgh_o$$



$$v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}$$

The cylinder with the **smaller** moment of inertia will have a **greater** final translational speed.

Since $I_{solid} = (1/2)mr^2$ and $I_{hollow} = mr^2$

Then, $I_{solid} < I_{hollow}$ ➔ $v_{f\,solid} > v_{f\,hollow}$

DEFINITION OF ANGULAR MOMENTUM

The angular momentum *L* of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

$$L = I\omega$$

Requirement: The angular speed must be expressed in rad/s.

SI Unit of Angular Momentum: $kg \cdot m^2/s$

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

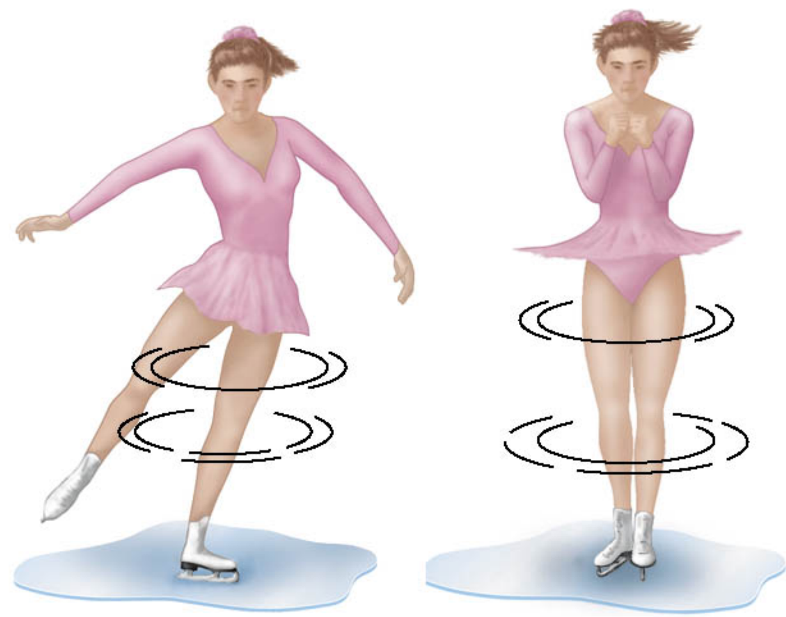
The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.

$$L_f = L_0$$

Conceptual Example 14 A Spinning Skater

An ice skater is spinning with both arms and a leg outstretched. She pulls her arms and leg inward and her spinning motion changes dramatically.

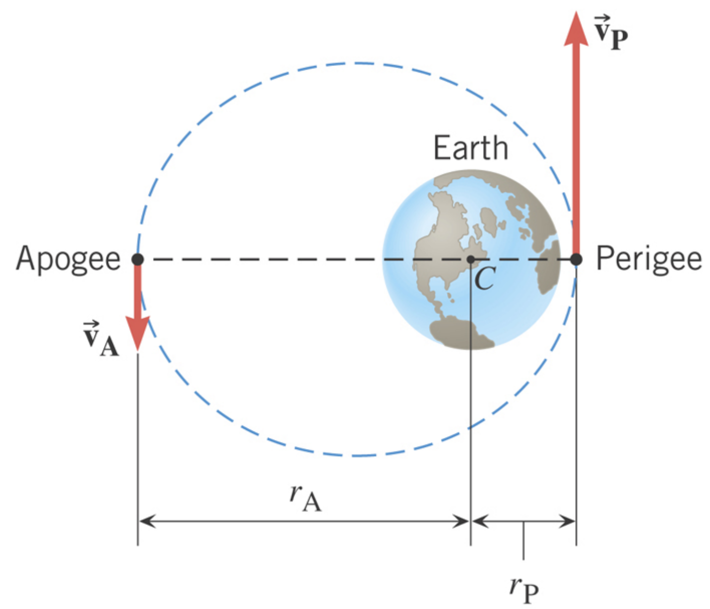
Use the principle of conservation of angular momentum to explain how and why her spinning motion changes.



(a) (b)

Example 15 A Satellite in an Elliptical Orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37 x 10^6 m from the center of the earth, and its point of greatest distance is 25.1 x 10^6 m from the center of the earth.



The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.

9.6 *Angular Momentum*

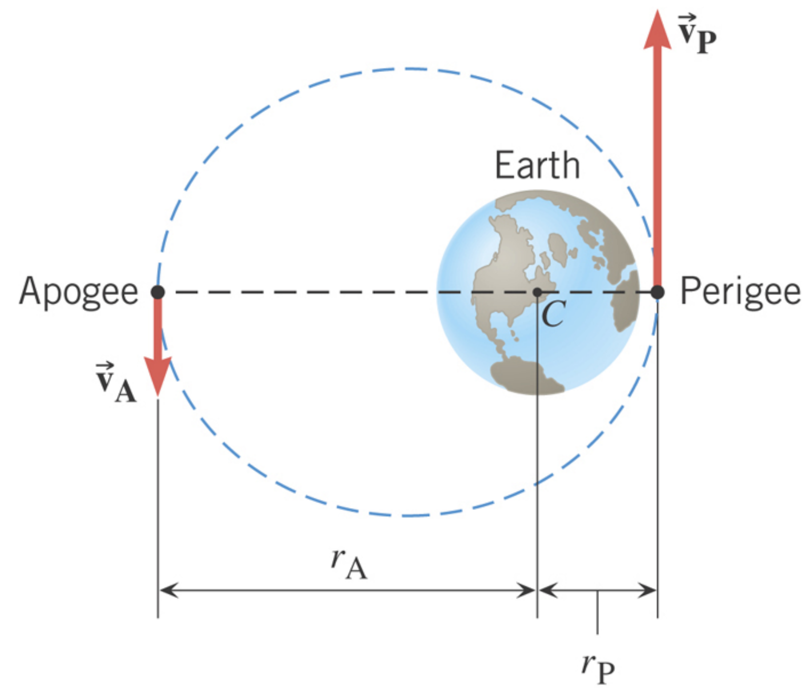
$$L = I\omega$$

Since no external torques are present in this case, we have angular momentum conservation

$$I_A\omega_A = I_P\omega_P$$

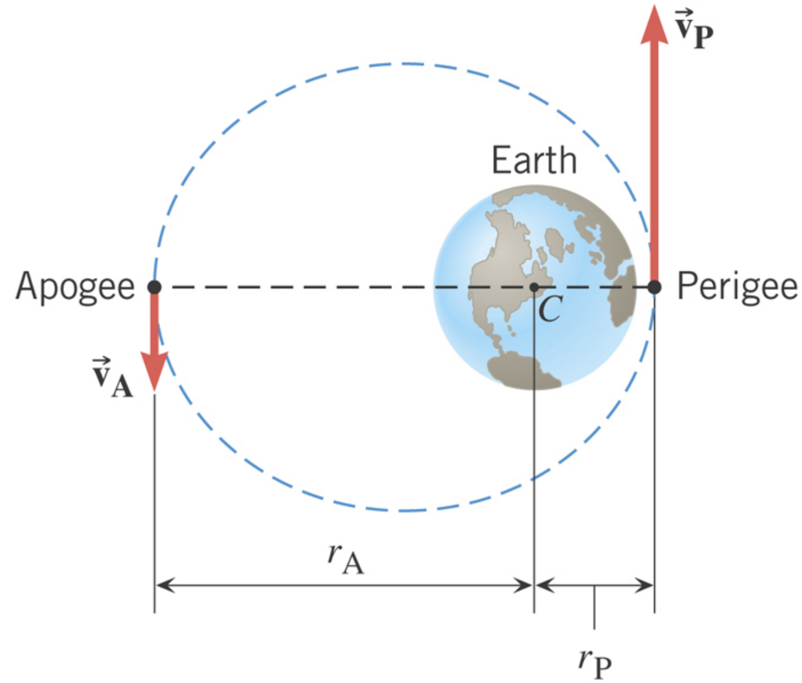
$$I = mr^2 \qquad \omega = v/r$$

$$mr_A^2\frac{v_A}{r_A} = mr_P^2\frac{v_P}{r_P}$$



9.6 Angular Momentum

$$mr_A^2 \frac{v_A}{r_A} = mr_P^2 \frac{v_P}{r_P}$$



$$r_A v_A = r_P v_P$$

$$v_A = \frac{r_P v_P}{r_A} = \frac{\left(8.37\times10^6\ \text{m}\right)\left(8450\,\text{m/s}\right)}{25.1\times10^6\ \text{m}} = 2820\,\text{m/s}$$