Chapter 7

Linear Momentum
The Principle of Conservation of Linear Momentum

WORK-ENERGY THEOREM $\Leftrightarrow$ CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM $\Leftrightarrow$ ???

Apply the impulse-momentum theorem to the midair collision between two objects.....
The Principle of Conservation of Linear Momentum

The midair collision between two objects.

*Internal forces* – Forces that objects within the system exert on each other.

*External forces* – Forces exerted on objects by agents external to the system.
The Principle of Conservation of Linear Momentum

\[ \left( \sum \overrightarrow{F} \right) \Delta t = m \overrightarrow{v}_f - m \overrightarrow{v}_o \]

\[ \left( \overrightarrow{W}_1 + \overrightarrow{F}_{12} \right) \Delta t = m_1 \overrightarrow{v}_{f1} - m_1 \overrightarrow{v}_{o1} \]

\[ \left( \overrightarrow{W}_2 + \overrightarrow{F}_{21} \right) \Delta t = m_2 \overrightarrow{v}_{f2} - m_2 \overrightarrow{v}_{o2} \]

(a) Before collision
(b) During collision
(c) After collision
The Principle of Conservation of Linear Momentum

\[
\left( \vec{W}_1 + \vec{F}_{12} \right) \Delta t = m_1 \vec{v}_{f1} - m_1 \vec{v}_{o1} \\
+ \\
\left( \vec{W}_2 + \vec{F}_{21} \right) \Delta t = m_2 \vec{v}_{f2} - m_2 \vec{v}_{o2}
\]

\[
\left( \vec{W}_1 + \vec{W}_2 + \vec{F}_{12} + \vec{F}_{21} \right) \Delta t = \left( m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2} \right) - \left( m_1 \vec{v}_{o1} + m_2 \vec{v}_{o2} \right)
\]

\[
\vec{F}_{12} = -\vec{F}_{21}
\]

Internal forces cancel from Newton’s 3rd law

\[
\vec{P}_f
\]
Total final momentum

\[
\vec{P}_o
\]
Total initial momentum
The Principle of Conservation of Linear Momentum

The internal forces cancel out.

\[
\left( \vec{W}_1 + \vec{W}_2 \right) \Delta t = \vec{P}_f - \vec{P}_o
\]

More generally,

\[
\left( \text{sum of average external forces} \right) \Delta t = \vec{P}_f - \vec{P}_o
\]

\[
\left( \sum \vec{F}_{EXT} \right) \Delta t = \vec{P}_f - \vec{P}_0
\]
The Principle of Conservation of Linear Momentum

\[
\left( \sum \vec{F}_{\text{EXT}} \right) \Delta t = \vec{P}_f - \vec{P}_0
\]

If the sum of the external forces is zero, then

\[
0 = \vec{P}_f - \vec{P}_0 \quad \rightarrow \quad \vec{P}_f = \vec{P}_0
\]

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.
**Conceptual Example:** Is the Total Momentum Conserved?

Imagine two balls colliding on a billiard table that is friction-free. Use the momentum conservation principle in answering the following questions. (a) Is the total momentum of the two-ball system the same before and after the collision? (b) Answer part (a) for a system that contains only one of the two colliding balls.
The Principle of Conservation of Linear Momentum

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

2. Relative to the system, identify the internal and external forces.

3. Verify that the system is isolated.

4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.
Example: Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54 kg woman and one is a 88 kg man. The woman moves away with a velocity of +2.5 m/s. Find the recoil velocity of the man.
The Principle of Conservation of Linear Momentum

\[ \vec{P}_f = \vec{P}_0 \]

\[ m_1 v_{f1} + m_2 v_{f2} = 0 \]

\[ v_{f2} = -\frac{m_1 v_{f1}}{m_2} \]

\[ v_{f2} = -\frac{(54 \text{ kg})(+ 2.5 \text{ m/s})}{88 \text{ kg}} = -1.5 \text{ m/s} \]
Collisions in One Dimension

The total linear momentum is conserved when two objects collide, provided they constitute an isolated system.

**Elastic collision** -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

**Inelastic collision** -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.
Elastic collision of an isolated system

→ both *momentum* and *kinetic energy* are conserved:

\[
\begin{align*}
\text{Initial} & \quad \text{Final} \\
\vec{v}_{01} & \quad \vec{v}_{02} & \quad \vec{v}_{f1} & \quad \vec{v}_{f2} \\
\end{align*}
\]

head-on collision

\[
m_1 \vec{v}_{01} + m_2 \vec{v}_{02} = m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2} \quad \Rightarrow \quad m_1 v_{01} + m_2 v_{02} = m_1 v_{f1} + m_2 v_{f2}
\]

\[
\frac{1}{2} m_1 v_{01}^2 + \frac{1}{2} m_2 v_{02}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2
\]

→ can be shown by combining these equations for *head-on collisions* (see book):

\[
v_{01} - v_{02} = -(v_{f1} - v_{f2})
\]
Example: Head-on elastic collision of an isolated system.
A softball of mass 0.200 kg that is moving with a speed 8.3 m/s collides head-on and elastically with another ball initially at rest. Afterward the incoming softball bounces backward with a speed of 3.2 m/s. a) Calculate the velocity of the target ball after the collision, and b) calculate the mass of the target ball.

a) \( v_{01} - v_{02} = -(v_{f1} - v_{f2}) \)

\( v_{01} = 8.3 \text{ m/s} \quad v_{02} = 0 \quad v_{f1} = -3.2 \text{ m/s} \quad v_{f2} = ? \)

\( v_{f2} = v_{f1} + v_{01} - v_{02} = -3.2 + 8.3 - 0 = 5.1 \text{ m/s} \)

b) \( m_1v_{01} + m_2v_{02} = m_1v_{f1} + m_2v_{f2} \quad m_1 = 0.200 \text{ kg} \quad m_2 = ? \)

\( m_2 \left( v_{f2} - v_{02} \right) = m_1 \left( v_{01} - v_{f1} \right) \quad \Rightarrow \quad m_2 = m_1 \left( \frac{v_{01} - v_{f1}}{v_{f2} - v_{02}} \right) \)

\[ m_2 = 0.200 \left( \frac{8.3 - (-3.2)}{5.1 - 0} \right) = 0.45 \text{ kg} \]
Example: Inelastic collision -- Ballistic Pendulum

The mass of the block of wood is 2.50 kg and the mass of the bullet is 0.0100 kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.
Collisions in One Dimension

Apply conservation of momentum to the collision:

\[ m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2} \]

\[ (m_1 + m_2) v_f = m_1 v_{o1} \]

\[ v_{o1} = \frac{(m_1 + m_2) v_f}{m_1} \]
Applying conservation of energy to the swinging motion:

\[ mgh = \frac{1}{2} m v^2 \]

\[ (m_1 + m_2)gh_f = \frac{1}{2}(m_1 + m_2)v_f^2 \]

\[ gh_f = \frac{1}{2} v_f^2 \]

\[ v_f = \sqrt{2gh_f} \]
Collisions in One Dimension

\[ v_f = \sqrt{2gh_f} \]

\[ v_{o1} = \frac{(m_1 + m_2)}{m_1} v_f \]

\[ v_{o1} = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh_f} \]

\[ v_{o1} = \left( \frac{0.0100 \text{ kg} + 2.50 \text{ kg}}{0.0100 \text{ kg}} \right) \sqrt{2 \left( 9.80 \text{ m/s}^2 \right) (0.650 \text{ m})} = +896 \text{ m/s} \]
The center of mass is a point that represents the average location for the total mass of a system.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
Center of Mass

The motion of the center-of-mass is related to conservation of momentum.

Consider the change in the positions of the masses in some short time $\Delta t$:

Divide by $\Delta t$

\[
\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}
\]

\[
v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}
\]
Center of Mass

\[ \mathbf{v}_{cm} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{P}{M} \]

In vector form,

\[ \mathbf{v}_{cm} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{\mathbf{P}}{M} \]

In an isolated system, the **total linear momentum** does not change, therefore the velocity of the center of mass does not change.
\[
\left( \sum \vec{F}_{EXT} \right) \Delta t = \vec{P}_f - \vec{P}_0 = \Delta \vec{P}
\]

Divide both sides by \( \Delta t \)

\[
\sum \vec{F}_{EXT} = \frac{\Delta \vec{P}}{\Delta t}
\]

\[
\vec{P} = M \vec{v}_{CM}
\]

\[
\frac{\Delta \vec{P}}{\Delta t} = \Delta \left( M \vec{v}_{CM} \right) = M \frac{\Delta \vec{v}_{CM}}{\Delta t} = M \vec{a}_{CM}
\]

\[
\therefore \sum \vec{F}_{EXT} = M \vec{a}_{CM}
\]

Newton’s 2\textsuperscript{nd} Law for a system of particles which act like a single particle of mass \( M \) concentrated at the center of mass
Center of Mass

Consider the “Ice Skater” example which was an isolated system:

**BEFORE**

\[ v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0 \]

**AFTER**

\[ v_{cm} = \frac{(54 \text{ kg})(+2.5 \text{ m/s}) + (88 \text{ kg})(-1.5 \text{ m/s})}{54 \text{ kg} + 88 \text{ kg}} = 0.02 \approx 0 \]