Defining wave fronts and rays.

Consider a sound wave since it is easier to visualize.

Shown is a hemispherical view of a sound wave emitted by a pulsating sphere.

The rays are perpendicular to the wave fronts (e.g. crests) which are separated from each other by the wavelength of the wave, $\lambda$. 
Wave Fronts and Rays

The positions of two spherical wave fronts are shown in (a) with their diverging rays.

At large distances from the source, the wave fronts become less and less curved and approach the limiting case of a plane wave shown in (b). A plane wave has flat wave fronts and rays parallel to each other.
Reflection of string pulses at boundaries and interfaces

Reflection inverted

Reflection not inverted

From less dense medium to more dense medium

From more dense medium to less dense medium
Law of reflection of waves at a boundary or interface:

\[ \theta_i = \theta_r \]
Chapters 11 and 12

Sound and Standing Waves
The Nature of Sound Waves

LONGITUDINAL SOUND WAVES

Speaker making sound waves in a tube
The distance between adjacent condensations is equal to the wavelength of the sound wave.
Individual air molecules are not carried along with the wave.

When the sound hits your ear it causes your eardrum to vibrate and your brain interprets the vibration as the pitch and loudness of the sound.
The Nature of Sound Waves

THE FREQUENCY OF A SOUND WAVE

The **frequency** is the number of cycles per second.

A sound with a single frequency is called a **pure tone**.

The brain interprets the frequency in terms of the subjective quality called **pitch**.
Loudness is an attribute of a sound that depends primarily on the pressure amplitude of the wave.
The Speed of Sound

Sound travels through gases, liquids, and solids at considerably different speeds.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gases</strong></td>
<td></td>
</tr>
<tr>
<td>Air (0 °C)</td>
<td>331</td>
</tr>
<tr>
<td>Air (20 °C)</td>
<td>343</td>
</tr>
<tr>
<td>Carbon dioxide (0 °C)</td>
<td>259</td>
</tr>
<tr>
<td>Oxygen (0 °C)</td>
<td>316</td>
</tr>
<tr>
<td>Helium (0 °C)</td>
<td>965</td>
</tr>
<tr>
<td><strong>Liquids</strong></td>
<td></td>
</tr>
<tr>
<td>Chloroform (20 °C)</td>
<td>1004</td>
</tr>
<tr>
<td>Ethyl alcohol (20 °C)</td>
<td>1162</td>
</tr>
<tr>
<td>Mercury (20 °C)</td>
<td>1450</td>
</tr>
<tr>
<td>Fresh water (20 °C)</td>
<td>1482</td>
</tr>
<tr>
<td>Seawater (20 °C)</td>
<td>1522</td>
</tr>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>5010</td>
</tr>
<tr>
<td>Glass (Pyrex)</td>
<td>5640</td>
</tr>
<tr>
<td>Lead</td>
<td>1960</td>
</tr>
<tr>
<td>Steel</td>
<td>5960</td>
</tr>
</tbody>
</table>
Conceptual Example: Lightning, Thunder, and a Rule of Thumb

There is a rule of thumb for estimating how far away a thunderstorm is. After you see a flash of lighting, count off the seconds until the thunder is heard. Divide the number of seconds by five. The result gives the approximate distance (in miles) to the thunderstorm. Why does this rule work?
The Speed of Sound

\[ v_{\text{sound}} \left(20^\circ \text{ C}\right) = 343 \text{ m/s} \times \frac{1 \text{ mile}}{1600 \text{ m}} = 0.214 \text{ miles/s} \]

\[ v_{\text{light}} = c = 3.00 \times 10^8 \text{ m/s} \times \frac{1 \text{ mile}}{1600 \text{ m}} = 188,000 \text{ miles/s} \]

1) Time for a light flash to travel distance \( x \) in miles

\[ t_{\text{light}} = \frac{x}{c} = \frac{x}{188,000 \text{ miles/s}} = \left(5.32 \times 10^{-6} \text{ s/mile}\right)x \]

light travels 1 mile in 5.32 \( \mu \text{s} \) \( \Rightarrow \) \( \approx \) instantaneous

2) Distance thunder travels \( x \) in miles in \( t \) seconds

\[ x = v_{\text{sound}}t = \left(0.214 \text{ miles/s}\right)t = \frac{t}{4.67 \text{ s}} \text{ miles} \approx \frac{t}{5 \text{ s}} \text{ miles} \]
For a 1000 Hz tone, the smallest sound intensity that the human ear can detect is about $1 \times 10^{-12}$ W/m$^2$. This intensity is called the *threshold of hearing*.

On the other extreme, continuous exposure to intensities greater than 1 W/m$^2$ can be painful.

As we saw before, if the source emits sound *uniformly in all directions*, the intensity depends on the distance from the source in a simple way:

$$I = \frac{P}{4\pi r^2}, \quad P = \text{power emitted from source}$$

$$r = \text{distance from source}$$
Decibels

The **decibel (dB)** is a measurement unit used when comparing two sound intensities.

Because of the way in which the human hearing mechanism responds to intensity, it is appropriate to use a logarithmic scale called the **intensity level**:

$$
\beta = (10 \text{ dB}) \log\left(\frac{I}{I_o}\right)
$$

$$
I_o = 1.00 \times 10^{-12} \text{ W/m}^2
$$

Note that $\log(1)=0$, so when the intensity of the sound is equal to the threshold of hearing, the intensity level is zero.
Quick review of logarithms:

\[
\log x \equiv \log_{10} x = y \quad \Rightarrow \quad 10^y = x
\]

For example, \( \log 1 = 0 \) since \( 10^0 = 1 \)

\[
\log A - \log B = \log \left( \frac{A}{B} \right) \quad \log A + \log B = \log (AB)
\]

\[
\log \left( A^N \right) = N \log A
\]
Decibels

\[ \beta = (10 \text{ dB}) \log \left( \frac{I}{I_o} \right) \]

\[ I_o = 1.00 \times 10^{-12} \text{ W/m}^2 \]

<table>
<thead>
<tr>
<th>Threshold of hearing</th>
<th>Intensity $I$ (W/m$^2$)</th>
<th>Intensity Level $\beta$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rustling leaves</td>
<td>$1.0 \times 10^{-11}$</td>
<td>10</td>
</tr>
<tr>
<td>Whisper</td>
<td>$1.0 \times 10^{-10}$</td>
<td>20</td>
</tr>
<tr>
<td>Normal conversation (1 meter)</td>
<td>$3.2 \times 10^{-6}$</td>
<td>65</td>
</tr>
<tr>
<td>Inside car in city traffic</td>
<td>$1.0 \times 10^{-4}$</td>
<td>80</td>
</tr>
<tr>
<td>Car without muffler</td>
<td>$1.0 \times 10^{-2}$</td>
<td>100</td>
</tr>
<tr>
<td>Live rock concert</td>
<td>1.0</td>
<td>120</td>
</tr>
<tr>
<td>Threshold of pain</td>
<td>10</td>
<td>130</td>
</tr>
</tbody>
</table>
Example: Comparing Sound Intensities

Audio system 1 produces a sound intensity level of 90.0 dB, and system 2 produces an intensity level of 93.0 dB. Determine the ratio of intensities.

\[
\beta = (10 \text{ dB}) \log \left( \frac{I}{I_o} \right)
\]
Decibels

\[ \beta = (10 \text{ dB}) \log \left( \frac{I}{I_o} \right) \]

\[ \beta_1 = (10 \text{ dB}) \log \left( \frac{I_1}{I_o} \right) \quad \beta_2 = (10 \text{ dB}) \log \left( \frac{I_2}{I_o} \right) \]

\[ \beta_2 - \beta_1 = (10 \text{ dB}) \log \left( \frac{I_2}{I_o} \right) - (10 \text{ dB}) \log \left( \frac{I_1}{I_o} \right) = (10 \text{ dB}) \log \left( \frac{I_2/I_o}{I_1/I_o} \right) = (10 \text{ dB}) \log \left( \frac{I_2}{I_1} \right) \]

3.0 dB = (10 dB) log \left( \frac{I_2}{I_1} \right)

0.30 = \log \left( \frac{I_2}{I_1} \right) \quad \frac{I_2}{I_1} = 10^{0.30} = 2.0
In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.
Transverse Standing Waves

Transverse standing wave patterns on a string.

One-half of the longest wavelength, $\lambda_1$, can fit on the string of length $L$

\[
\Rightarrow L = \frac{\lambda_1}{2} \quad \Rightarrow \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}
\]
Transverse Standing Waves

String fixed at both ends

\[ f_n = n\left(\frac{v}{2L}\right) \quad n = 1, 2, 3, 4, \ldots \]
### Note frequencies (Hz) of the chromatic music scale

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>C#</th>
<th>D</th>
<th>Eb</th>
<th>E</th>
<th>F</th>
<th>F#</th>
<th>G</th>
<th>G#</th>
<th>A</th>
<th>Bb</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16.35</td>
<td>17.32</td>
<td>18.35</td>
<td>19.45</td>
<td>20.60</td>
<td>21.83</td>
<td>23.12</td>
<td>24.50</td>
<td>25.96</td>
<td>27.50</td>
<td>29.14</td>
<td>30.87</td>
</tr>
<tr>
<td>1</td>
<td>32.70</td>
<td>34.65</td>
<td>36.71</td>
<td>38.89</td>
<td>41.20</td>
<td>43.65</td>
<td>46.25</td>
<td>49.00</td>
<td>51.91</td>
<td>55.00</td>
<td>58.27</td>
<td>61.74</td>
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<tr>
<td>2</td>
<td>65.41</td>
<td>69.30</td>
<td>73.42</td>
<td>77.78</td>
<td>82.41</td>
<td>87.31</td>
<td>92.50</td>
<td>98.00</td>
<td>103.8</td>
<td>110.0</td>
<td>116.5</td>
<td>123.5</td>
</tr>
<tr>
<td>3</td>
<td>130.8</td>
<td>138.6</td>
<td>146.8</td>
<td>155.6</td>
<td>164.8</td>
<td>174.6</td>
<td>185.0</td>
<td>196.0</td>
<td>207.7</td>
<td>220.0</td>
<td>233.1</td>
<td>246.9</td>
</tr>
<tr>
<td>4</td>
<td>261.6</td>
<td>277.2</td>
<td>293.7</td>
<td>311.1</td>
<td>329.6</td>
<td>349.2</td>
<td>370.0</td>
<td>392.0</td>
<td>415.3</td>
<td>440.0</td>
<td>466.2</td>
<td>493.9</td>
</tr>
<tr>
<td>5</td>
<td>523.3</td>
<td>554.4</td>
<td>587.3</td>
<td>622.3</td>
<td>659.3</td>
<td>698.5</td>
<td>740.0</td>
<td>784.0</td>
<td>830.6</td>
<td>880.0</td>
<td>932.3</td>
<td>987.8</td>
</tr>
<tr>
<td>6</td>
<td>1047</td>
<td>1109</td>
<td>1175</td>
<td>1245</td>
<td>1319</td>
<td>1397</td>
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<td>1568</td>
<td>1661</td>
<td>1760</td>
<td>1865</td>
<td>1976</td>
</tr>
<tr>
<td>7</td>
<td>2093</td>
<td>2217</td>
<td>2349</td>
<td>2489</td>
<td>2637</td>
<td>2794</td>
<td>2960</td>
<td>3136</td>
<td>3322</td>
<td>3520</td>
<td>3729</td>
<td>3951</td>
</tr>
<tr>
<td>8</td>
<td>4186</td>
<td>4435</td>
<td>4699</td>
<td>4978</td>
<td>5274</td>
<td>5588</td>
<td>5920</td>
<td>6272</td>
<td>6645</td>
<td>7040</td>
<td>7459</td>
<td>7902</td>
</tr>
</tbody>
</table>

The octave number is in the left column so to find the frequency of middle C which is C4, look down the "C" column til you get to the "4" row : so middle C is 261.6 Hz.
Example: The A string on a violin has a fundamental frequency of 440 Hz. The length of the string is 32.0 cm and it has a mass of 0.450 g. Under what tension must the string be placed?

\[ v = \sqrt{\frac{F_T}{m/L}} \quad \Rightarrow \quad F_T = v^2 \frac{m}{L} \]

\[ f_1 = \frac{v}{2L} \quad \Rightarrow \quad v = 2Lf_1 \]

\[ \therefore F_T = \left(2Lf_1\right)^2 \frac{m}{L} = 4Lf_1^2 m = 4 \times 0.320 \times 440^2 \times 0.450 \times 10^{-3} = 112 \text{ N} \]
Transverse Standing Waves

Changing the pitch of a guitar string by fingering it: the smaller you make $L$, the higher the pitch.

$$f_n = n\left(\frac{v}{2L}\right) \quad n = 1, 2, 3, 4, \ldots$$
Longitudinal Standing Waves

A longitudinal standing wave pattern on a slinky.
Longitudinal Standing Waves

Standing sound waves in a tube open at both ends

The anti-nodes occur at the open ends of the tube.

One-half of the longest wavelength, $\lambda_1$, can fit in the tube of length $L$

$$\Rightarrow L = \frac{\lambda_1}{2} \quad \Rightarrow \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Tube open at both ends

$$f_n = n\left(\frac{v}{2L}\right) \quad n = 1, 2, 3, 4, \ldots$$

Note that the string fixed at both ends and the tube open at both ends have the same equation for the standing wave frequencies!
**Example:** Playing a Flute

When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance $L$. 

---

Image of a person playing a flute, labeled as "Head joint."
Longitudinal Standing Waves

\[ f_n = n \left( \frac{v}{2L} \right) \quad n = 1, 2, 3, 4, \ldots \]

\[ L = \frac{nv}{2f_n} = \frac{1(343 \text{ m/s})}{2(261.6 \text{ Hz})} = 0.656 \text{ m} \]
Standing sound waves in a tube open at one end

1\textsuperscript{st} harmonic

3\textsuperscript{rd} harmonic, 1\textsuperscript{st} overtone

One-quarter of the longest wavelength, \( \lambda_1 \), can fit in the tube of length \( L \)

\[
\Rightarrow L = \frac{\lambda_1}{4} \quad \Rightarrow \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}
\]

\[ f_n = n\left(\frac{v}{4L}\right) \quad n = 1, 3, 5, \ldots \]
Example: The fundamental frequency of an open organ pipe corresponds to the note D2 \((f_1 = 73.42 \text{ Hz on the chromatic musical scale})\). The third harmonic of another organ pipe that is closed at one end has the same frequency. Compare the lengths of these two pipes.

Tube open at both ends \(f_n = n\left(\frac{v}{2L}\right)\) \(n = 1, 2, 3, 4, \ldots\)

\[ f_1 = \frac{v}{2L} \quad \Rightarrow \quad L = \frac{v}{2f_1} = \frac{343}{2(73.42)} = 2.34 \text{ m} \]

Tube closed at one end \(f'_n = n'\left(\frac{v}{4L'}\right)\) \(n' = 1, 3, 5, \ldots\)

\[ f'_3 = 3\left(\frac{v}{4L'}\right) = f_1 \quad \Rightarrow \quad L' = \frac{3v}{4f_1} = \frac{3(343)}{4(73.42)} = 3.50 \text{ m} \]
Longitudinal Standing Waves

Conceptual Example: Why does inhaling Helium raise the pitch of a voice?

! Warning: It is dangerous to inhale Helium!

Assume that the mouth/larynx system acts as a tube open at one end of length 0.25 m.

\[
f_n = n\left(\frac{v}{4L}\right)
\]

\(n = 1, 3, 5, \ldots\)

In air, where \(v_{\text{sound}}^{\text{air}} = 343 \text{ m/s}\), the fundamental frequency of a voice is

\[
f_1^{\text{air}} = \frac{v_{\text{sound}}^{\text{air}}}{4L} = \frac{343}{4(0.25)} = 343 \text{ Hz} \quad \Rightarrow \quad \approx \text{ an octave 4 F note}
\]

In Helium, where \(v_{\text{sound}}^{\text{Helium}} = 965 \text{ m/s}\), the fundamental frequency of a voice is

\[
f_1^{\text{Helium}} = \frac{v_{\text{sound}}^{\text{Helium}}}{4L} = \frac{965}{4(0.25)} = 965 \text{ Hz} \quad \Rightarrow \quad \approx \text{ an octave 5 B note}
\]
Complex Sound Waves

Complex pressure pattern

Air pressure

Time

Amplitude

1 2 3
Harmonic number
Complex Sound Waves

Pressure pattern

Spectrum analyzer

Fourier spectrum
Complex Sound Waves

Fourier spectra of some sounds

- Pure tone
- White noise
- Violin note