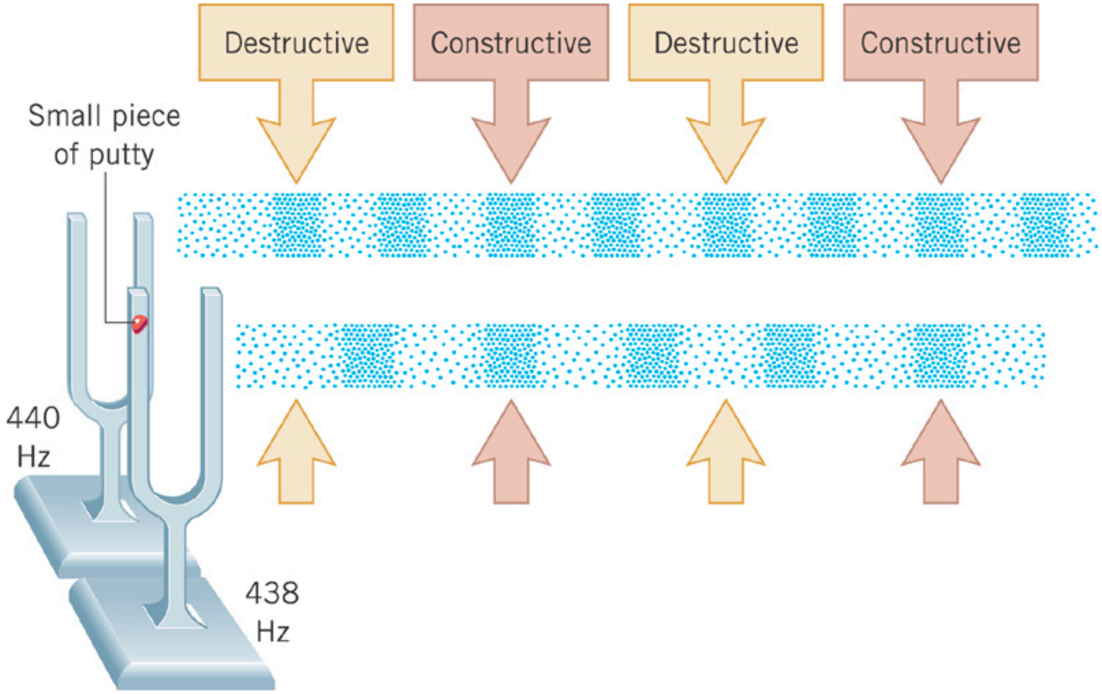
Chapters 11, 12, 24

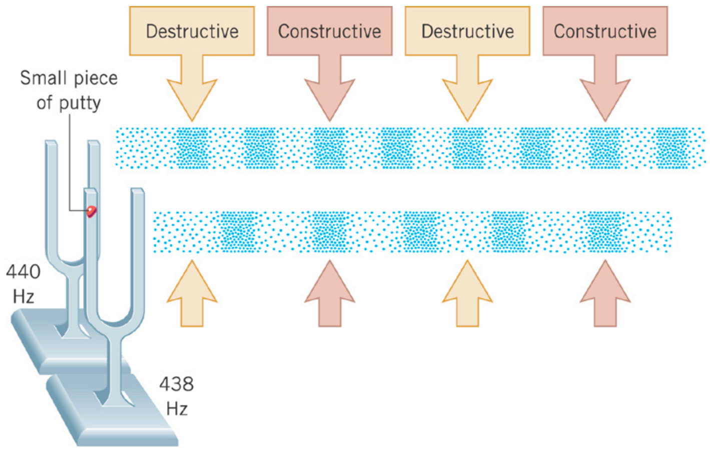
Refraction and Interference of Waves

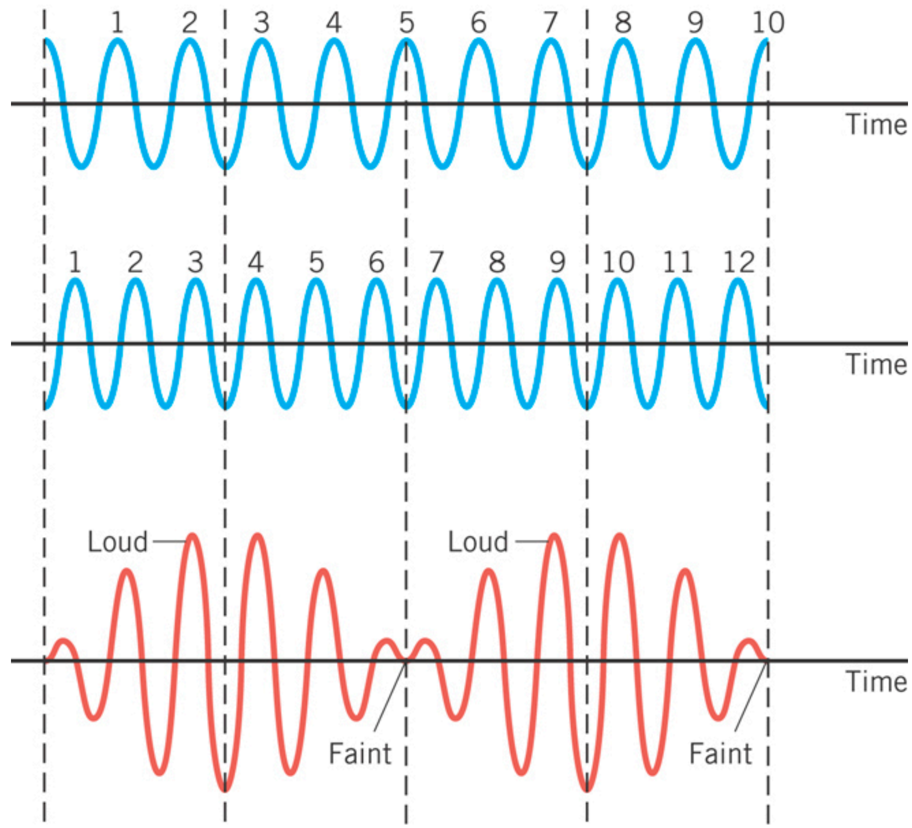
Beats



Two overlapping waves with *slightly different frequencies* gives rise to the phenomena of beats.

Beats





The ***beat frequency*** is the ***difference*** between the two sound frequencies.

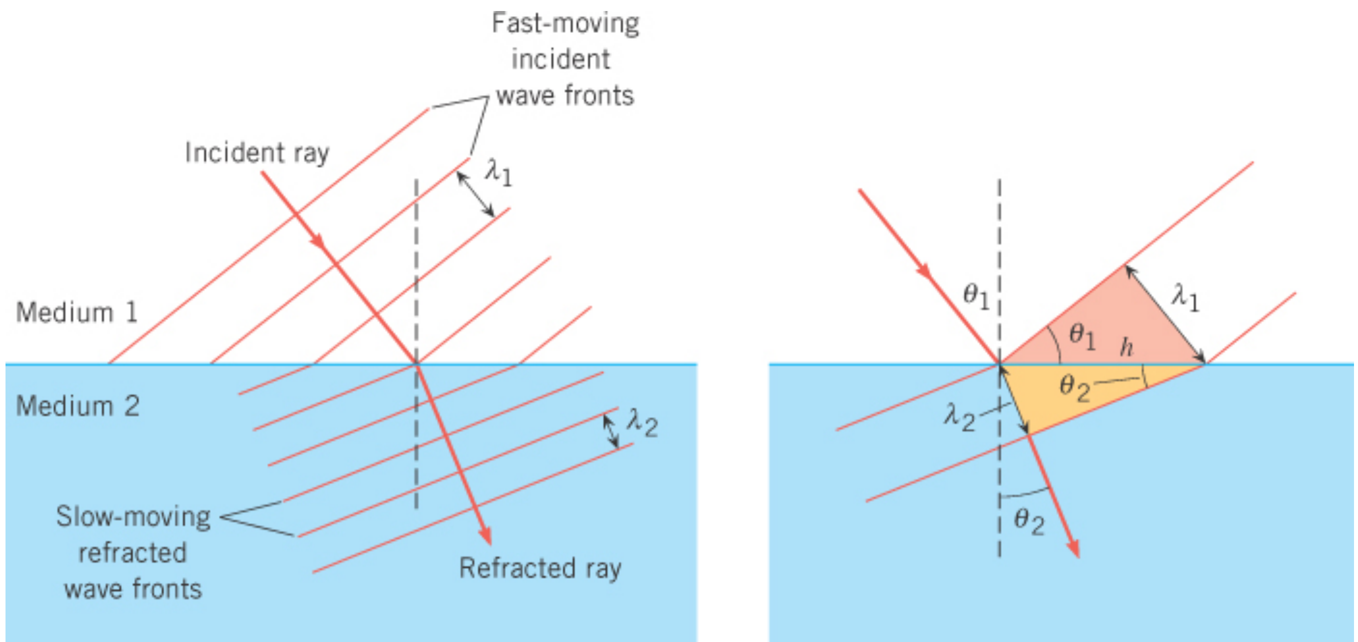
$$f_{beat} = |f_1 - f_2|$$

In this case, $f_{beat} = |440 - 438| = 2 \text{ Hz}$

Refraction

Wave transmitted from one medium to another → Refraction

In this figure,
$v_1 > v_2$



(a) (b)

f is same in each medium but v and λ are different

$$\lambda = \frac{v}{f} \Rightarrow \lambda_1 = \frac{v_1}{f} \quad and \quad \lambda_2 = \frac{v_2}{f}$$

$$\sin\theta_1 = \frac{\lambda_1}{h} = \frac{v_1}{fh} \qquad \sin\theta_2 = \frac{\lambda_2}{h} = \frac{v_2}{fh}$$

$$\frac{\sin\theta_1}{v_1} = \frac{1}{fh} = \frac{\sin\theta_2}{v_2} \Rightarrow \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

Snell's Law

Example: A sound wave in air is incident on a pool of water at an angle of 10° with respect to the normal to the surface. Find the angle of the wave with respect to the normal to the surface after it is transmitted into the water.

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \quad \Rightarrow \quad \frac{\sin\theta_{water}}{v_{water}} = \frac{\sin\theta_{air}}{v_{air}}$$

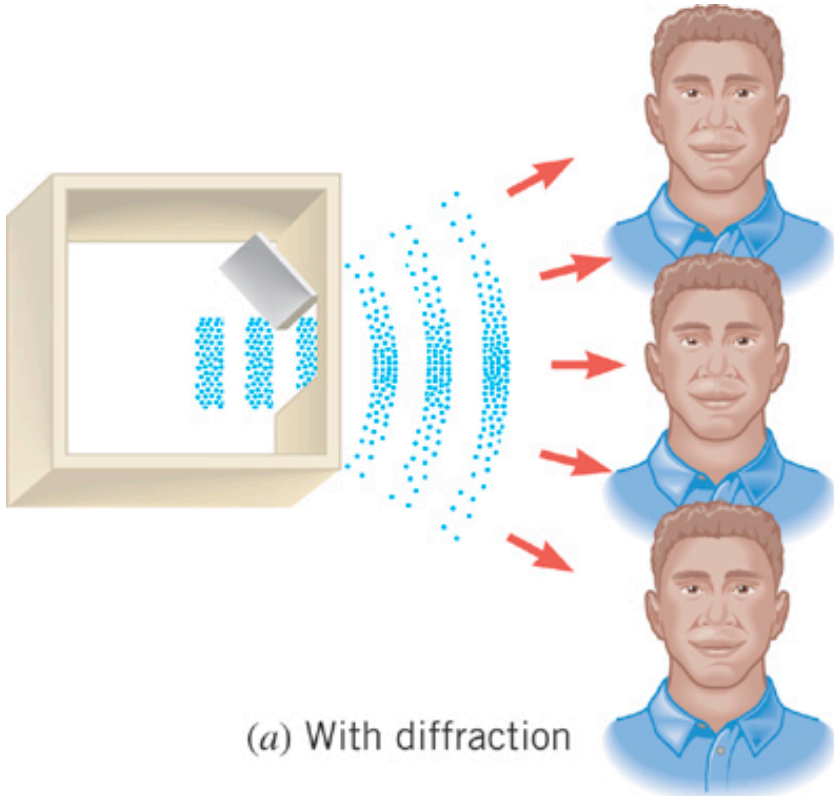
$$\sin\theta_{water} = \frac{v_{water}}{v_{air}}\sin\theta_{air} = \frac{1482}{343}\sin 10^o = 0.750$$

$$\therefore \theta_{water} = \sin^{-1} 0.750 = 49^o$$

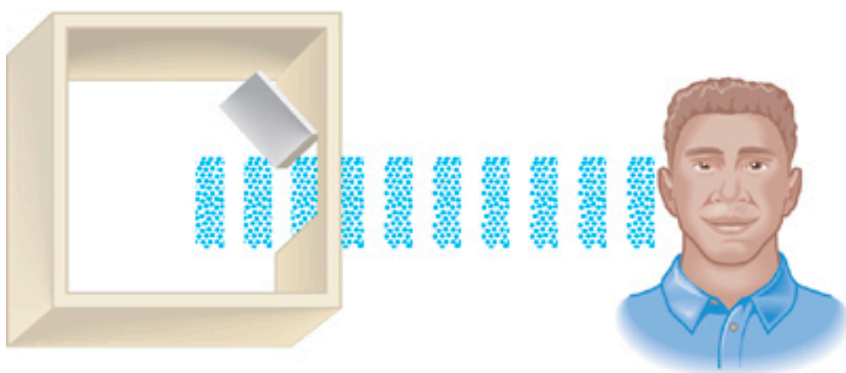
Chapters 11 and 24

Diffraction of Waves

17.3 Diffraction



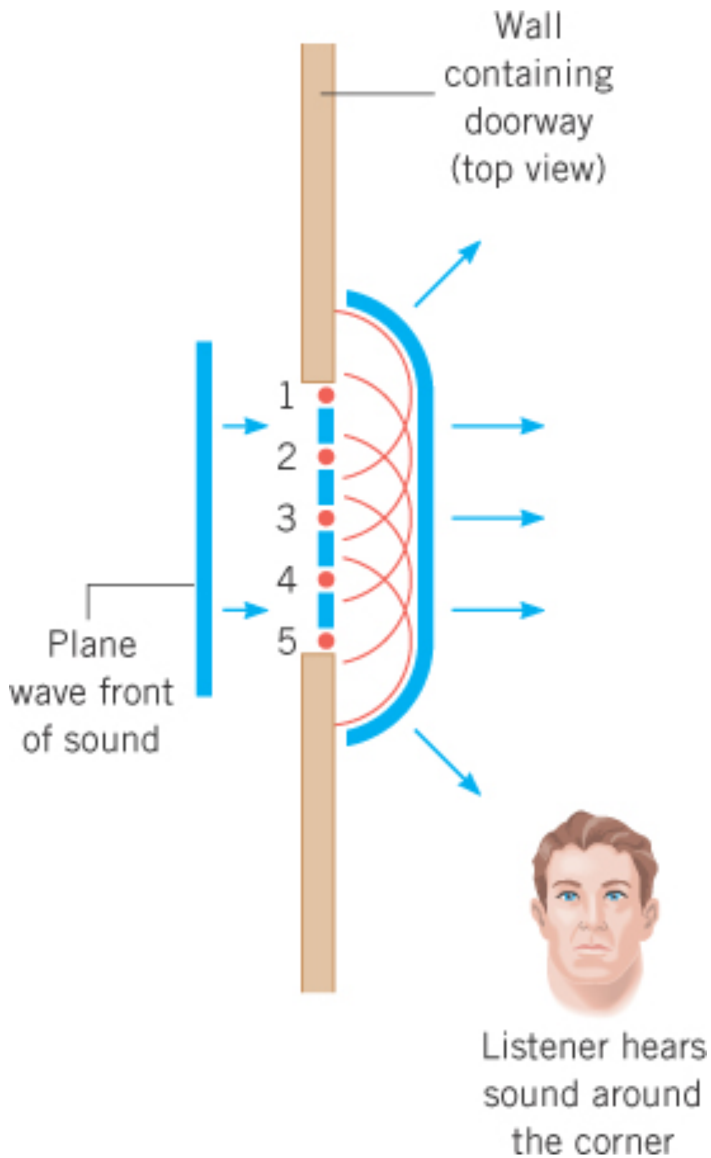
(*a*) With diffraction



(*b*) Without diffraction

The bending of a wave around an obstacle or the edges of an opening is called ***diffraction***.

Diffraction

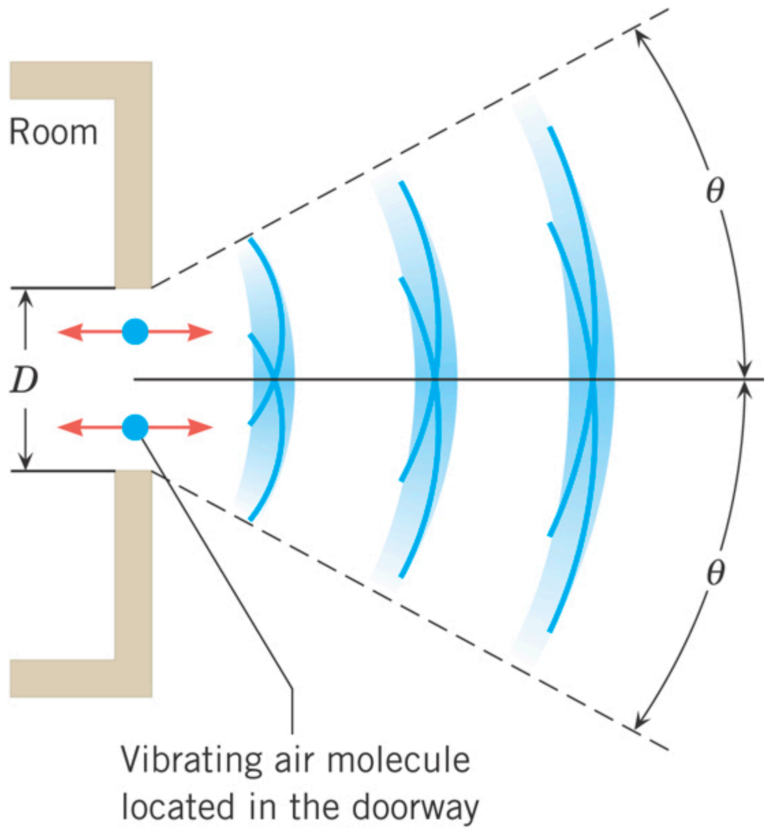


Diffraction is the bending of waves around obstacles or the edges of an opening.

Huygens' principle

Every point on a wave front acts as a source of tiny wavelets that move forward with the same speed as the wave; the wave front at a latter instant is the surface that is tangent to the wavelets.

Diffraction



Approximate expression for angular bending of waves of wavelength λ around an object or opening of size D

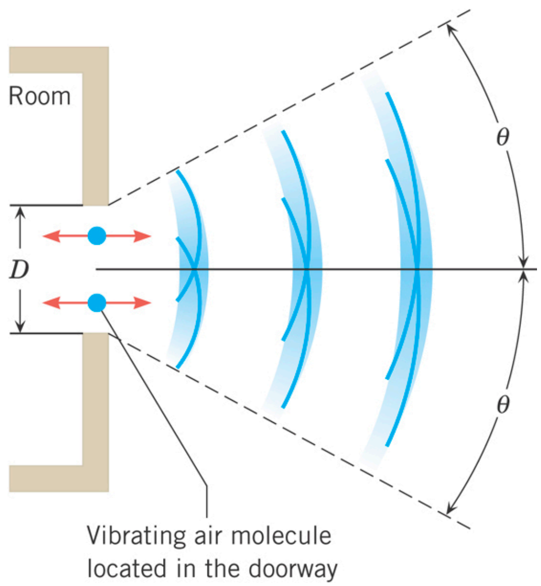
$$\theta \text{ (radians)} \approx \frac{\lambda}{D}$$

Diffraction

Example: **Assume the doorway is *D* = 1.0 m wide and the sound in the room has a frequency of 500 Hz. Find the diffraction angle of the sound through the doorway.**

$D = 1.0$ m

$f = 500$ Hz



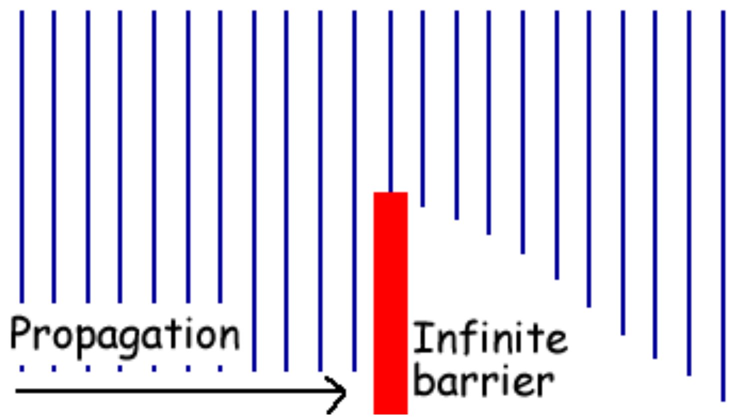
θ?

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{500 \text{ Hz}} = 0.686 \text{ m}$$

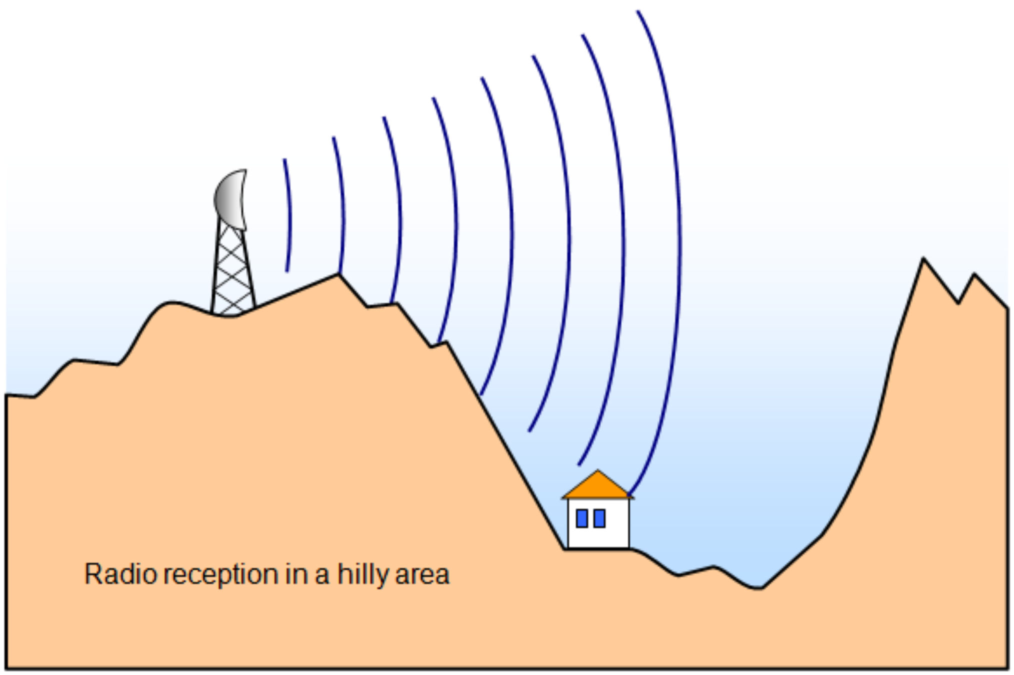
$$\theta \text{ (radians)} \approx \frac{\lambda}{D} = \frac{0.686 \text{ m}}{1.0 \text{ m}} = 0.686 \text{ rad} \approx 39^{o}$$


Image © 2011 GeoEye
a)

Diffraction around barriers



Shadow region infringed upon by diffracting waves

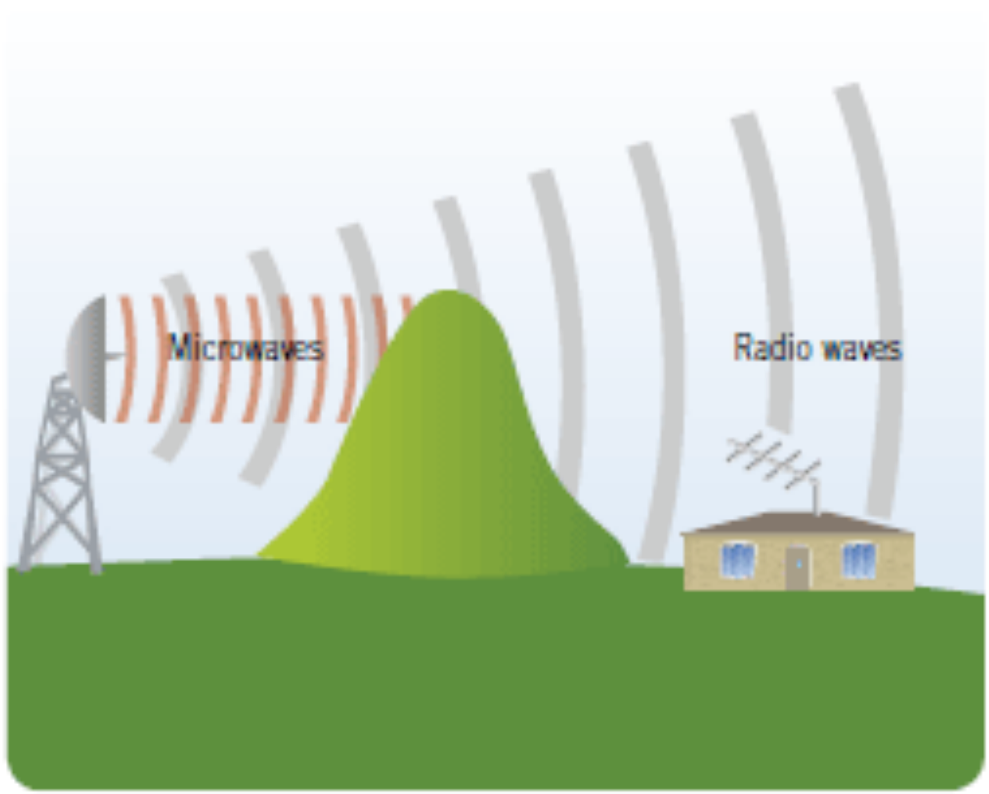


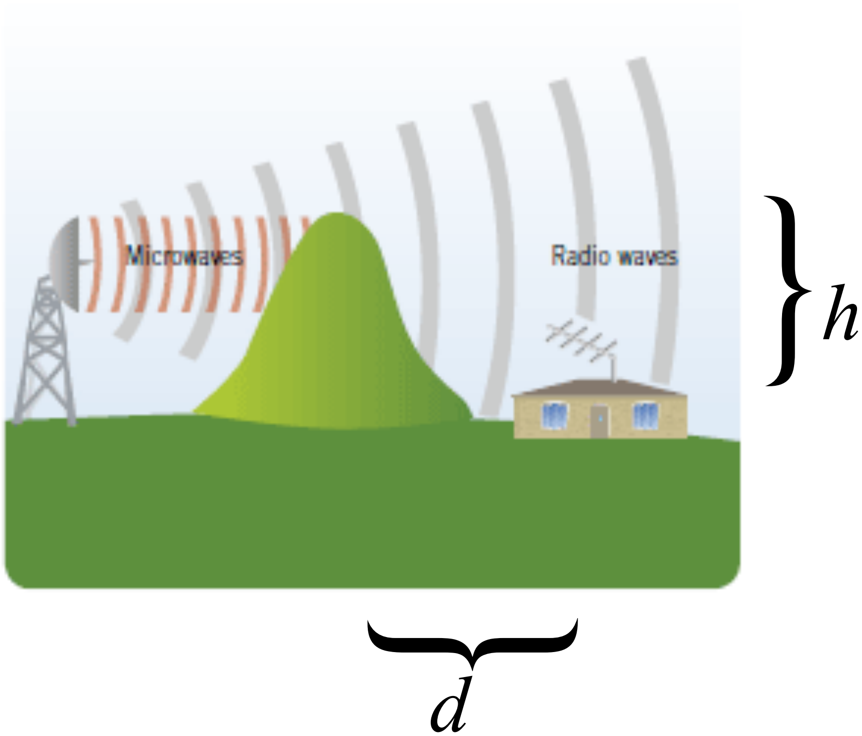
Diffraction around a hill allows radio reception in the valley


52 m
Image © 2011 DigitalGlobe
Google

Example of diffraction of electromagnetic waves around a hill.

An AM radio station broadcasts at a frequency is 1230×10^3 Hz, and a microwave transmitter broadcasts waves at 2.50 GHz. Approximately how much will each of the waves be bent when they encounter a hill that is 1000 ft high? How far behind the hill must a receiver antenna be located to pick up the AM radio broadcast? Will it pick up the microwave signal?





$$\lambda_{AM} = \frac{v}{f_{AM}} = \frac{3.00\times10^{8}\ \text{m/s}}{1230\times10^{3}\text{Hz}} = 244\ \text{m}$$

$$\lambda_{\mu W} = \frac{v}{f_{\mu W}} = \frac{3.00\times10^{8}\ \text{m/s}}{2.5\times10^{9}\text{Hz}} = 0.12\ \text{m}$$

$$\theta_{AM} \approx \frac{\lambda_{AM}}{h} = \frac{244\ \text{m}}{1000\ \text{ft}\times0.305\ \text{m/ft}} = 0.80\ \text{rad} \rightarrow 46^{o}$$

A lot of diffraction!

$$\theta_{\mu W} \approx \frac{\lambda_{\mu W}}{h} = \frac{0.12\ \text{m}}{1000\ \text{ft}\times0.305\ \text{m/ft}} = 3.9\times10^{-4}\ \text{rad} \rightarrow 0.023^{o}$$

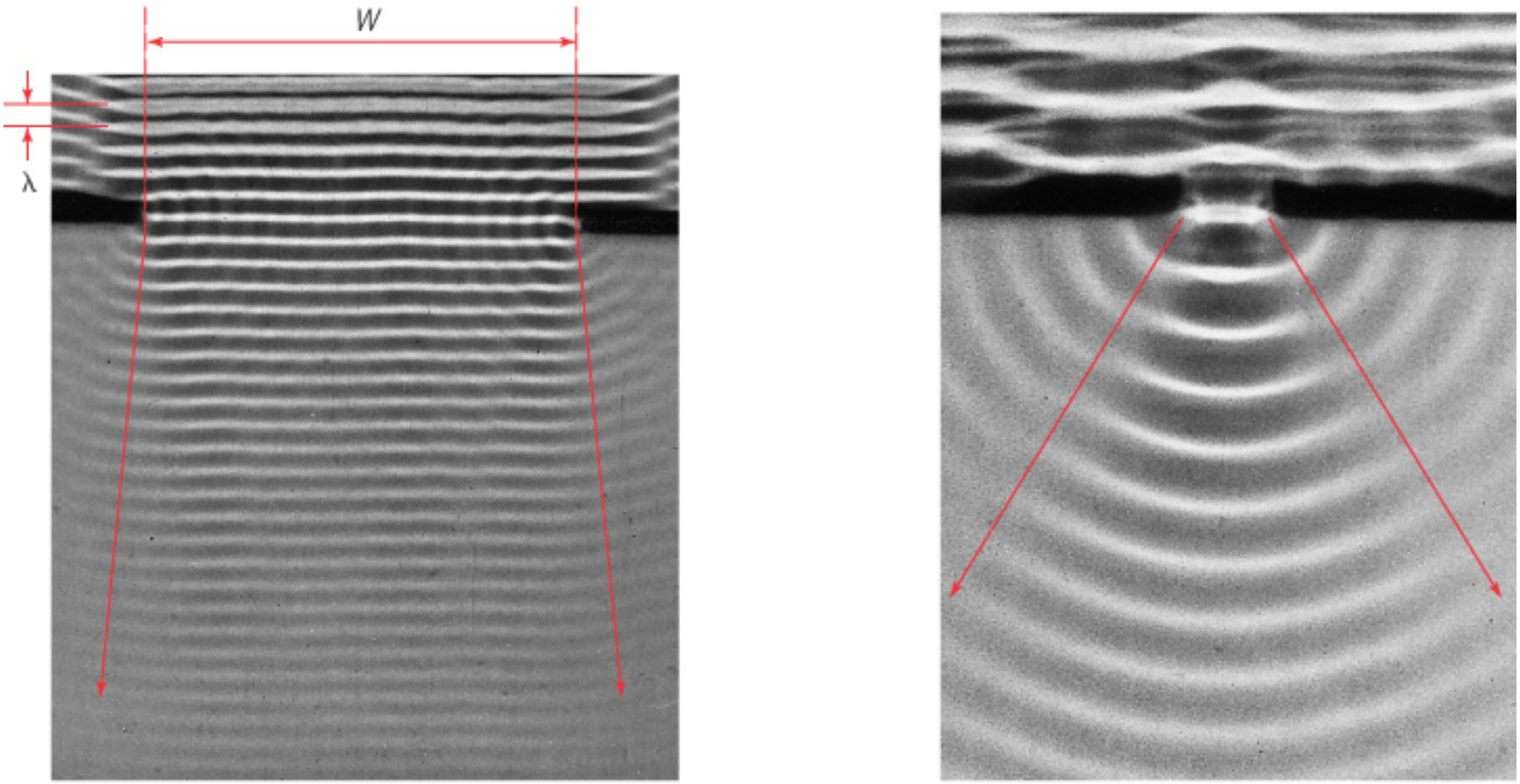
hardly any diffraction!

$$\tan\theta_{AM} = \frac{h}{d} \quad\Rightarrow\quad d = \frac{h}{\tan\theta_{AM}} = \frac{1000\ \text{ft}}{\tan 46^{o}} = 970\ \text{ft}$$

For the microwave: $d = \frac{h}{\tan\theta_{\mu W}} = \frac{1000\ \text{ft}}{\tan 0.023^{o}} = 470\ \text{miles}$

Won't pick up the µW signal!

Diffraction



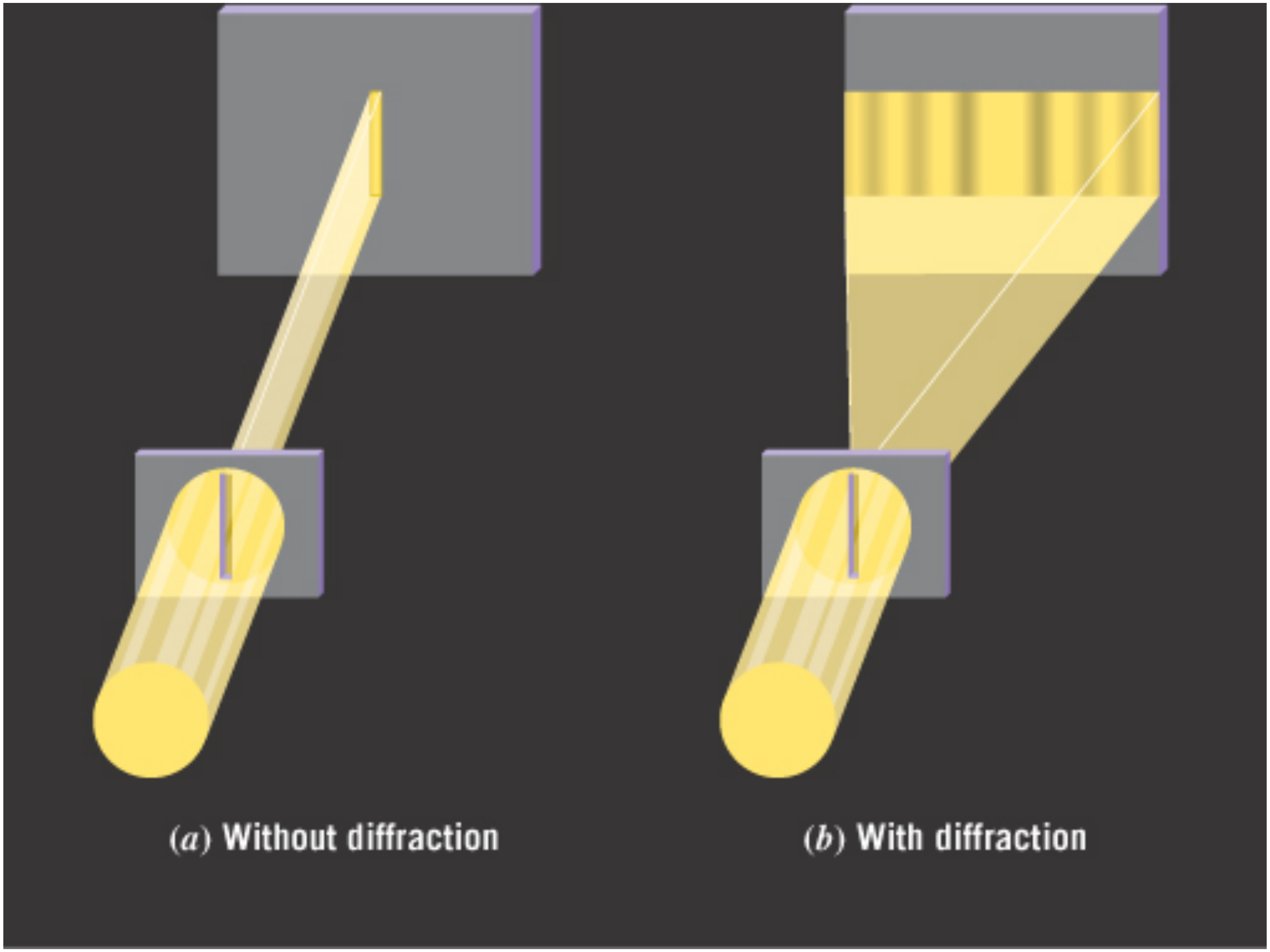
(*a*) Smaller value for λ/*W*, less diffraction.

(*b*) Larger value for λ/*W*, more diffraction.

The extent of the diffraction increases as the ratio of the wavelength to the width of the opening increases.

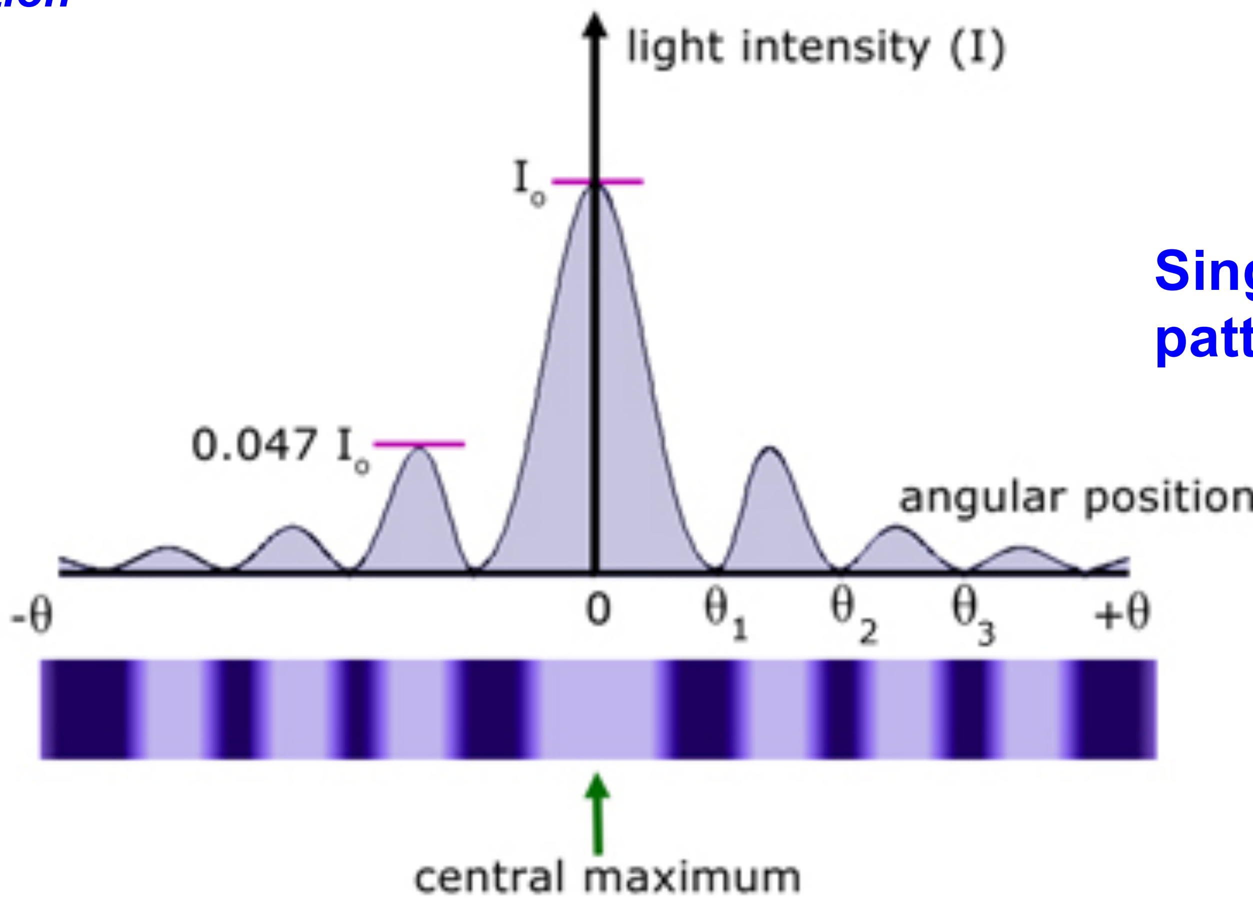
$$\theta \text{ (radians)} \approx \frac{\lambda}{W}$$

Diffraction

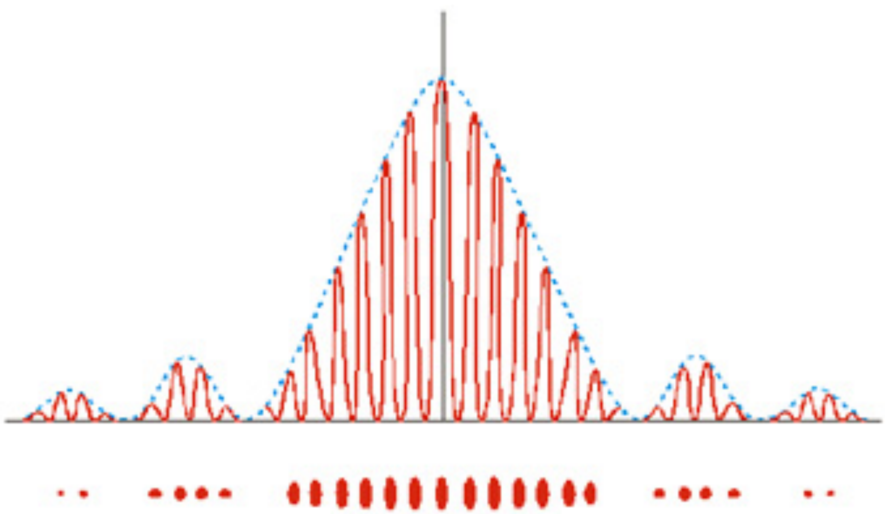


Light waves also exhibit diffraction effects.

Diffraction

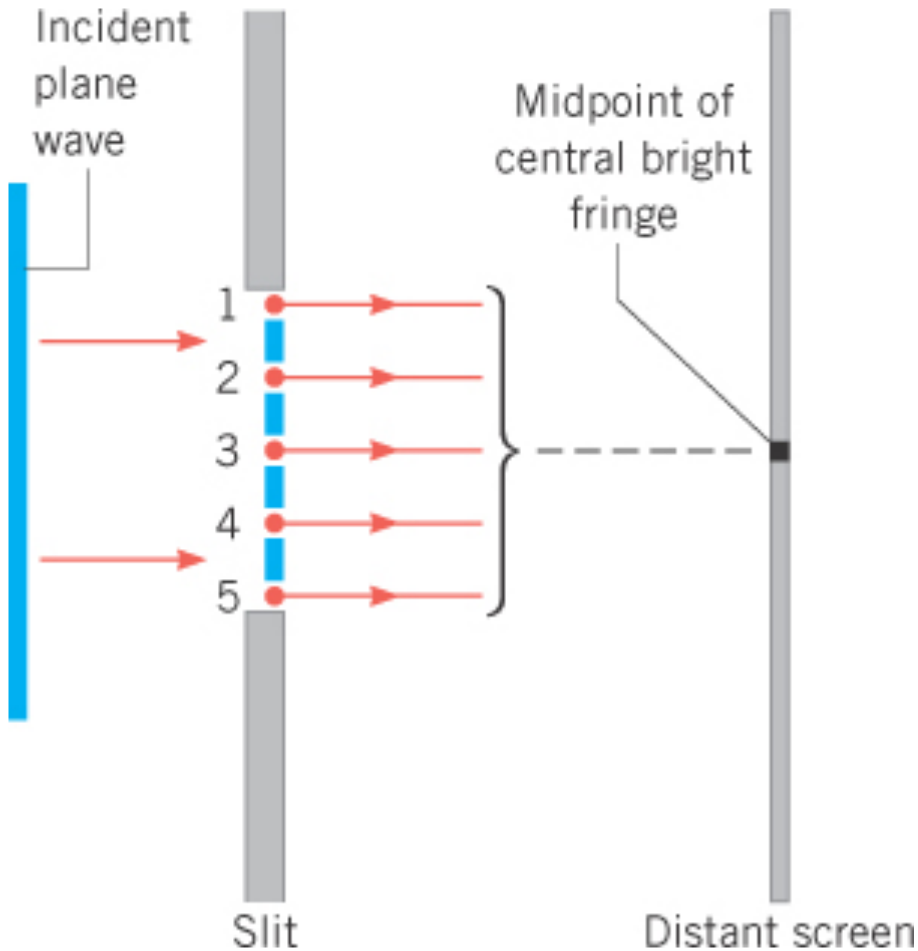


Single-slit diffraction pattern



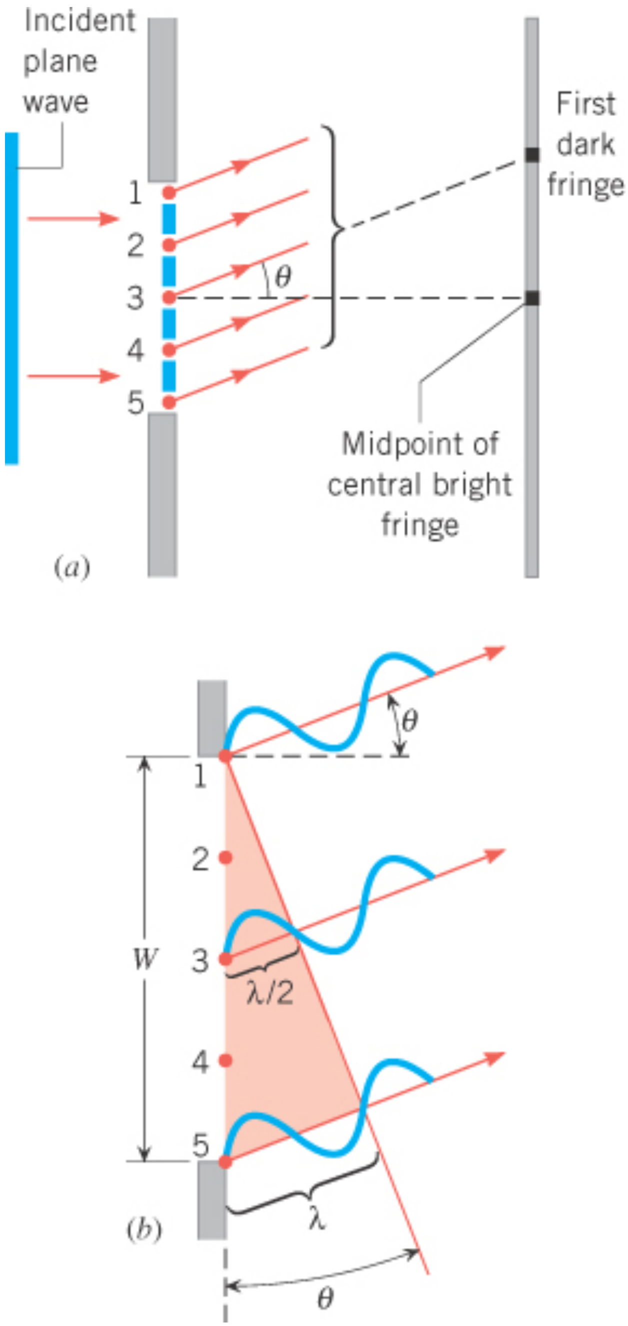
Double-slit interference pattern
(note the single-slit diffraction pattern superimposed on it)

Diffraction



This top view shows five sources of Huygens' wavelets.

Diffraction

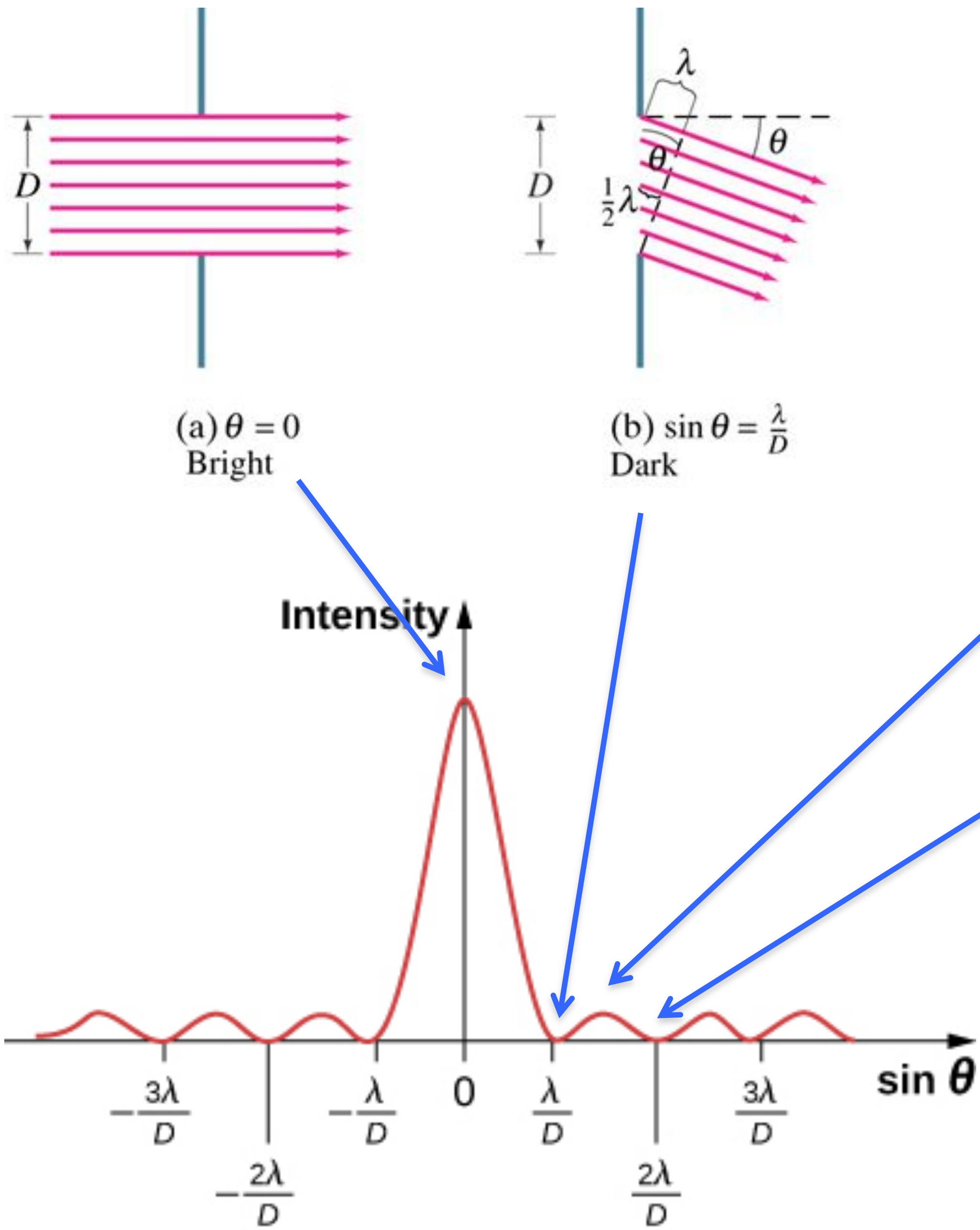


These drawings show how destructive interference leads to the first dark fringe on either side of the central bright fringe.

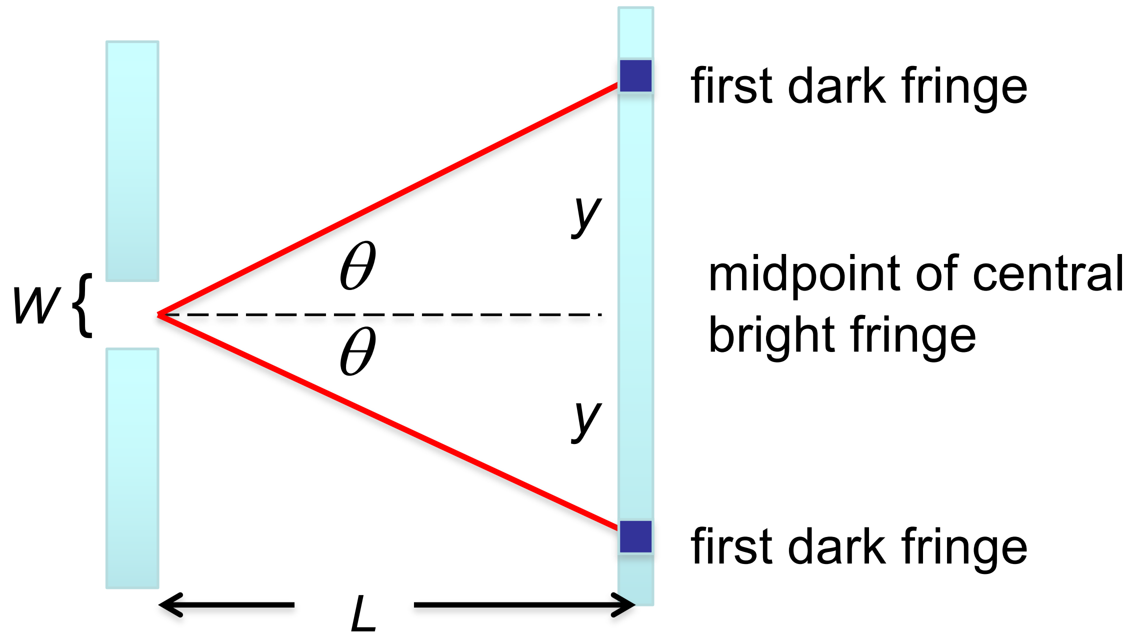
Dark fringes for single-slit diffraction

$$\sin\theta = m\frac{\lambda}{W} \qquad m = 1,2,3\ldots\ldots$$

Single-slit diffraction pattern



Example for single-slit diffraction. Light passes through a slit and shines on a flat screen that is located L = 0.40 m away. The wavelength of the light in a vacuum is λ = 410 nm. The distance between the midpoint of the central bright fringe and the first dark fringe is y. Determine the width $2y$ of the central bright fringe when the width of the slit is (a) $W = 5.0 \times 10^{-6}$ m and (b) $W = 2.5 \times 10^{-6}$ m.

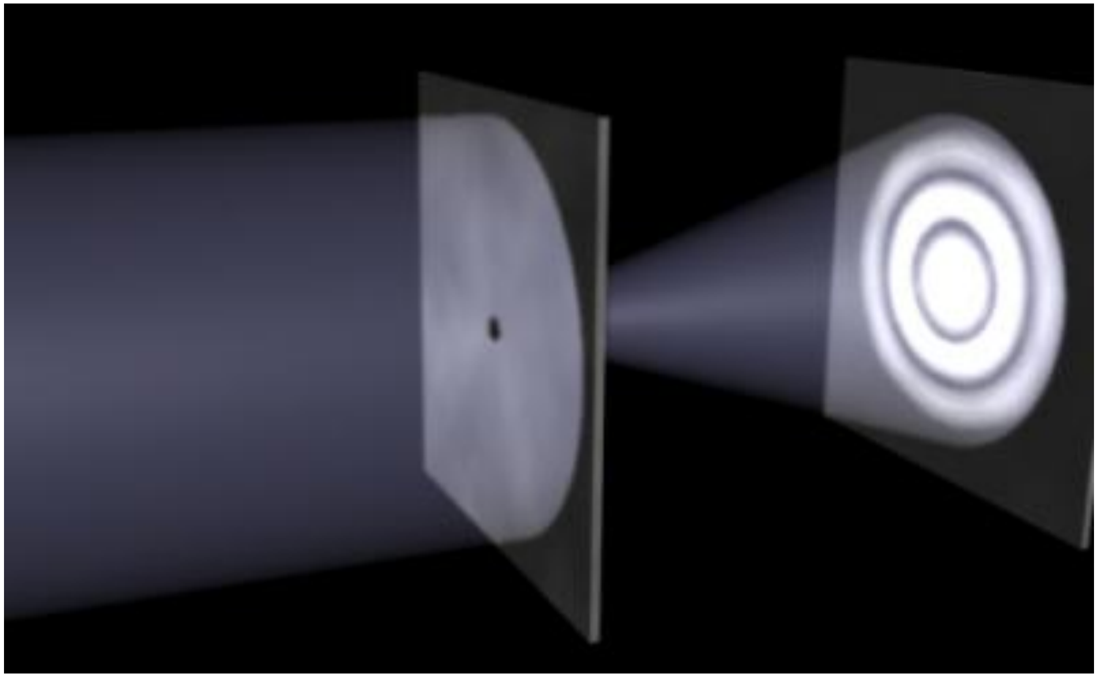


$$(a) \quad \theta = \sin^{-1}\left(\frac{\lambda}{W}\right) = \sin^{-1}\left(\frac{410\times10^{-9}}{5.0\times10^{-6}}\right) = 4.7^{o}$$

$$2y = 2L\tan\theta = 2(0.40)\tan 4.7^{o} = 0.066\ m$$

$(b) \quad 2y = 0.13\ m$ → The width of the central bright fringe is greater for smaller W

Diffraction through a circular aperture of diameter D by a wave of wavelength λ



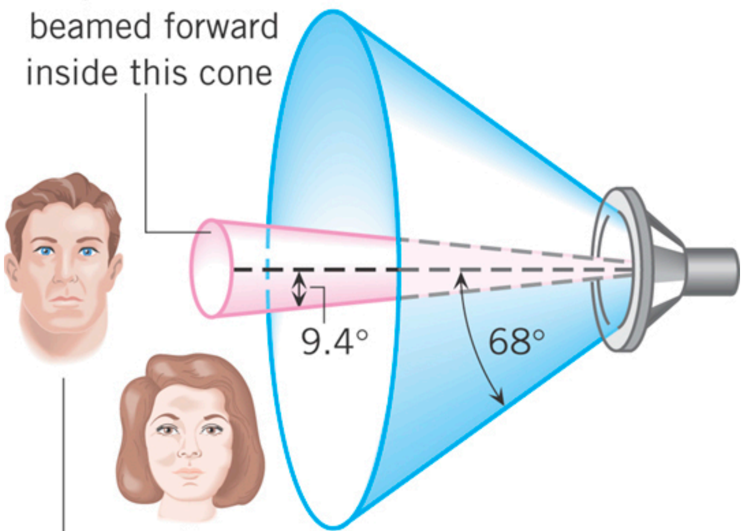
Diffraction pattern of light incident on a circular aperture

$$\sin\theta = 1.22\frac{\lambda}{D}$$

where θ is the angle from the center of the central maximum to the first minimum

Diffraction

Example. A 1500 Hz sound and a 8500 Hz sound each emerges from a loudspeaker through a circular opening that has a diameter of 0.30 m. Find the diffraction angle θ for each sound (assume v_{sound} = 343 m/s).



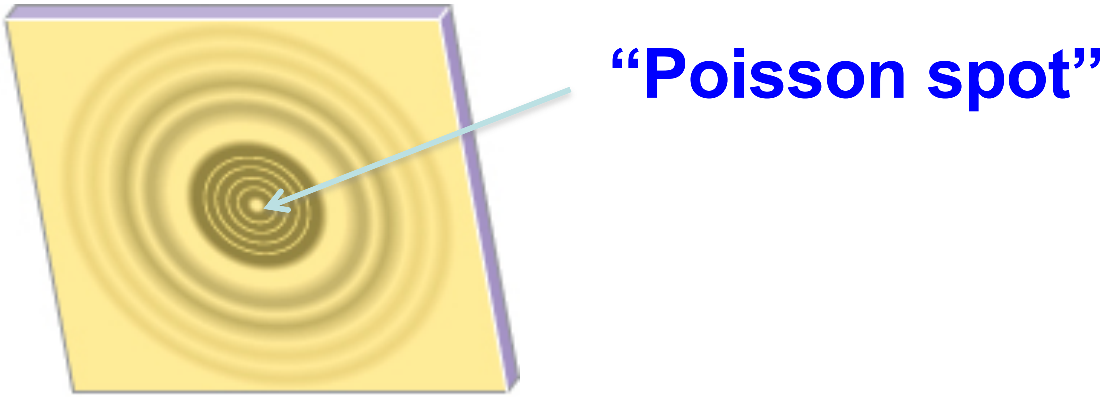
$$\lambda = \frac{v}{f}$$

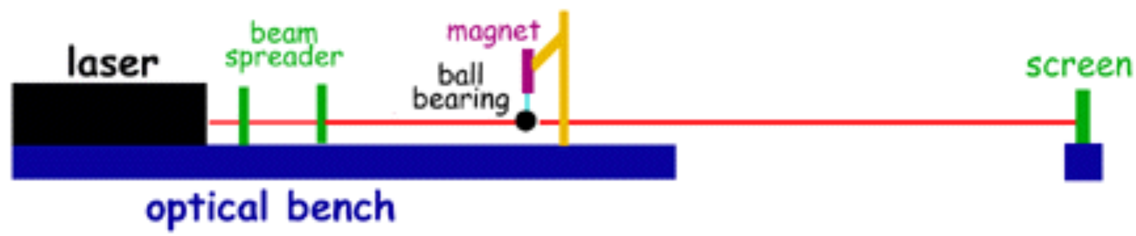
$$\sin\theta = 1.22\frac{\lambda}{D} = 1.22\frac{v}{fD}$$

$$\text{1500 Hz:} \quad \theta = \sin^{-1}\left(1.22\frac{v}{fD}\right) = \sin^{-1}\left[1.22\frac{343}{(1500)(0.30)}\right] = 68^o$$

$$\text{8500 Hz:} \quad \theta = \sin^{-1}\left(1.22\frac{v}{fD}\right) = \sin^{-1}\left[1.22\frac{343}{(8500)(0.30)}\right] = 9.4^o$$

Diffraction





Diffraction pattern formed by an opaque disk or sphere.