Chapter 23

The Refraction of Light: Lenses and Optical Instruments

The Index of Refraction

Light travels through a **vacuum** at a speed $c = 3.00 \times 10^{8}\ \mathrm{m/s}$

Light travels through **materials** at a speed **less than its speed in a vacuum.**

DEFINITION OF THE INDEX OF REFRACTION

The index of refraction of a material is the ratio of the speed of light in a vacuum to the speed of light in the material:

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the material}} = \frac{c}{v}$$

The Index of Refraction

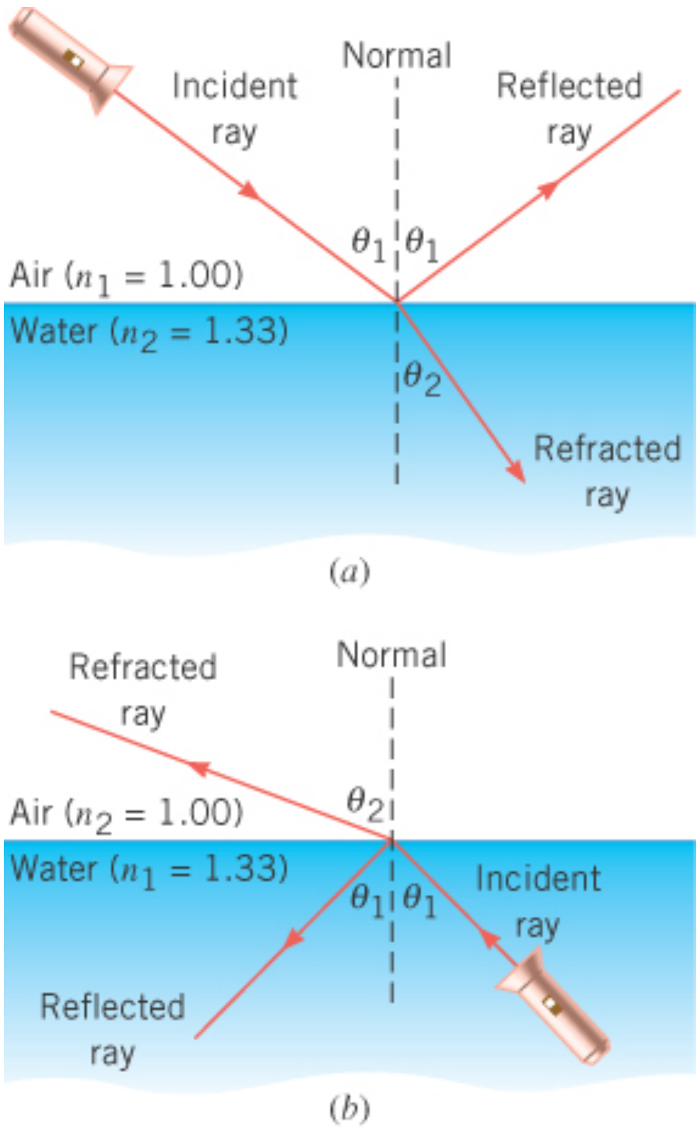
Table 26.1 **Index of Refraction[a] for Various Substances**

Substance	Index of Refraction, n
Solids at 20 °C	
Diamond	2.419
Glass, crown	1.523
Ice (0 °C)	1.309
Sodium chloride	1.544
Quartz	
Crystalline	1.544
Fused	1.458
Liquids at 20 °C	
Benzene	1.501
Carbon disulfide	1.632
Carbon tetrachloride	1.461
Ethyl alcohol	1.362
Water	1.333
Gases at 0 °C, 1 atm	
Air	1.000 293
Carbon dioxide	1.000 45
Oxygen, O_2	1.000 271
Hydrogen, H_2	1.000 139

[a] Measured with light whose wavelength in a vacuum is 589 nm.

Snell's Law and the Refraction of Light

SNELL'S LAW -- When light strikes an interface between two materials it breaks up into two pieces - one **reflected** and one **refracted** (transmitted).



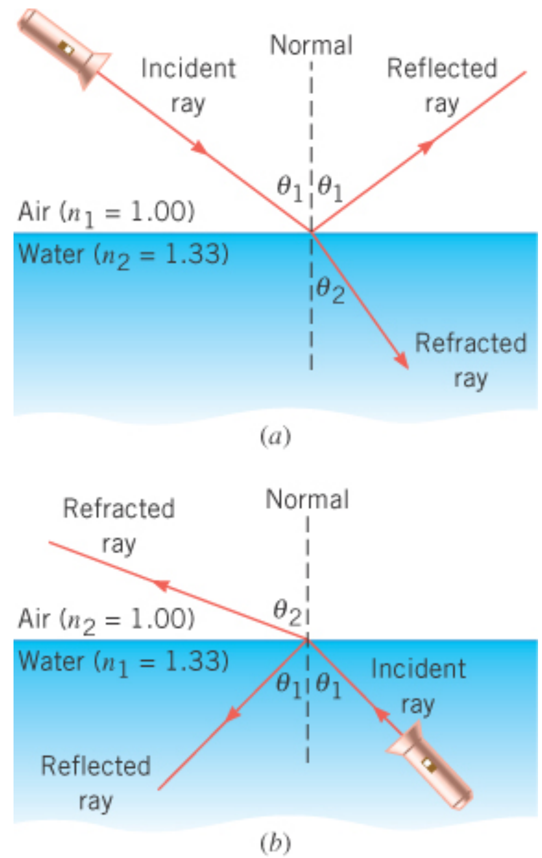
SNELL'S LAW OF REFRACTION

When light travels from a material with one index of refraction to a material with a different index of refraction, the angle of incidence is related to the angle of refraction by

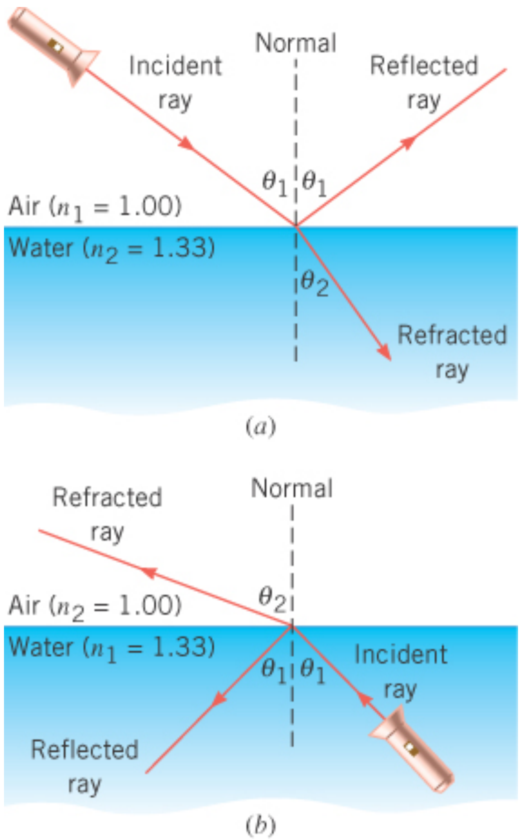
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Example: Determining the Angle of Refraction

A light ray strikes an air/water surface at an angle of 46 degrees with respect to the normal. Find the angle of refraction when the direction of the ray is (a) from air to water and (b) from water to air.



Snell's Law and the Refraction of Light



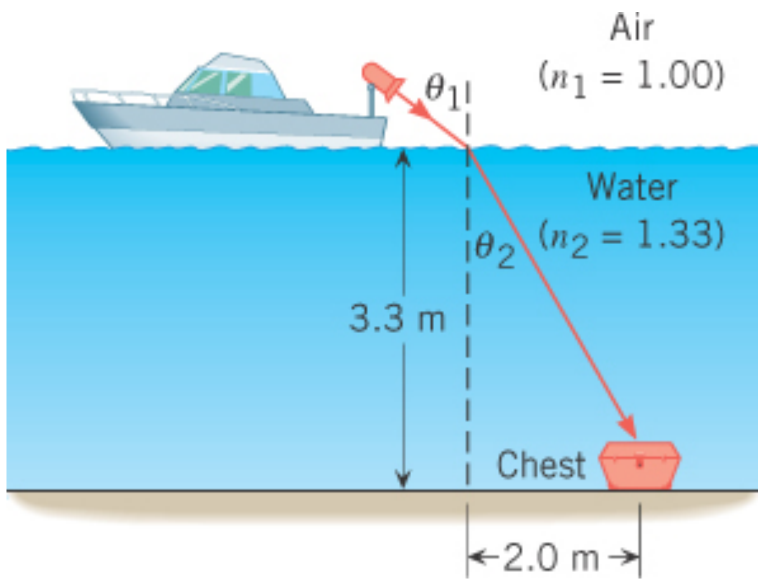
(a) $$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{(1.00)\sin 46^\circ}{1.33} = 0.54$$

$$\theta_2 = 33^\circ$$

(b) $$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{(1.33)\sin 46^\circ}{1.00} = 0.96$$

$$\theta_2 = 74^\circ$$

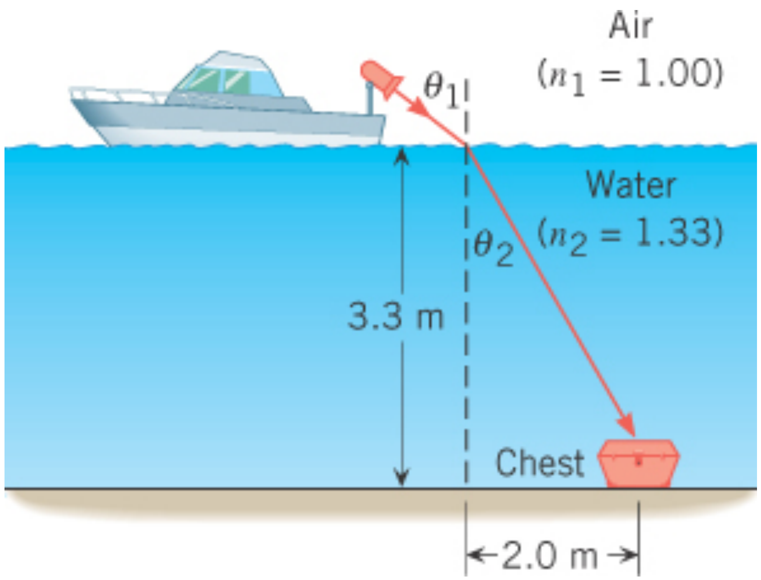
APPARENT DEPTH



***Example:* Finding a Sunken Chest**

The searchlight on a yacht is being used to illuminate a sunken chest. At what angle of incidence should the light be aimed?

Snell's Law and the Refraction of Light



First find θ_2 from the geometry and then use Snell's Law to find θ_1:

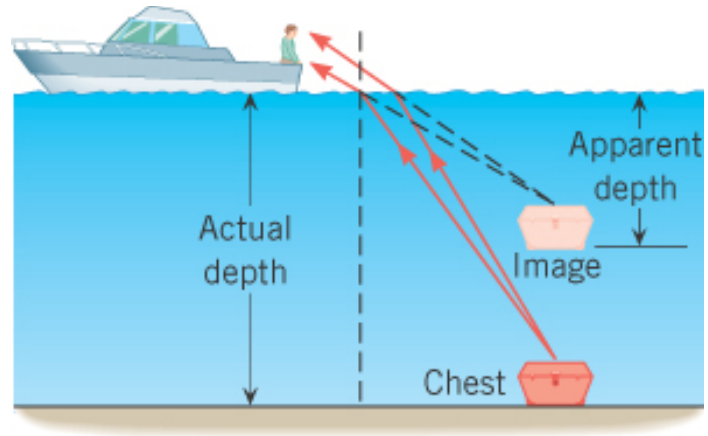
$$\theta_2 = \tan^{-1}\left(\frac{2.0}{3.3}\right) = 31^o$$

$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1} = \frac{(1.33)\sin 31^\circ}{1.00} = 0.69$$

$$\theta_1 = 44^\circ$$

Snell's Law and the Refraction of Light

Because light from the chest is refracted away from the normal when the light enters the air, the apparent depth of the image is less than the actual depth.

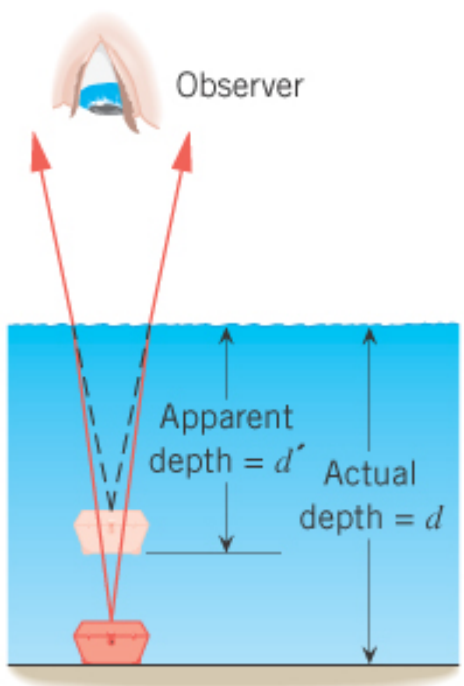


(a)

Simpler case -- look directly above the object.

Apparent depth, observer directly above object

$$d' = d\left(\frac{n_2}{n_1}\right)$$



(b)

n_1 -- medium of object n_2 -- medium of observer

Example. On the Inside Looking Out

A swimmer is under water and looking up at the surface. Someone holds a coin in the air, directly above the swimmer's eyes at a distance of 50 cm above the water. Find the apparent height of the coin as seen by the swimmer (assume n = 1.33 for water).

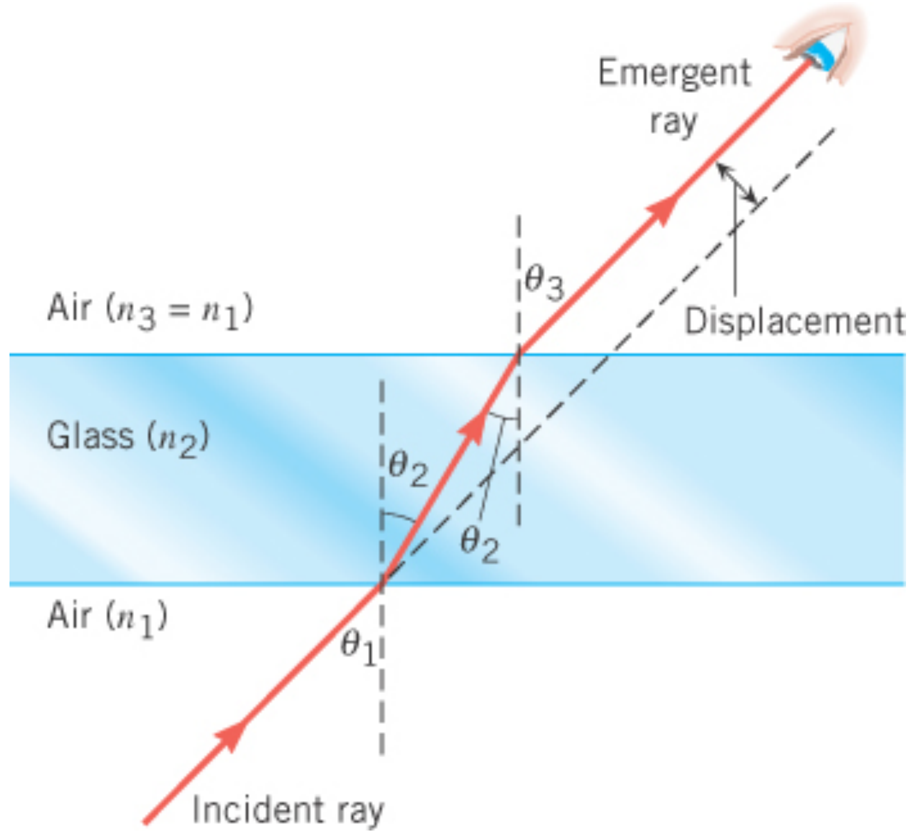
Use the equation $$d' = d\left(\frac{n_2}{n_1}\right)$$

In this case, d' will be the apparent height of the coin, d is the actual height above the water, n_1 = 1.00 for air (object), and n_2 = 1.33 for water (the observer),

$$d' = 50\left(\frac{1.33}{1.00}\right) = 66.5\ cm$$ ➔ greater than the actual height

THE DISPLACEMENT OF LIGHT BY A SLAB OF MATERIAL

When a ray of light passes through a pane of glass that has parallel surfaces and is surrounded by air, the emergent ray is parallel to the incident ray, $\theta_3 = \theta_1$, but is displaced from it.



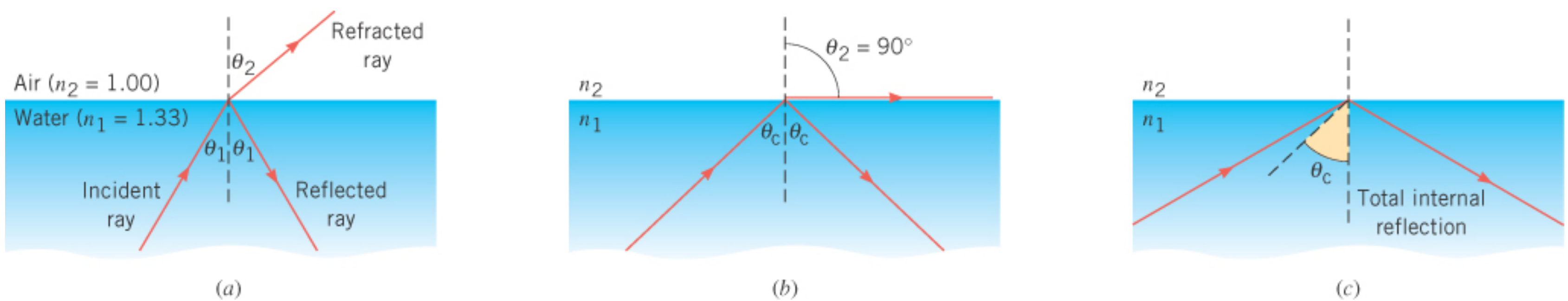
1st interface: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

2nd interface: $n_2 \sin\theta_2 = n_1 \sin\theta_3$

➔ $n_1 \sin\theta_1 = n_1 \sin\theta_3$ ➔ $\theta_3 = \theta_1$

Total Internal Reflection

When light passes from a medium of larger refractive index into one of smaller refractive index, the refracted ray bends away from the normal.



$n_1 \sin \theta_1 = n_2 \sin \theta_2$

$n_1 \sin \theta_c = n_2 \sin 90^o$

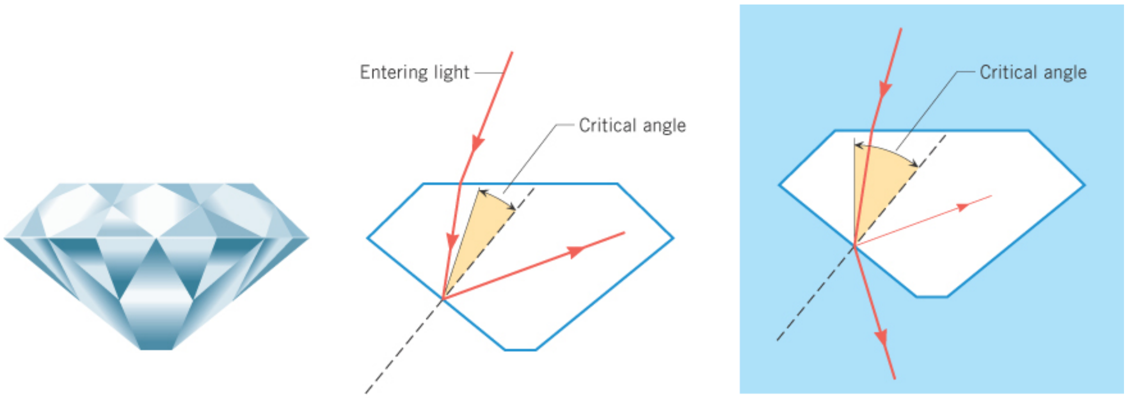
For $\theta_1 > \theta_c$ the ray is totally reflected

Critical angle $$\sin\theta_c = \frac{n_2}{n_1} \qquad n_1 > n_2$$

For this water/air interface: $\sin\theta_c = \dfrac{1.00}{1.33} = 0.752 \rightarrow \theta_c = 48.8^o$

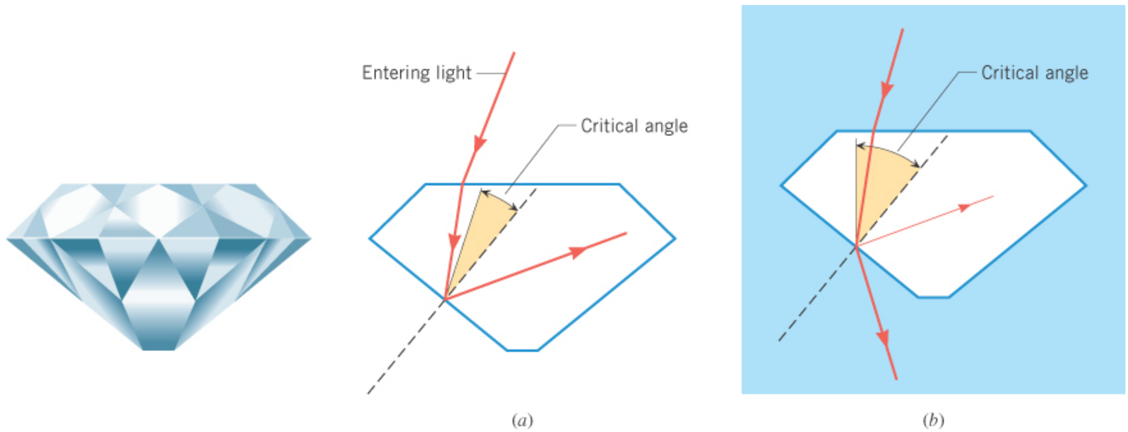
Example. **Total Internal Reflection**

A beam of light is propagating through diamond and strikes the diamond-air interface at an angle of incidence of 28 degrees. (a) Will part of the beam enter the air or will there be total internal reflection? (b) Repeat part (a) assuming that the diamond is surrounded by water.



(a) (b)

Total Internal Reflection



Incident angle in diamond = θ_1 = 28°

(a) Diamond in air: $$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{2.42}\right) = 24.4^\circ$$

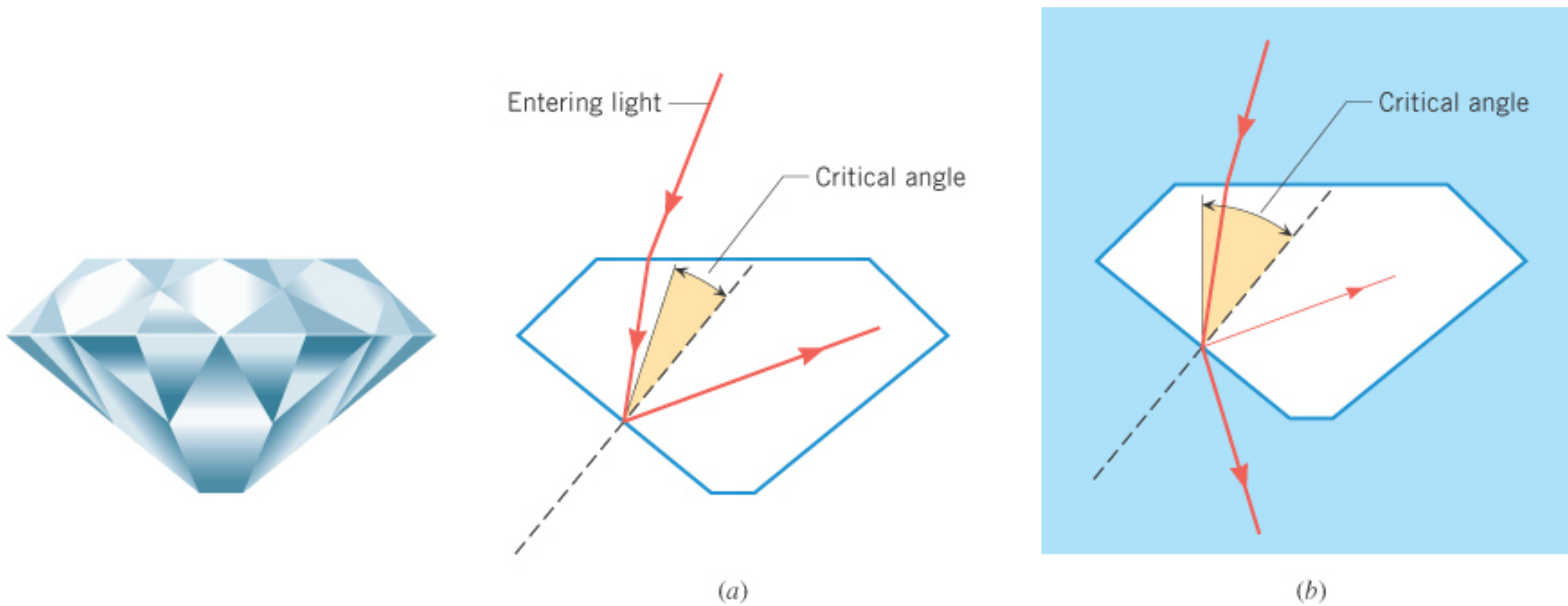
$\boldsymbol{\theta_1 > \theta_c}$ ➔ ray is totally reflected back into the diamond

(b) Diamond in water: $$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.33}{2.42}\right) = 33.3^\circ$$

$\theta_1 < \theta_c$ ➔ some light is reflected back into the diamond and some light is transmitted into the water

Conceptual Example. The Sparkle of a Diamond

The diamond is famous for its sparkle because the light coming from it glitters as the diamond is moved about. Why does a diamond exhibit such brilliance? Why does it lose much of its brilliance when placed under water?



As seen in the last example, θ_c is relatively small for the diamond in air so much of the light incident on its back surface reflects back through the top of the diamond, making it sparkle. In water θ_c is larger so less light is reflected through the top, reducing its sparkle.

Total internal reflection at a glass-air interface.

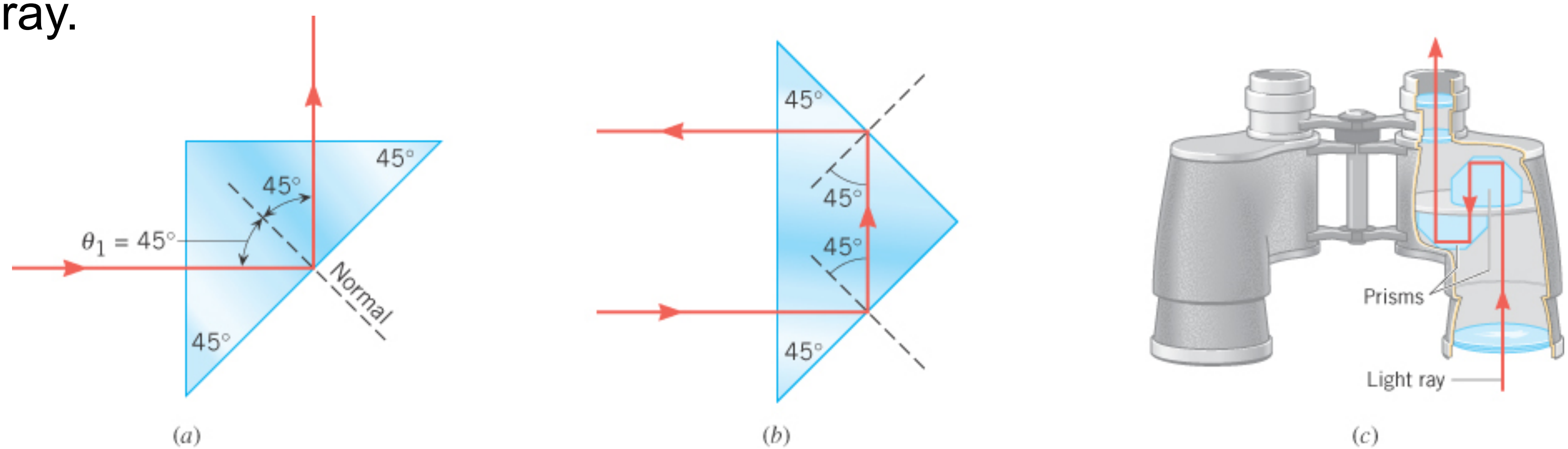
Since n = 1.5 for glass, at a glass-air interface the critical angle is

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.5} = 0.667 \quad \Rightarrow \quad \theta_c = 42^o$$

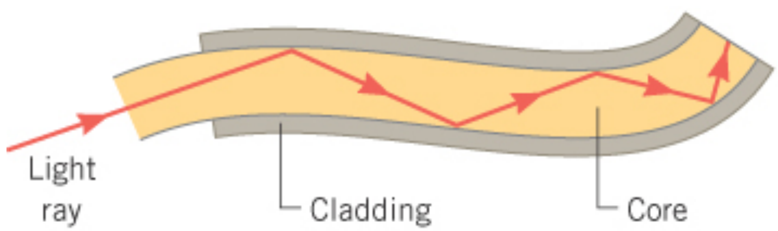
This can be used to turn a ray of light through an angle of 90° or 180° with total internal reflection using a prism of glass and keeping θ_1 = 45°

➔ useful in the design of optical instruments.

Two prisms, each reflecting the light twice by total internal reflection, are sometimes used in binoculars to produce a lateral displacement of a light ray.



Total internal reflection in optical fibers.



(a)

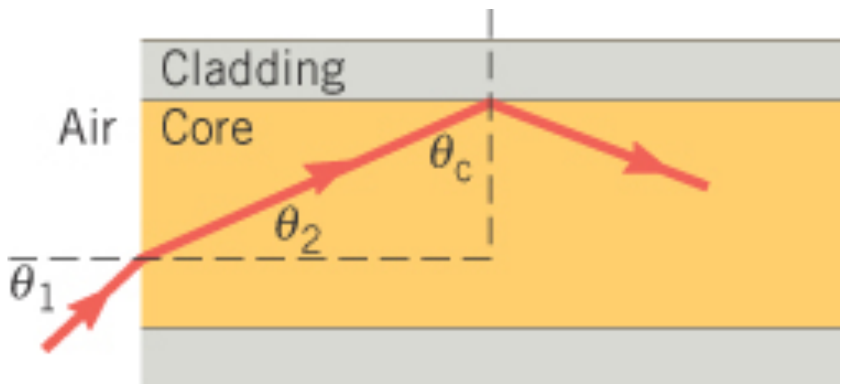

(b)

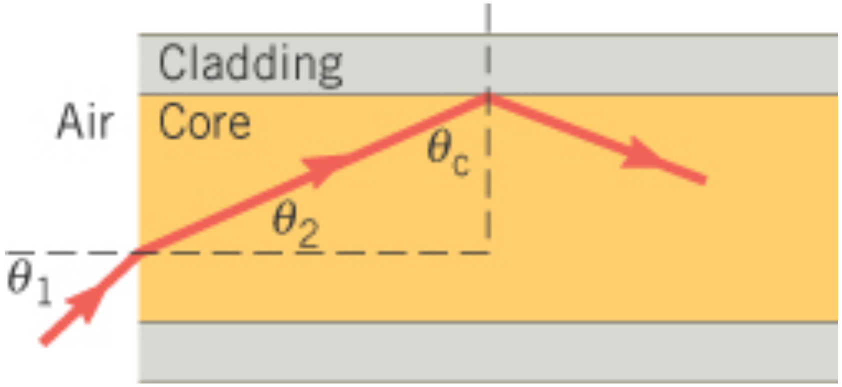
Light can travel with little loss in a curved optical fiber made of glass or plastic (“**light pipe**”) because the light is totally reflected whenever it strikes the core-cladding Interface and since the absorption of light by the core itself is small.

Using optical fibers, light can be piped from one place to another for many applications, e.g. telecommunications.

Example. An optical fiber consists of a core made of flint glass (n_{flint} = 1.667) surrounded by a cladding made of crown glass (n_{crown} = 1.523). A ray of light in air enters the fiber at an angle θ_1 with respect to the normal.

What is θ_1 if this light also strikes the core-cladding interface at an angle that just barely exceeds the critical angle?





$n_{cladding} = n_{crown} = 1.523$

$n_{core} = n_{flint} = 1.667$

Strategy:

- find θ_c using the known n_{core} and $n_{cladding}$
- find θ_2 using θ_c and geometry
- find θ_1 from θ_2 , n_{core} and Snell's Law

$$\sin\theta_c = \frac{n_{cladding}}{n_{core}} = \frac{1.523}{1.667} = 0.9136 \quad \Rightarrow \quad \theta_c = 66.01^o$$

From figure, since right triangle ➔ $\theta_2 = 90^o - \theta_c = 90^o - 66.01^o = 23.99^o$

Using Snell's Law at the air-core interface ➔ $n_{air} \sin\theta_1 = n_{core} \sin\theta_2$

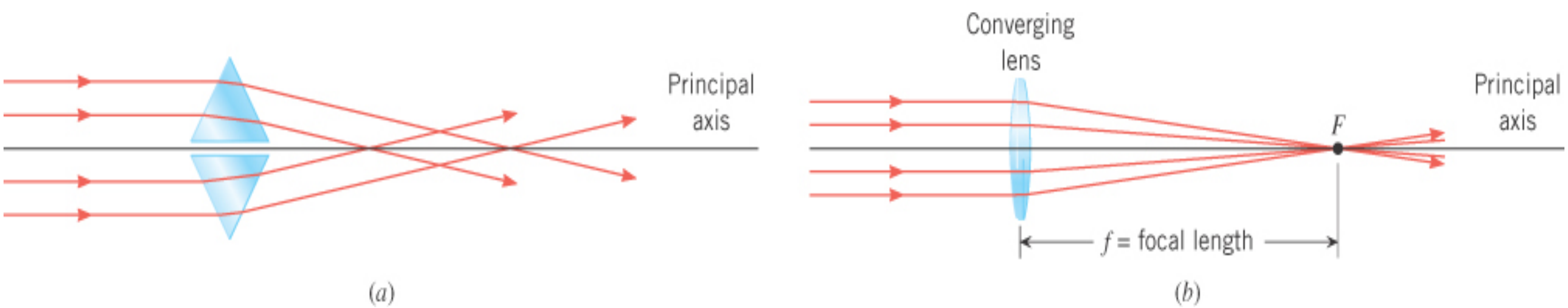
$(1.000) \sin\theta_1 = (1.667) \sin 23.99^o$ ➔ $\sin\theta_1 = 0.6778$ ➔ $\theta_1 = 42.67^o$

Lenses

Converging and diverging lenses.

Lenses refract light in such a way that an image of the light source is formed.

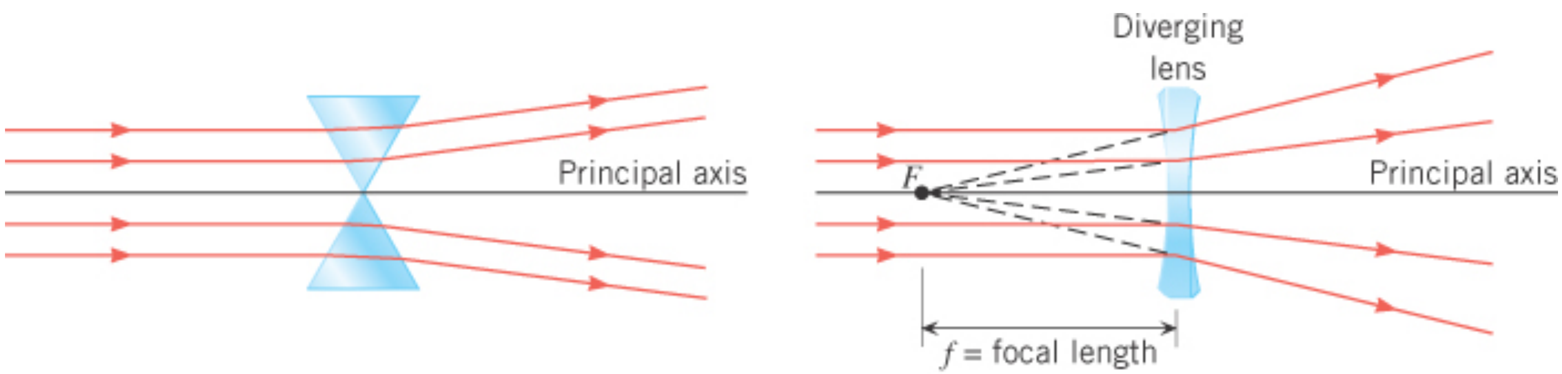
With a **converging lens**, paraxial rays that are parallel to the principal axis converge to the focal point, F. The focal length, f, is the distance between F and the lens.



Two prisms can bend light toward the principal axis acting like a crude converging lens but cannot create a sharp focus.

Lenses

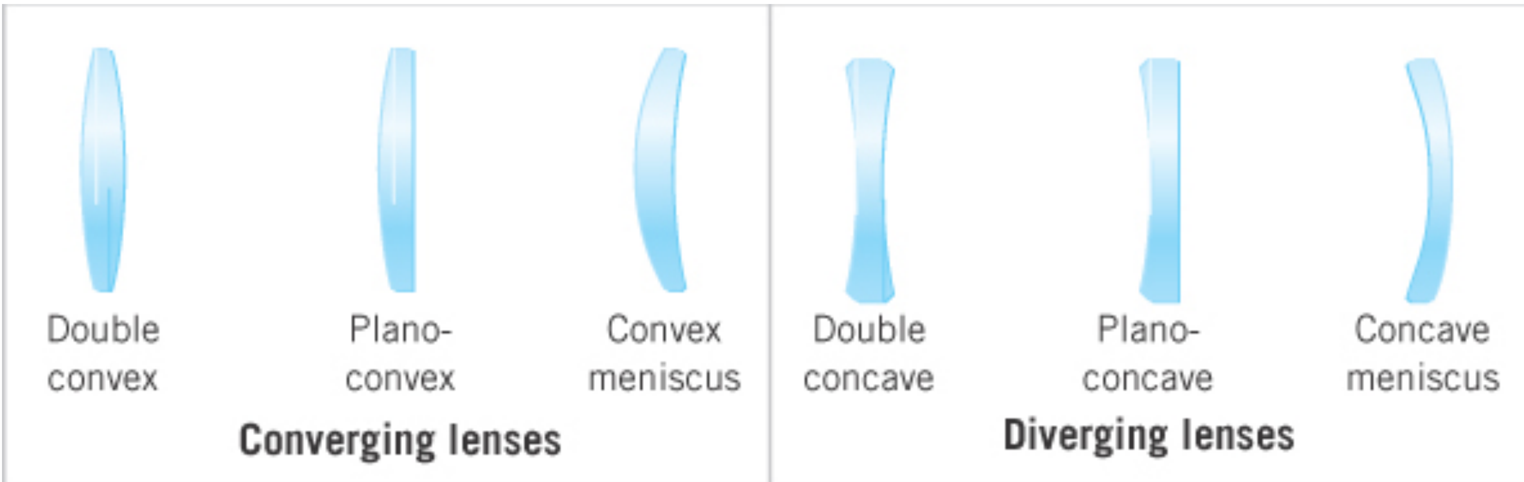
With a **diverging lens**, paraxial rays that are parallel to the principal axis appear to originate from the focal point, F. The focal length, f, is the distance between F and the lens.



(a) (b)

Two prisms can bend light away from the principal axis acting like a crude diverging lens, but the apparent focus is not sharp.

Lenses



Converging and diverging lens come in a variety of shapes depending on their application.

We will assume that the thickness of a lens is small compared with its focal length ➔ **Thin Lens Approximation**

The Formation of Images by Lenses

RAY DIAGRAMS. Here are some useful rays in determining the nature of the images formed by converging and diverging lens.

Since lenses pass light through them (unlike mirrors) it is useful to draw a focal point on each side of the lens for ray tracing.

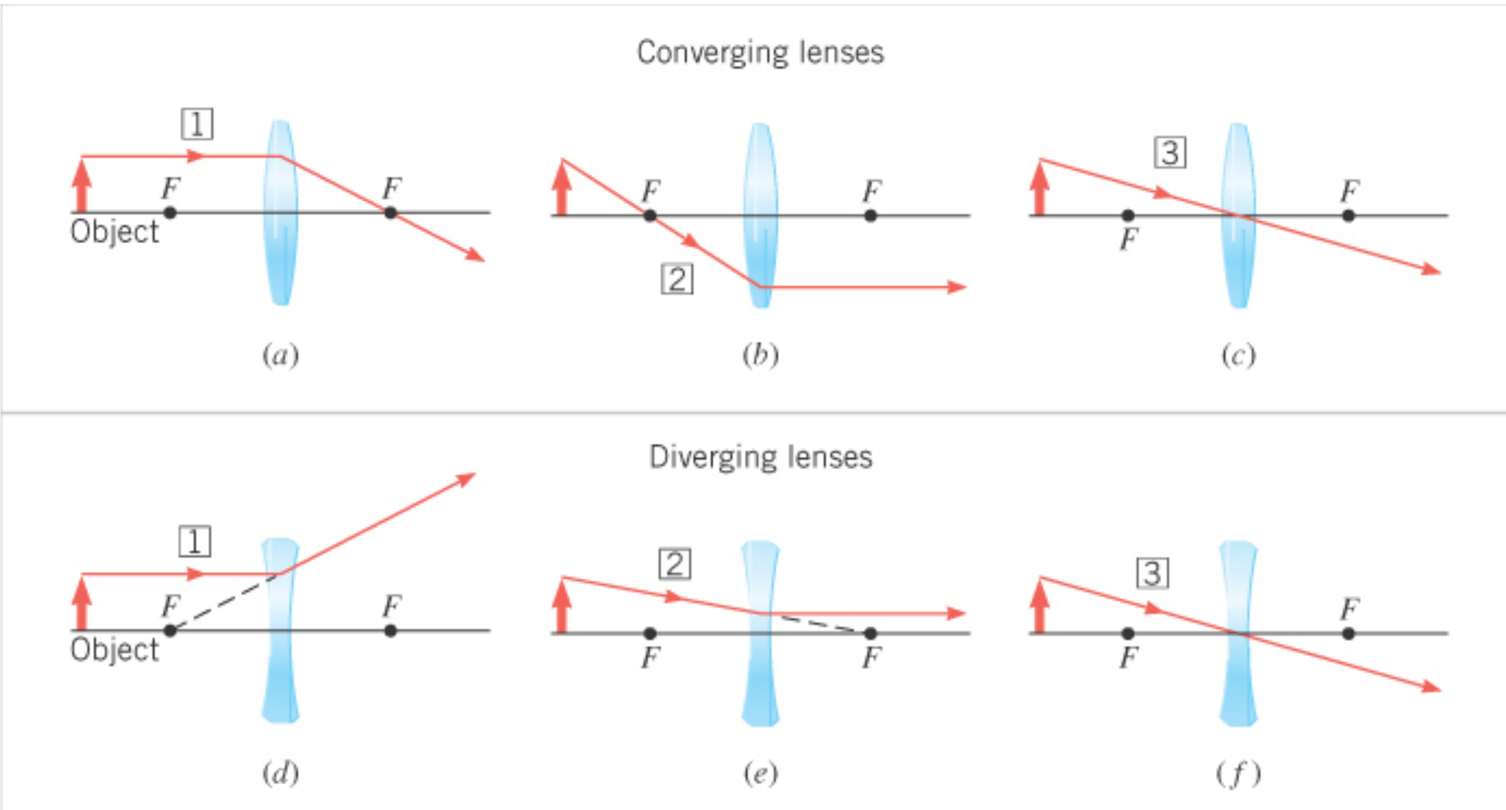
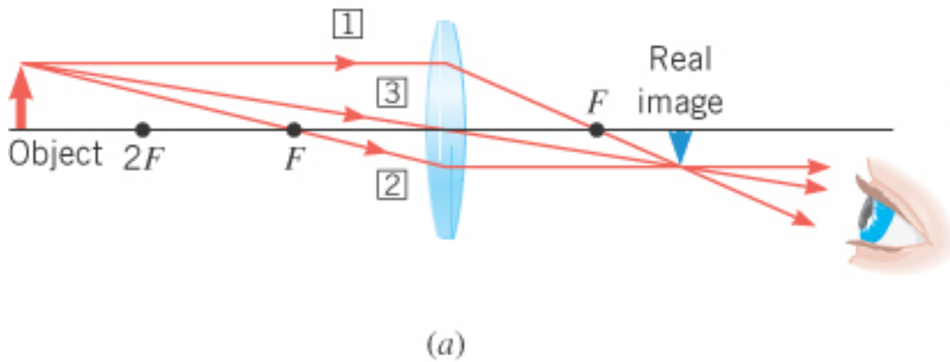
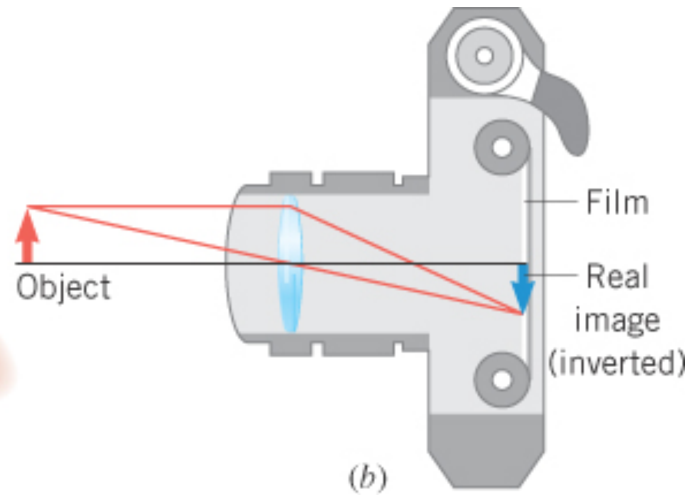


IMAGE FORMATION BY A CONVERGING LENS

$$d_o > 2f$$



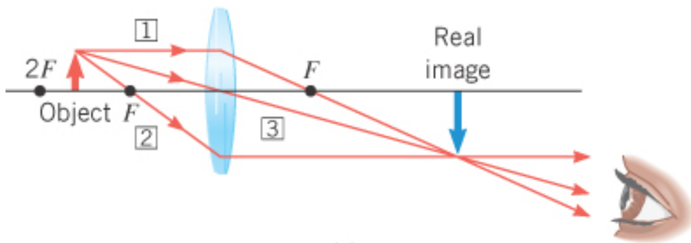


When the object is placed further than twice the focal length from the lens, the real image is inverted and smaller than the object.

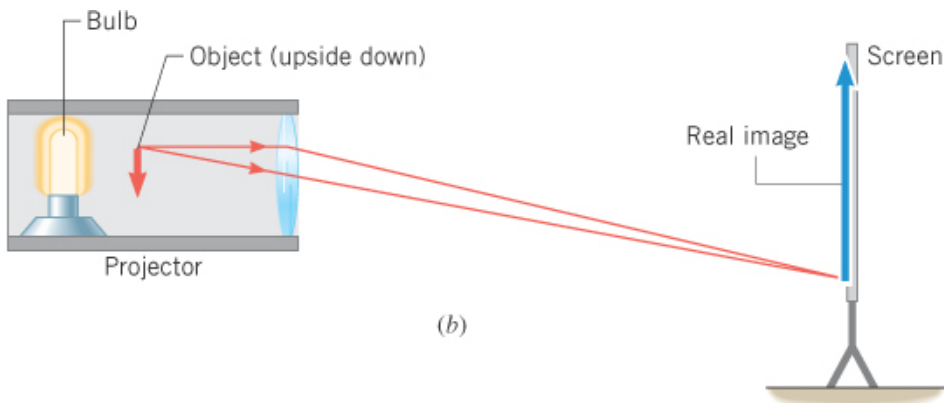
This is the configuration for a camera. The focal length of the lens system of a camera must be adjusted for a particular object distance so that the image distance is at the location of the film and thus the real image on the film is sharp (focused).

IMAGE FORMATION BY A CONVERGING LENS

$$2f > d_o > f$$



(a)



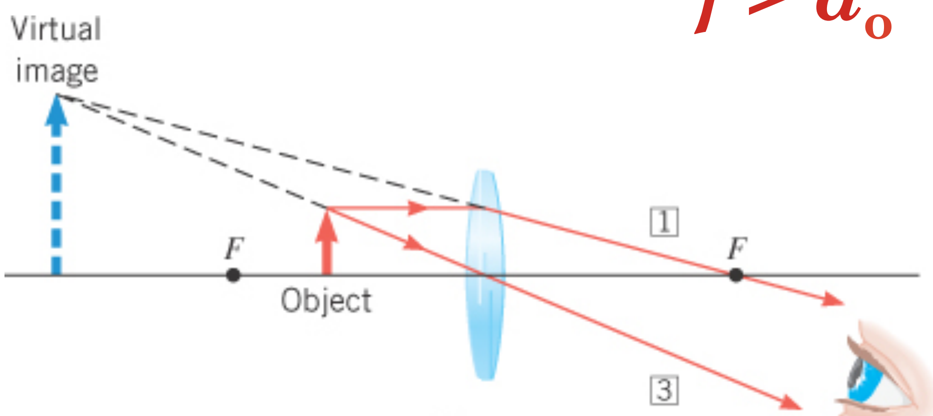
(b)

When the object is placed between F and 2F, the real image is inverted and larger than the object.

This is the configuration for a projector. Since you normally want the real image on the screen to be upright, the object (film or slide) is placed upside down in the projector.

IMAGE FORMATION BY A CONVERGING LENS

$$f > d_o$$



(a)

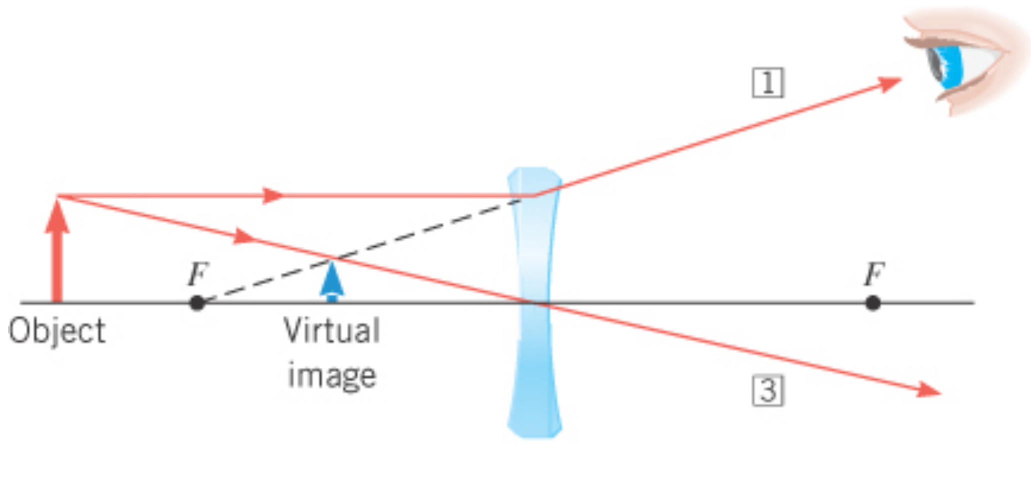
(b)

When the object is placed between F and the lens, the virtual image is upright and larger than the object.

This is the configuration for a magnifying glass. The magnifying glass must clearly be positioned so that the object distance is less than its focal length

IMAGE FORMATION BY A DIVERGING LENS

all d_o



(a)

(b)

A diverging lens always forms an upright, virtual, diminished image.