# *Chapter 26*

# ***Special Relativity***

## *Events and Inertial Reference Frames*

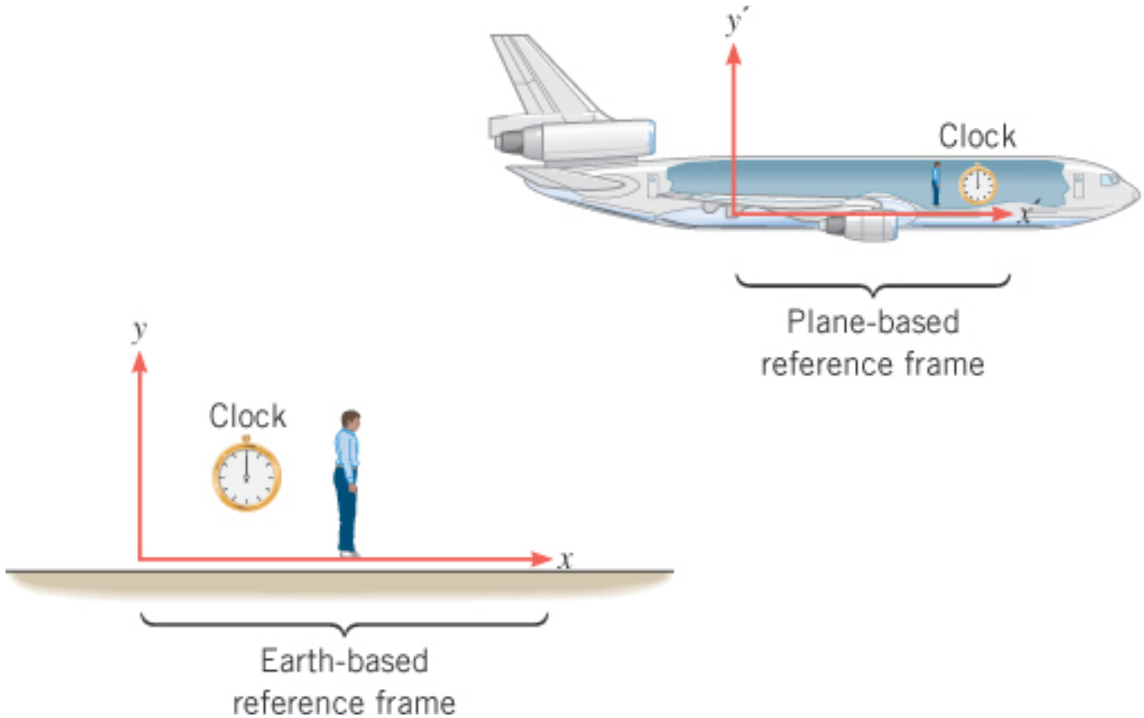
An ***event*** is a physical "happening" that occurs at a certain place and time.

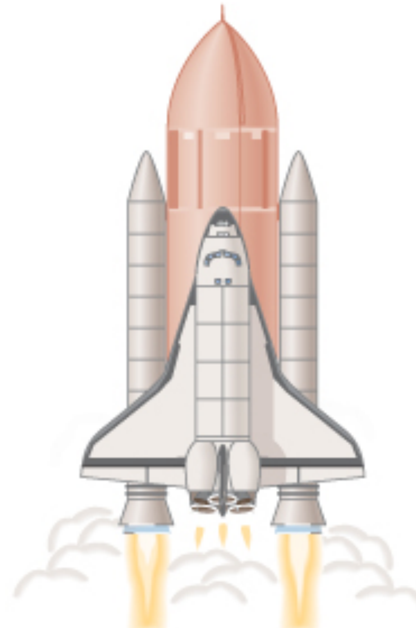
To record the event, each observer uses a ***reference frame*** that consists of a coordinate system and a clock.

Each observer is at rest relative to her own reference frame.

An ***inertial reference frame*** is one in which Newton's law of inertia is valid.

# Events and Inertial Reference Frames



In this example, the event is the space shuttle lift off.

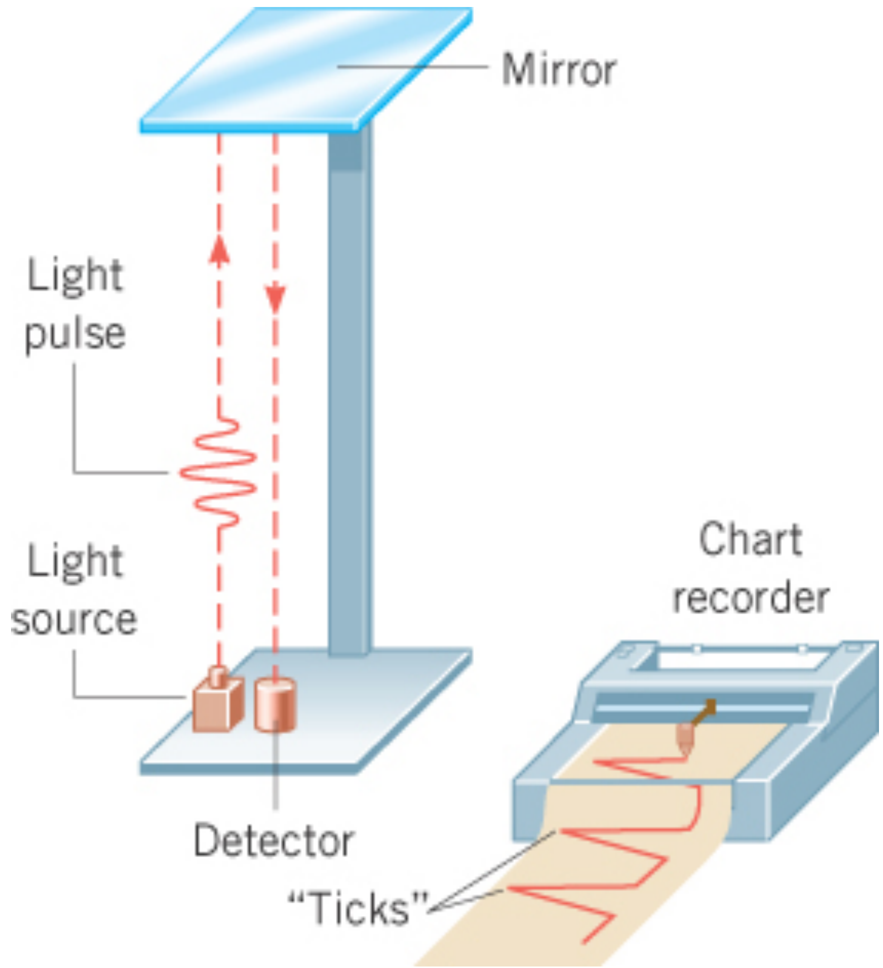
## THE POSTULATES OF EINSTEIN'S SPECIAL RELATIVITY

***1. The Relativity Postulate.*** The laws of physics are the same in every inertial reference frame.

***2. The Speed of Light Postulate.*** The speed of light in a vacuum, measured in any inertial reference frame, always has the same value of $c$, no matter how fast the source of light and the observer are moving relative to one another.

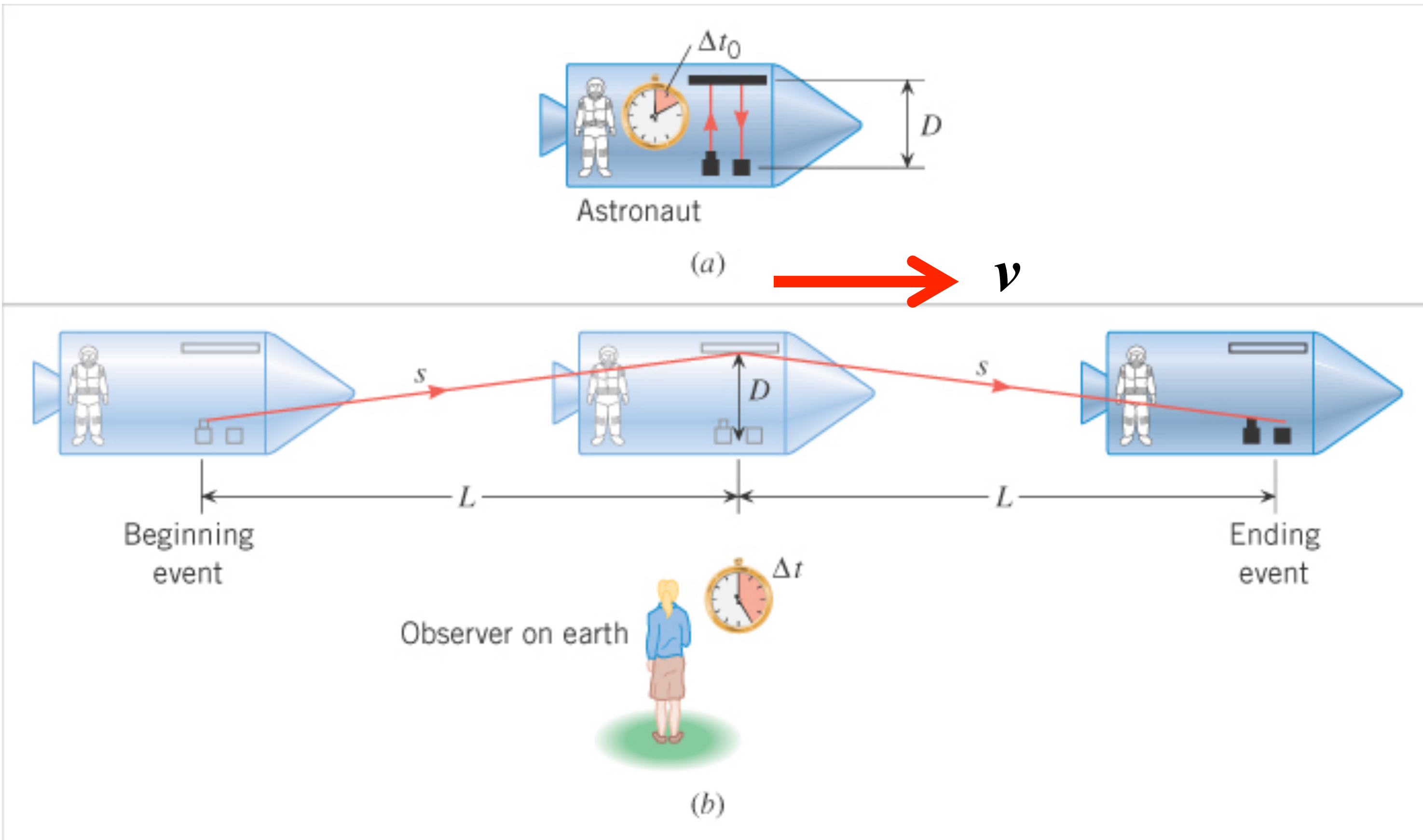
## TIME DILATION

***A light clock***



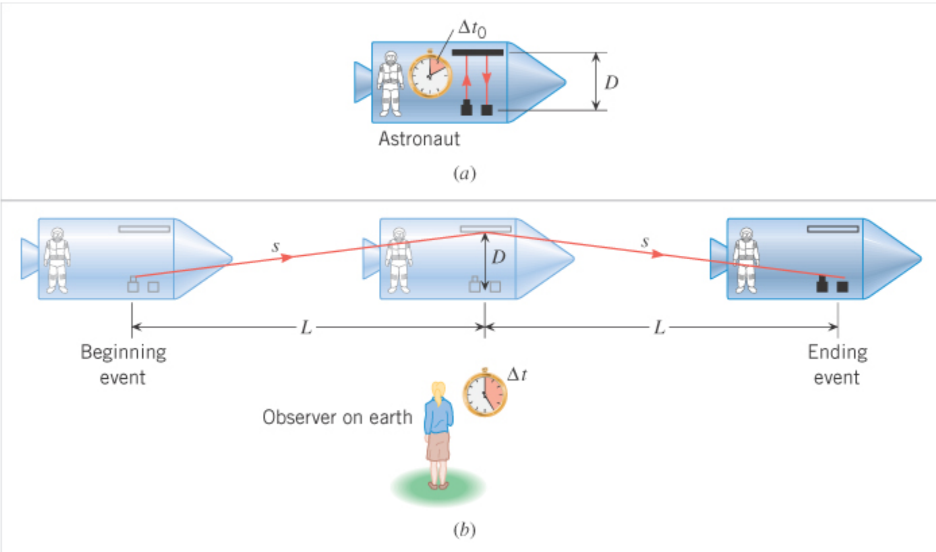
## *The Relativity of Time: Time Dilation*

Consider the light clock placed into spaceship moving at velocity, $v$, w.r.t. Earth




An observer on the Earth sees the light pulse travel a greater distance between ticks.

## The Relativity of Time: Time Dilation




**Time interval for astronaut**

$$\Delta t_0 = \frac{2D}{c}$$

**Time interval for Earth observer**

$$\Delta t = \frac{2s}{c} = \frac{2\sqrt{D^2 + L^2}}{c}, \quad 2L = v\Delta t$$

$$D = \frac{c\Delta t_0}{2}, \quad L = \frac{v\Delta t}{2}$$

$$\frac{c\Delta t}{2} = \sqrt{D^2 + L^2} = \sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{v\Delta t}{2}\right)^2 \quad \Rightarrow \quad \left(c^2 - v^2\right)\left(\Delta t\right)^2 = c^2\left(\Delta t_0\right)^2$$

**Time dilation**

$$\left(\Delta t\right)^2 = \frac{c^2\left(\Delta t_0\right)^2}{c^2 - v^2} = \frac{\left(\Delta t_0\right)^2}{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_0 \qquad \left(\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1\right)$$

## PROPER TIME INTERVAL

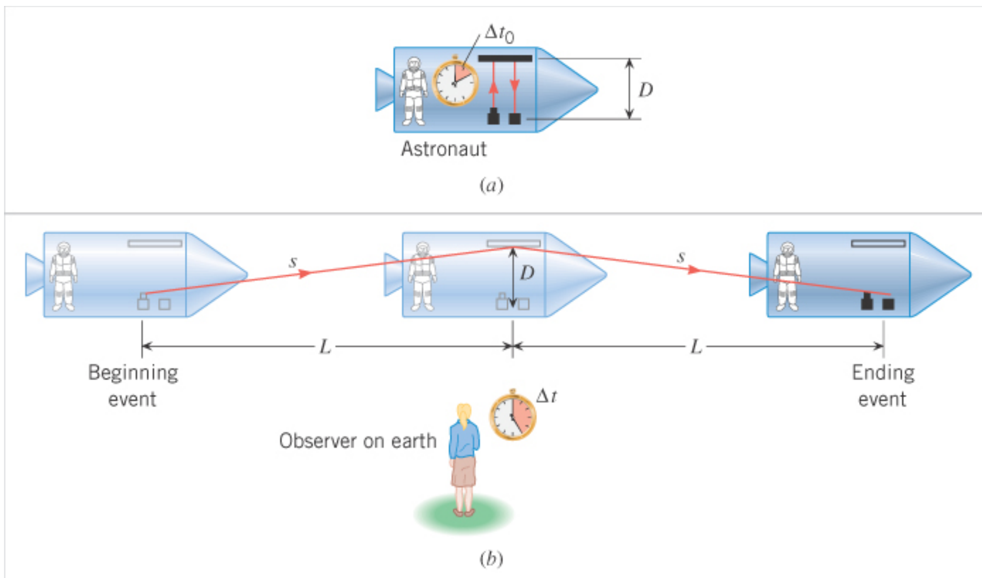
The time interval measured at rest with respect to the clock is called the ***proper time interval***.

In general, the proper time interval between events is the time interval measured by an observer who is at rest relative to the events.

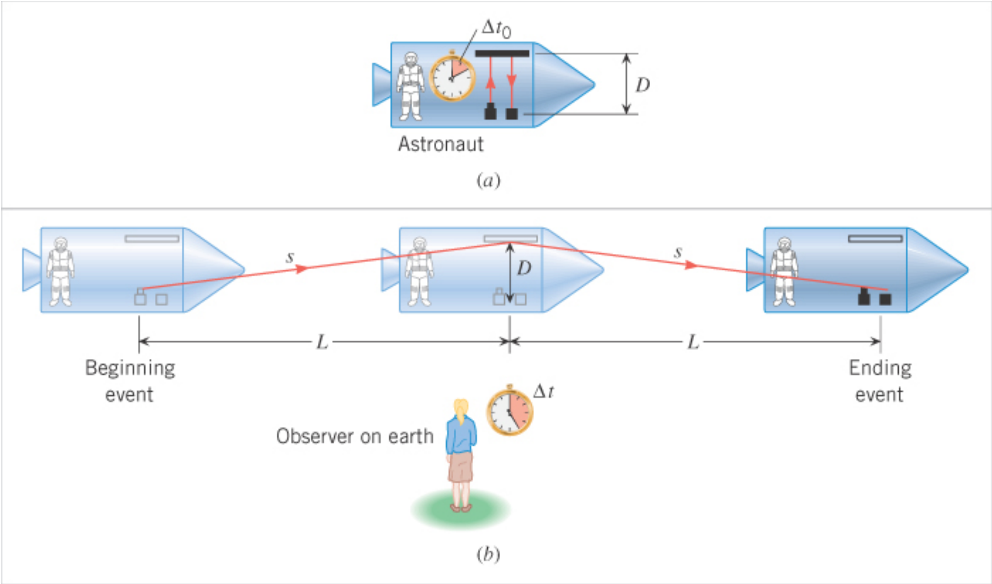
***Proper time interval*** $\Delta t_o$

## *Example:* Time Dilation

The spacecraft is moving past the earth at a constant speed of 0.92 times the speed of light. the astronaut measures the time interval between ticks of the spacecraft clock to be 1.0 s. What is the time interval that an earth observer measures?



## The Relativity of Time: Time Dilation




$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - v^2/c^2}} = \frac{1.0\,\text{s}}{\sqrt{1 - (0.92c/c)^2}} = 2.6\,\text{s}$$

**Example: Time dilation of a moving neutral kaon.**

A neutral kaon ($K^0_{short}$) is a subatomic particle with a mass about half that of the proton which has a mean lifetime when at rest, i.e. in its rest frame or its proper time, of 8.95 x $10^{-11}$ s. Such a kaon is produced in a proton on proton collision at the Large Hadron Collider with a speed 0.995 c. Find a) the lifetime of this kaon in the laboratory frame, i.e. to an observer standing in the lab, and b) how far it travels from its production point before it decays. c) repeat a) and b) for a speed 0.300 c.

$$a)\ proper\ time\ of\ kaon: \Delta t_0 = 8.95\times 10^{-11} s, \quad v = 0.995\, c$$

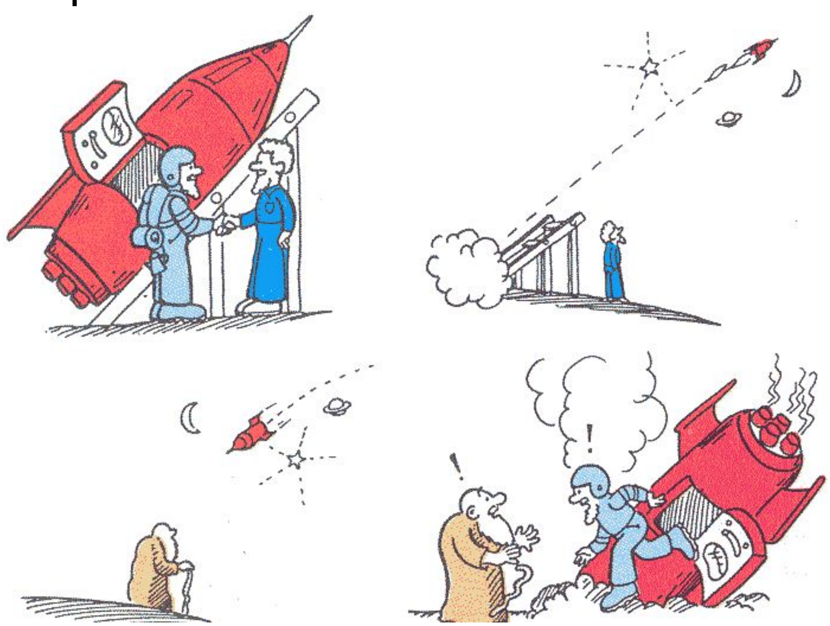
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{8.95\times 10^{-11}}{\sqrt{1-\frac{(0.995\, c)^2}{c^2}}} = 10.0\times 8.95\times 10^{-11} = 8.95\times 10^{-10} s$$

$$b)\ x = v\Delta t = (0.995)(3.00\times 10^{8})(8.95\times 10^{-10}) = 0.267\ m = 26.7\ cm$$

$$c)\ v = 0.300\, c \quad \Rightarrow \Delta t = \gamma\Delta t_0 = 1.05\times 8.95\times 10^{-11} = 9.38\times 10^{-11} s$$

$$x = v\Delta t = (0.300)(3.00\times 10^{8})(9.38\times 10^{-11}) = 0.0084\ m = 0.84\ cm$$

**Example: The Twin Paradox.** Twin brothers part company when one of the twins launches off in a spaceship for a trip to a star 30 light-years away. The ship traveling at a speed 0.99 c reaches the star, turns around and returns to Earth. Since the spaceship is traveling near c, to the Earth twin the trip will take 2x30 years=60 years, whereas for the astronaut twin as calculated in the Earth frame, the trip will take $\Delta t_0 = \Delta t/\gamma = 60/7.1 = 8.4$ years. When the twins meet up again, the twin who stayed on Earth is 60 years older than when the spaceship left whereas the astronaut twin is only 8.4 years older, as calculated. The astronaut twin is still surprised, since in the astronaut's frame, the Earth was moving with speed 0.99 c with respect to the spaceship's frame, so the astronaut expected his twin on Earth would be younger than him since time would pass more slowly in the moving frame of the Earth.
So the paradox is that both twins can't be correct.

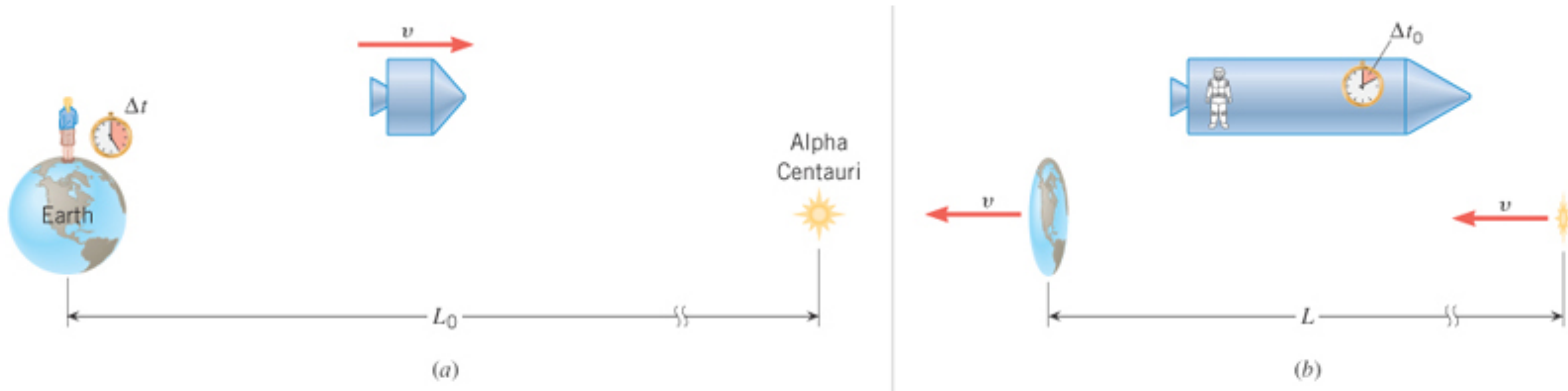


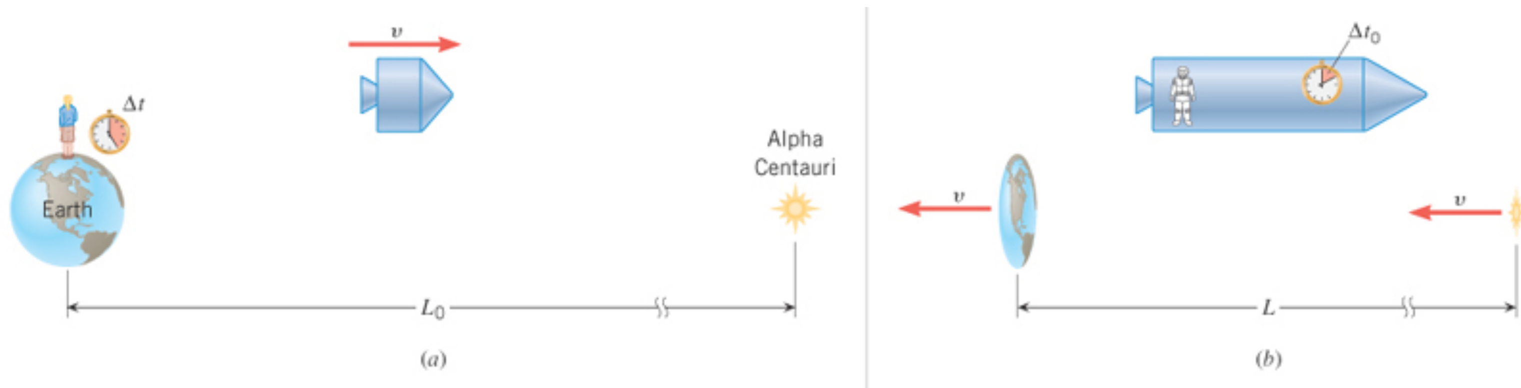
The resolution of the apparent paradox is that $\Delta t = \gamma\Delta t_0$ is only valid in an inertial frame, which the twin on Earth was in, so his calculation was correct. The twin in the spaceship was not in an inertial frame since he had to accelerate to leave Earth and then accelerate again at the star to turn around and return to Earth, so using $\Delta t = \gamma\Delta t_0$ is not valid for him.

## *The Relativity of Length: Length Contraction*

The shortening of the distance between two points is one example of a phenomenon known as ***length contraction***.

Consider a spacecraft traveling at velocity, $v$, w.r.t. Earth towards the star Alpha Centauri ($L_0$ = 4.3 light-years distance from Earth as measured from the Earth frame of reference).






Distance to Alpha Centauri determined by the Earth observer: $L_0 = v\Delta t$

Distance to Alpha Centauri determined by the astronaut: $L = v\Delta t_0$

$$\frac{L}{L_0} = \frac{v\Delta t_0}{v\Delta t} = \frac{\Delta t_0}{\Delta t} = \frac{\Delta t_0}{\gamma \Delta t_0} = \frac{1}{\gamma}$$

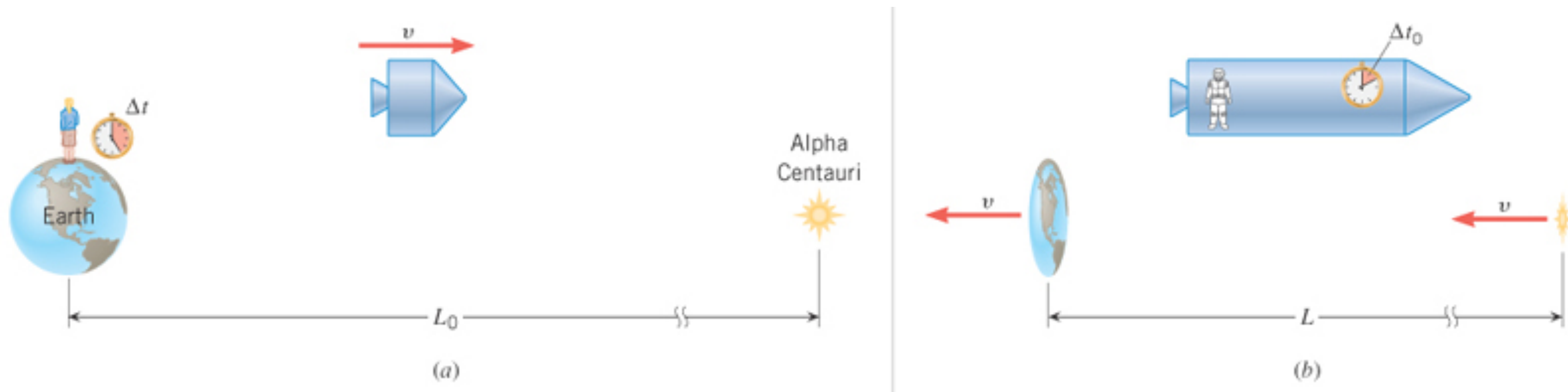
$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

***Length contraction***

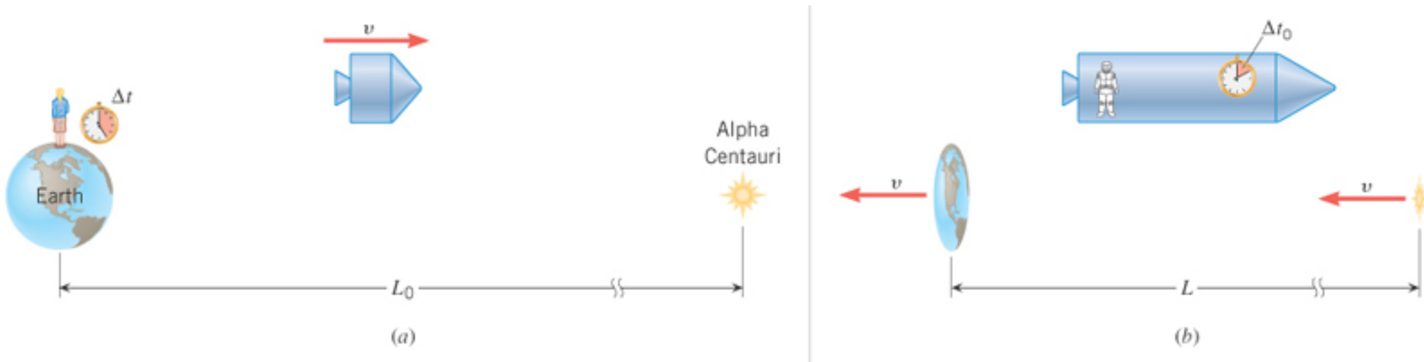
$$L_0 \equiv proper\ length$$

## *Example:* The Contraction of a Spacecraft

An astronaut, using a meter stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter to be 82 m and 21 m respectively. The spacecraft moves with a constant speed of 0.95*c* relative to the earth. What are the dimensions of the spacecraft, as measured by an observer on earth.



## *The Relativity of Length: Length Contraction*




$$L = L_o\sqrt{1-\frac{v^2}{c^2}} = (82\ \text{m})\sqrt{1-(0.95c/c)^2} = 26\ \text{m}$$

The diameter stays the same.