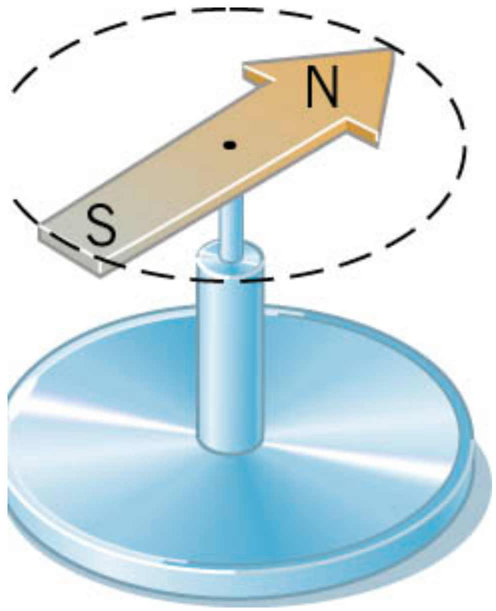
Chapter 20

Magnetic Forces and Magnetic Fields

Magnetic Fields

The most familiar example of magnetism for most people is a **magnet**.

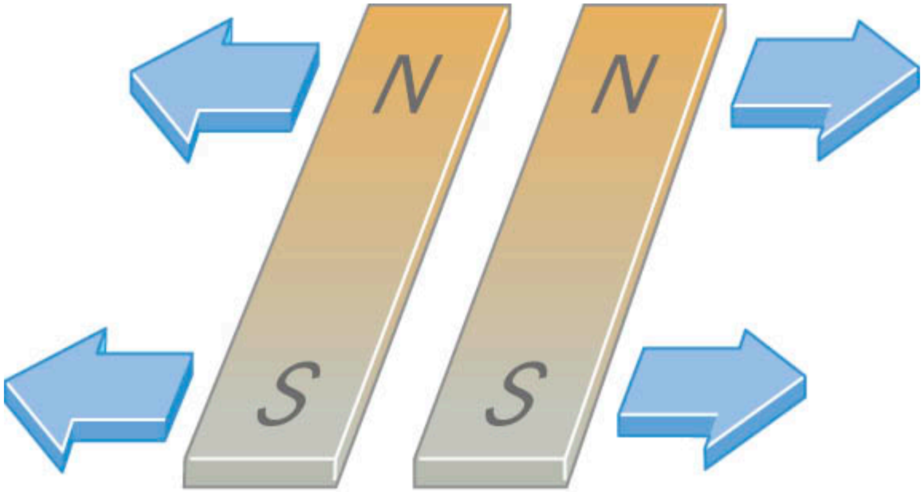
Every magnet has **two poles**, **North** and **South** --> called this since if the magnet is suspended, its North pole points more or less to the Earth's North.



The needle of a **compass** is a permanent magnet that has a north magnetic pole (N) at one end and a south magnetic pole (S) at the other.

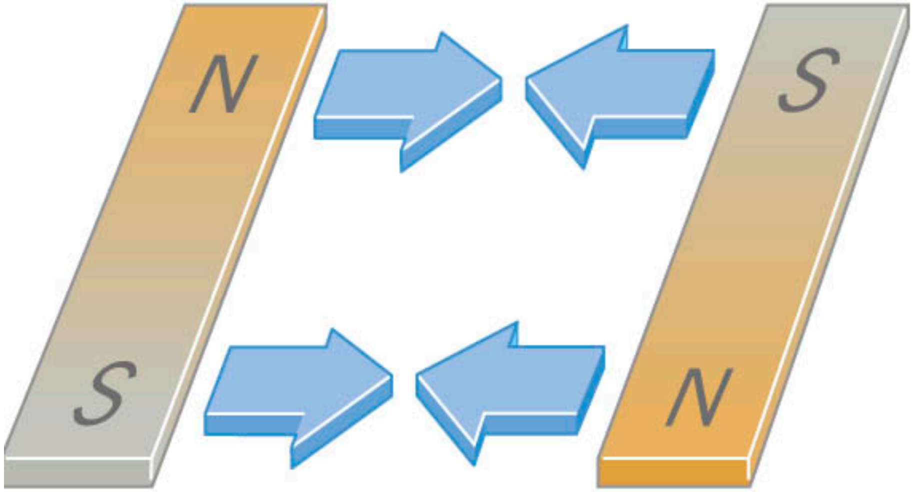
Magnetic Fields

The behavior of magnetic poles is similar to that of like and unlike electric charges.



Like poles repel

(*a*)



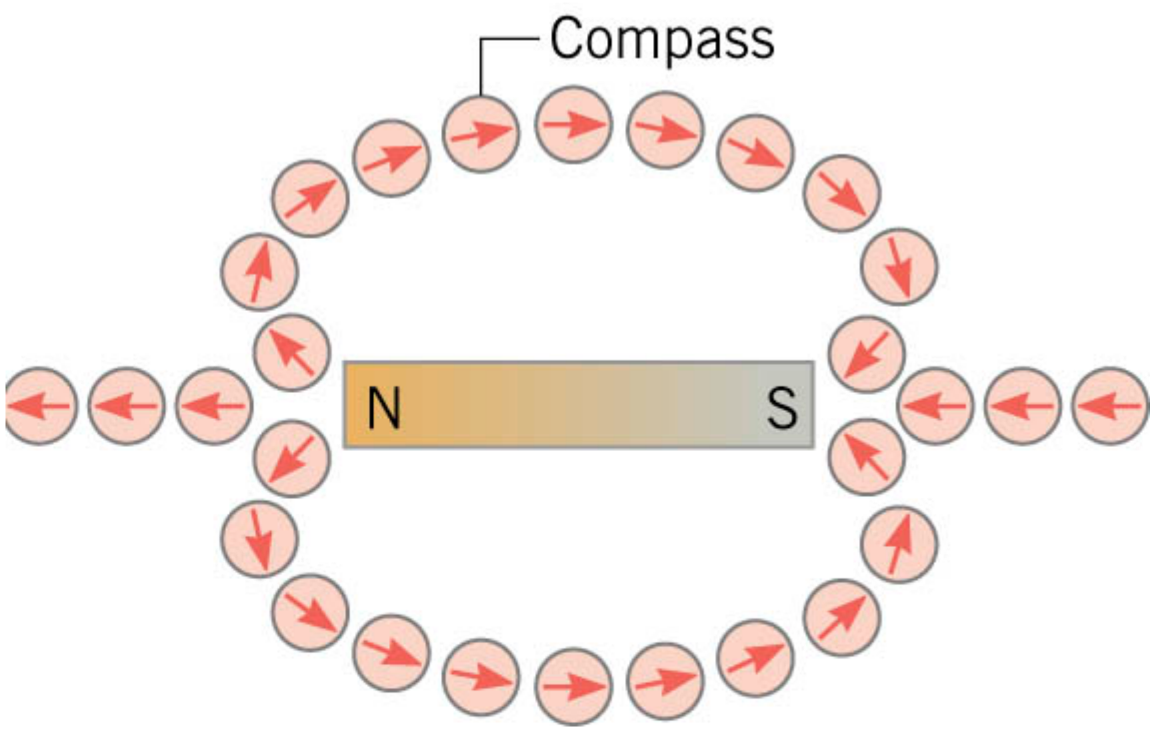
Unlike poles attract

(*b*)

Magnetic Fields

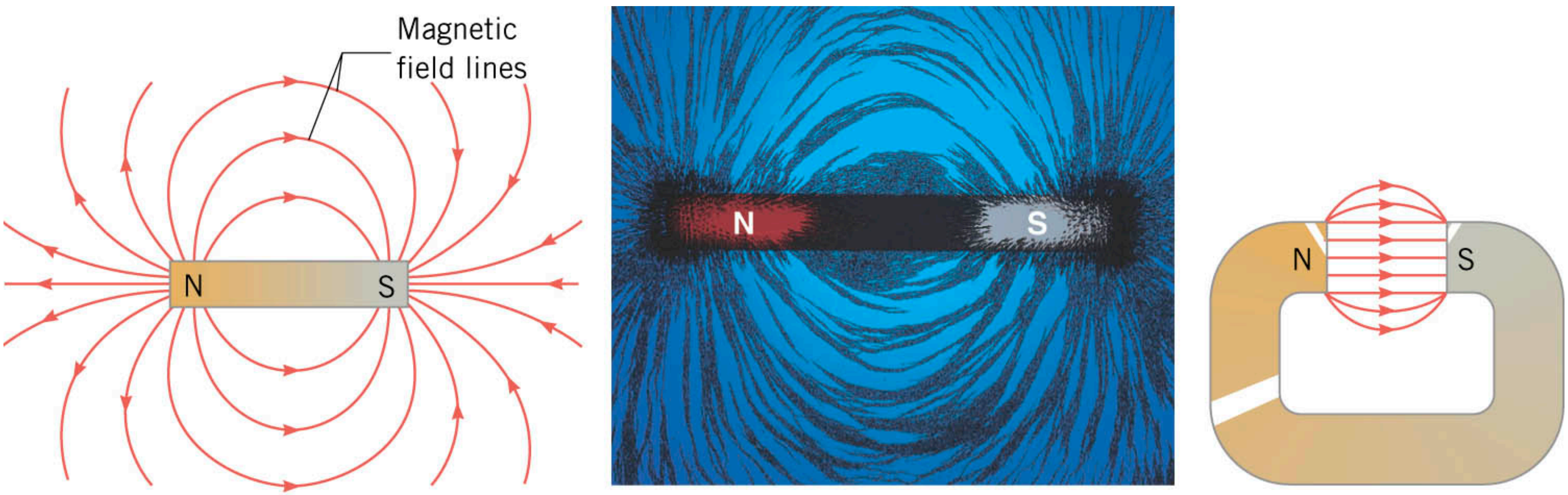
Since magnets can exert force at a distance like electric charge, in analogy with the electric field surrounding a charge we can associate a **magnetic field** surrounding a magnet.

The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point -- the compass acts analogously to a test charge mapping out an electric field.



Magnetic Fields

The magnetic field lines and pattern of iron filings in the vicinity of a bar magnet and the magnetic field lines in the gap of a horseshoe magnet.



(a) (b) (c)

Features of magnetic field lines:

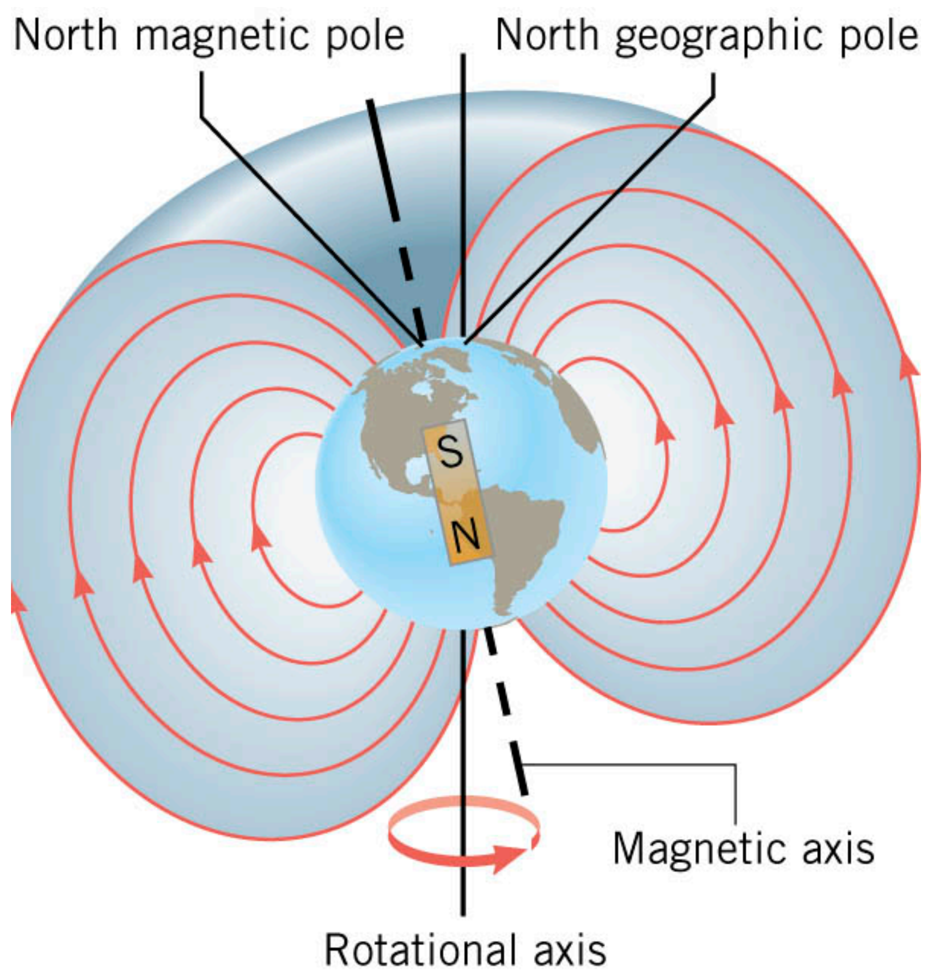
- They point out from the N-pole and point toward the S-pole.
- The direction of the magnetic field vector is tangent to them.
- Number of lines/area is proportional to the strength of the magnetic field.
- They form closed loops -- separate S or N poles do not exist in nature.

Magnetic Fields

The Earth behaves magnetically almost as if a bar magnet were located near its center. The axis of the fictitious bar magnet does not coincide with the Earth's rotational axis -- they are about 11.5^{o} apart. Magnetic North lies in northern Canada, not at the North Pole.

For the **Columbus area**, the compass needle points about 6.5^{o} West of true North (this angle is called the **angle of declination**).

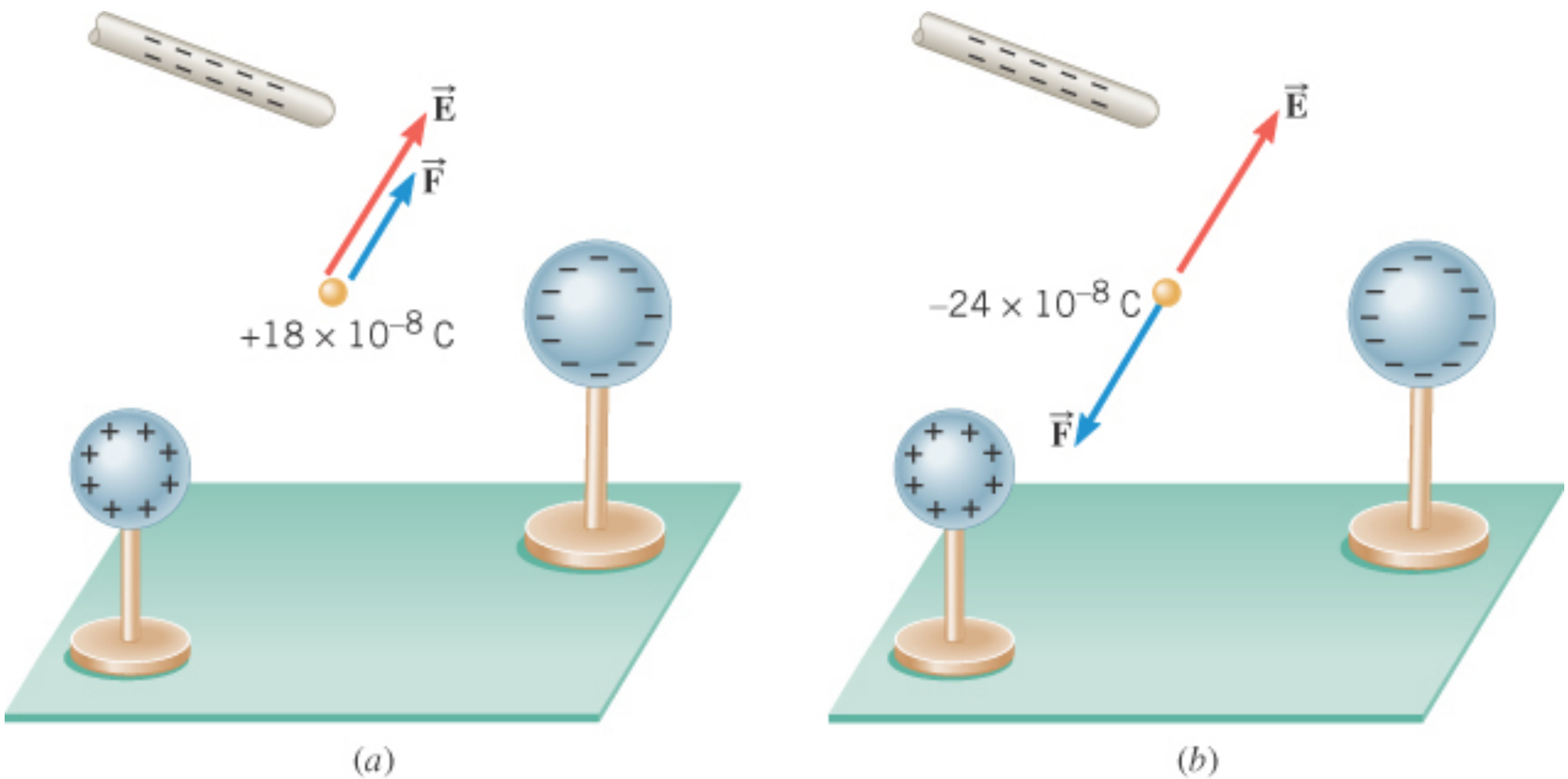
Note that Earth's magnetic field lines are not everywhere parallel to the surface. Near the North and South Poles they are almost perpendicular to the surface (**angle of dip** close to 90^{o}).



The Force That a Magnetic Field Exerts on a Charge

When a charge is placed in an electric field, it experiences a force, according to

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

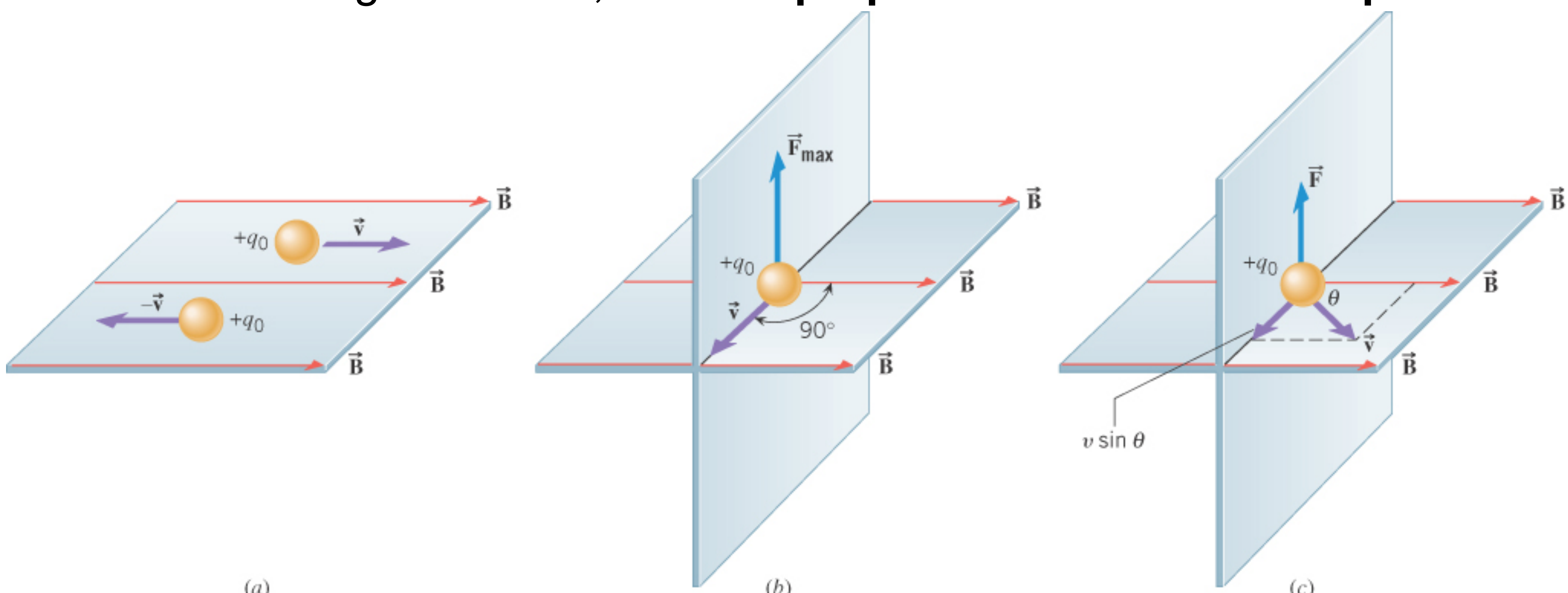


The Force That a Magnetic Field Exerts on a Charge

The following conditions must be met for a *charge* to experience a *magnetic force* when placed in a *magnetic field*:

1. The charge must be moving.
2. The velocity of the charge must have a component that is perpendicular to the direction of the magnetic field.

If there is a magnetic force, it will be **perpendicular to the v - B plane**



No magnetic force since **v** || **B**

Maximum $F = F_{max}$ since $\mathbf{v} \perp \mathbf{B}$

$0 < F < F_{max}$ since $0 < \theta < 90^o$

DEFINITION OF THE MAGNETIC FIELD

The **magnitude** of the magnetic field at any point in space is defined as

$$B = \frac{F}{|q_o|(v \sin\theta)}$$

where $\boldsymbol{F}$ is the magnetic force on a test charge q_0 moving with velocity $\boldsymbol{v}$ in a magnetic field **B**, and where θ ($0<\theta<180^o$) is the angle between $\boldsymbol{v}$ and $\boldsymbol{B}$.

SI Unit of Magnetic Field: $\frac{\text{newton} \cdot \text{second}}{\text{coulomb} \cdot \text{meter}} = 1 \text{ tesla (T)}$

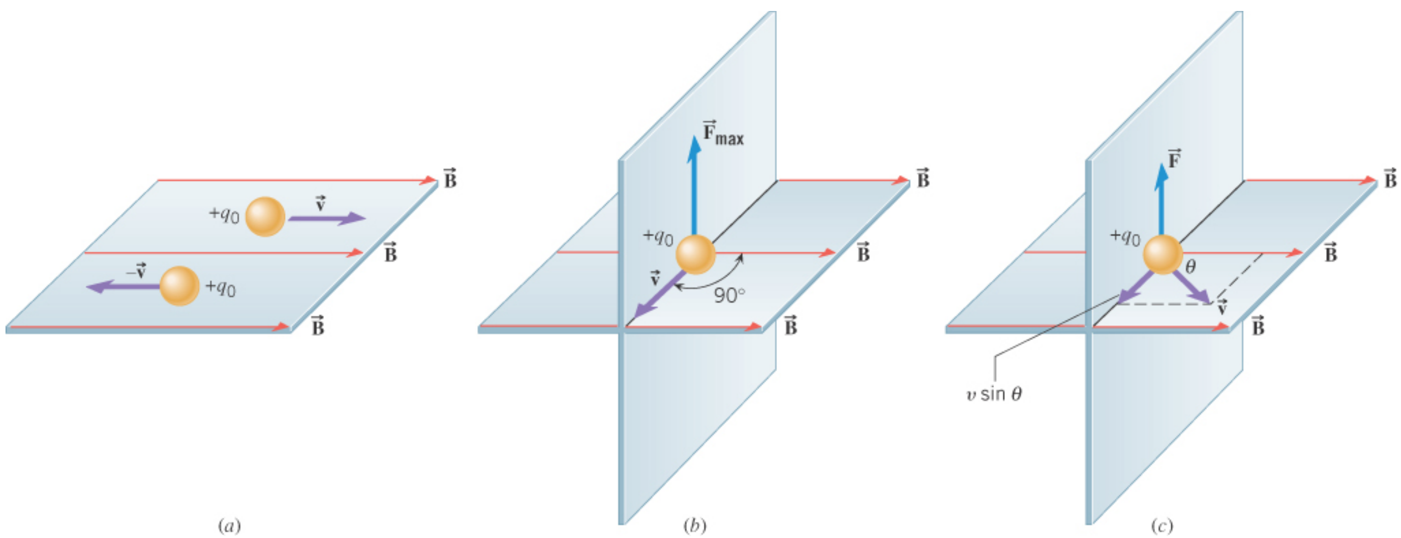
The non-SI unit, gauss, is also often used:

$$1 \text{ gauss} = 10^{-4} \text{ tesla}$$

(the Earth's magnetic field at the surface is ~ 0.5 gauss)

Example: **Magnetic Forces on Charged Particles**

A proton in a particle accelerator has a speed of 5.0×10^6 m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes and angle of 30.0 degrees with respect to the proton's velocity (see part (c) of the figure). Find (a) the magnitude and direction of the force on the proton and (b) the acceleration of the proton. (c) What would be the force and acceleration of the particle were an electron?



The Force That a Magnetic Field Exerts on a Charge

(a) **proton:**

$$F = |q_o| vB \sin\theta = \left(1.60\times10^{-19}\,\mathrm{C}\right)\left(5.0\times10^{6}\,\mathrm{m/s}\right)\left(0.40\mathrm{T}\right)\sin\left(30.0^\circ\right)$$
$$= 1.6\times10^{-13}\,\mathrm{N}$$

Using the RHR the direction of F is perpendicular to v-B plane and up.

(b) $$a = \frac{F}{m_\mathrm{p}} = \frac{1.6\times10^{-13}\,\mathrm{N}}{1.67\times10^{-27}\,\mathrm{kg}} = 9.6\times10^{13}\,\mathrm{m/s^2}$$

(c) **electron:**

The magnitude of F is the same, but the direction is opposite --> down.

$$a = \frac{F}{m_\mathrm{e}} = \frac{1.6\times10^{-13}\,\mathrm{N}}{9.11\times10^{-31}\,\mathrm{kg}} = 1.8\times10^{17}\,\mathrm{m/s^2}$$

Formal vector equation for the force, $\boldsymbol{F}$, on a charge, q, moving with velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$:

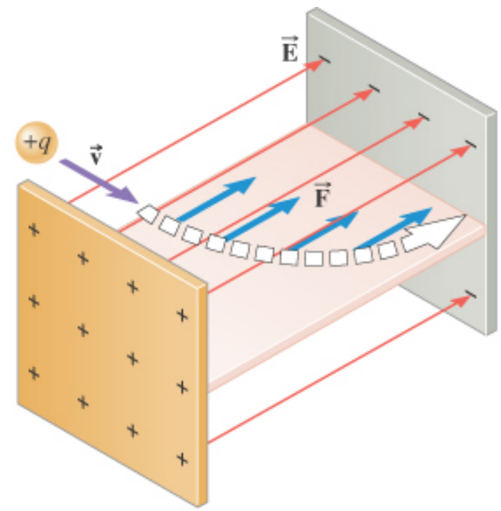
$$\vec{F} = q\vec{v} \times \vec{B}$$

Where "X" is the **cross product** between the vectors $\boldsymbol{v}$ and $\boldsymbol{B}$ and implies the right hand rule to find the direction of $\boldsymbol{F}$ and $qvB\sin\theta$ for the magnitude of $\boldsymbol{F}$.

The Motion of a Charged Particle in a Magnetic Field

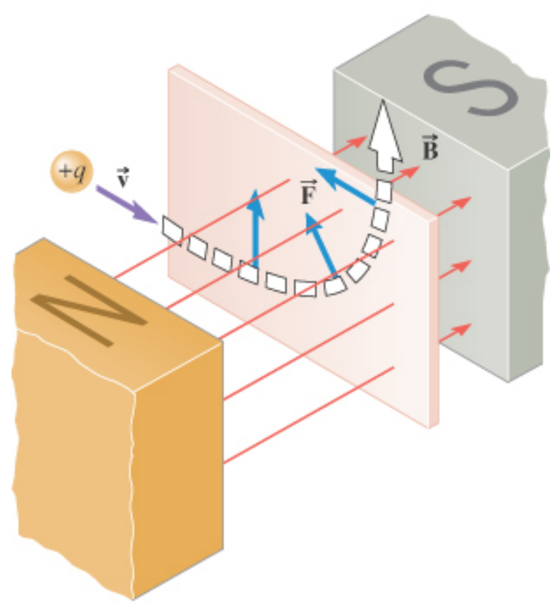
The motion of a charged particle in an electric field is very different from its motion in a magnetic field as shown:

Charged particle in an electric field.
F acts on the charge in the constant direction parallel to **E** and the particle's trajectory bends in the horizontal plane.



(a)

Charged particle in a magnetic field.
F acts on the charge in a direction perpendicular to **B** and its trajectory bends in the vertical plane with **F** continuously changing direction to remain perpendicular to **v**



(b)

Conceptual Example: A Velocity Selector

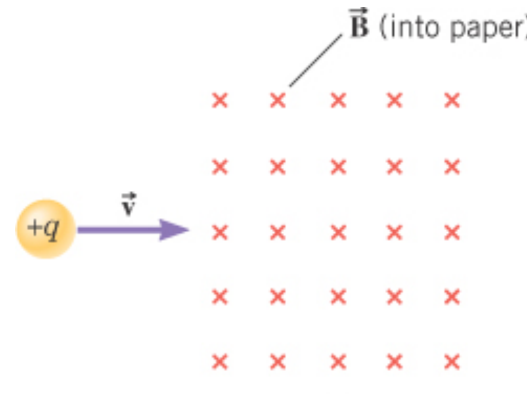
A velocity selector is a device for measuring the velocity of a charged particle. The device operates by applying electric and magnetic forces to the particle in such a way that these forces balance.

How should an electric field be applied so that the force it applies to the particle can balance the magnetic force?

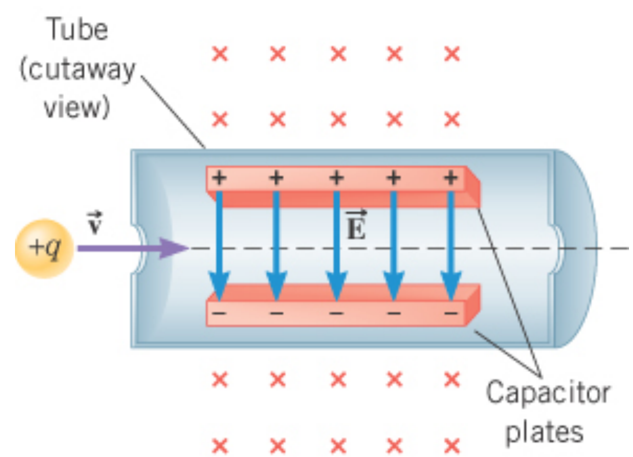
From RHR, F_M points upward, so F_E should point downward so the forces balance, i.e., $F_M - F_E = 0$ --> $F_M = F_E$

Since $F_M = qvB$ $F_E = qE$

Then, $qvB = qE$ --> $\mathbf{v = E/B}$, knowing E and B gives v independent of q



(a)

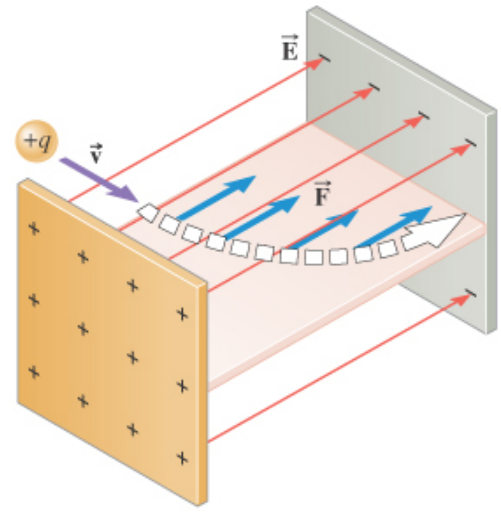


(b)

The Motion of a Charged Particle in a Magnetic Field

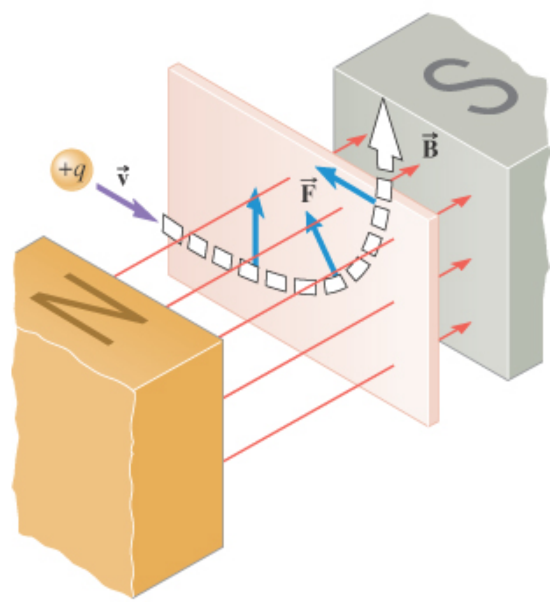
Work done on a charged particle moving through electric and magnetic fields.

For a constant force, F, --> $W_{AB} = F\Delta s = \Delta KE$
where Δs is the displacement along the direction of F and ΔKE is the change in the KE.



(a)

The **electrical force *can* do work** on a charged particle since it can displace the particle in the direction of the force and thus change its kinetic energy.



(b)

The **magnetic force *cannot* do work** on a charged particle since it acts perpendicular to the motion of the particle so that no displacement occurs along the direction of the force and thus its speed remains constant and its kinetic energy does not change.

The circular trajectory.

Since the magnetic force always remains perpendicular to the velocity, if a charged particle moves perpendicular to a uniform B-field its path will be circular. The magnitude of the force remains constant and it is directed toward the center of the circular path, i.e. it is the centripetal force for the motion.

Centripetal Force

$$F_c = m\frac{v^2}{r}$$

$$\Downarrow$$

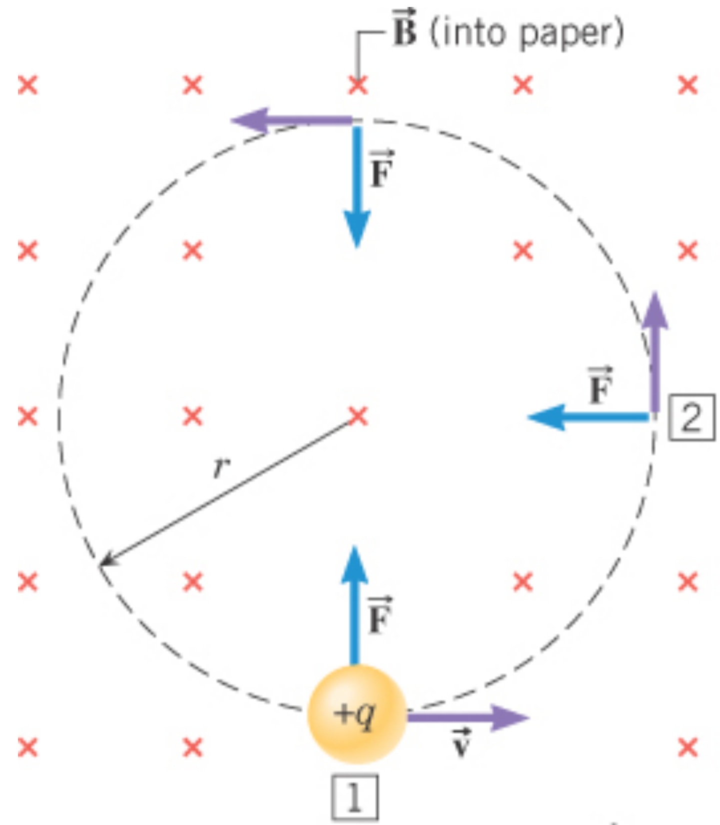
$F_M = F_c$

$$qvB = m\frac{v^2}{r}$$

$$\Downarrow$$

Solve for r

$$r = \frac{mv}{qB}$$



Example. Find the radius of curvature for a fast electron with speed $v = 2 \times 10^7$ m/s in a) the Earth's magnetic field at the surface, and b) in a 10 T magnetic field.

a) $B_{Earth} \sim 0.5$ gauss $= 0.5 \times 10^{-4}$ T

$$r = mv/(qB) = (9.11 \times 10^{-31})(2 \times 10^7)/[(1.6 \times 10^{-19})(0.5 \times 10^{-4})]$$

$$= \mathbf{2.3\ m}$$

b) $B = 10$ T

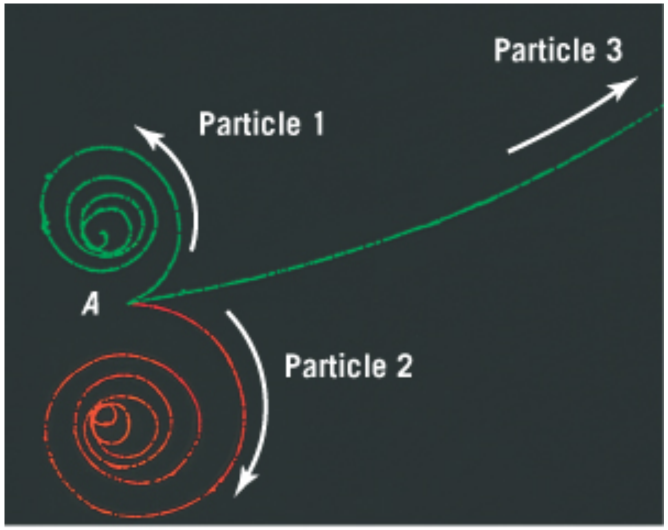
$$r = mv/(qB) = (9.11 \times 10^{-31})(2 \times 10^7)/[(1.6 \times 10^{-19})(10)]$$

$$= \mathbf{1.1 \times 10^{-5}\ m = 11\ \mu m}$$

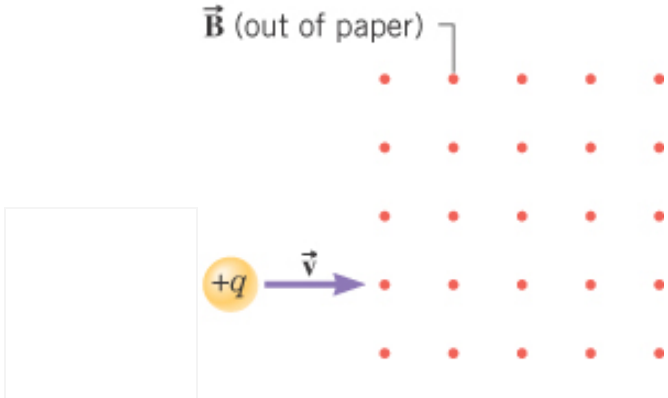
Conceptual Example: Particle Tracks in a Bubble Chamber

The figure shows the bubble-chamber tracks from an event that begins at point A. At this point a gamma ray travels in from the left, spontaneously transforms into two charged particles. The particles move away from point A, producing two spiral tracks. A third charged particle is knocked out of a hydrogen atom and moves forward, producing the long track.

The magnetic field is directed out of the paper. Determine the sign of each particle and which particle is moving most rapidly.



(a)



(b)

The Mass Spectrometer

Mass spectrometers can be used to measure the masses and relative abundances of **isotopes** (atoms which have nuclei with same number of protons but different number of neutrons).

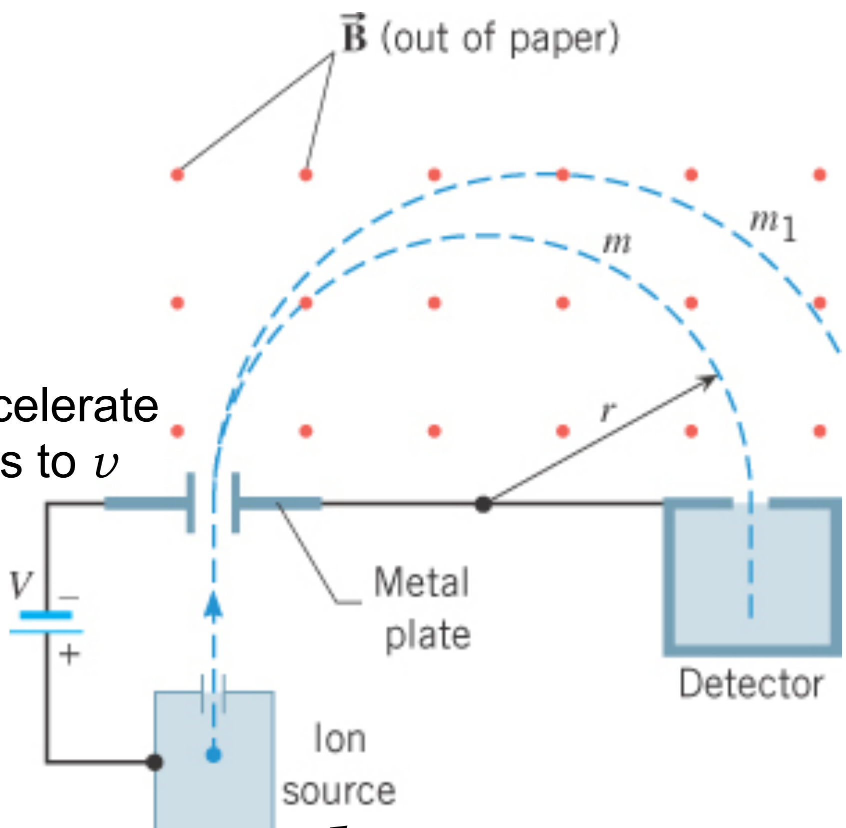
$$r = \frac{mv}{qB} = \frac{mv}{eB}$$

magnitude of electron charge

$$\tfrac{1}{2} m v^2 = eV$$

KE=PE

Accelerate Ions to v



$$m = \left(\frac{er^2}{2V}\right) B^2$$

can count ions hitting it

Put sample atoms here, ionize one electron --> q=+e

The Mass Spectrometer

The mass spectrum of naturally occurring **neon**, showing three isotopes.

For neon,

Z = 10 --> 10 protons

N = 10, 11, 12 --> number of neutrons for three isotopes

A = Z + N = 20, 21, 22 --> atomic mass numbers for neon isotopes -->