The ejection of electrons from the surface of a material by the action of light is known as the *photoelectric effect.*

Work function: This is the minimum energy required to remove an electron from the interior of a solid to a position outside.

The work function of the metal is typically a few electron volts and is a measure of the strength with which the metal confines the electrons.

Many subtleties associated with work function: surface contamination, roughness, dependence on surface orientation, double layers (the charge distribution on the surface cells differs from the interior and this distortion leads to substantial electric fields).

Assume monochromatic light is incident through a quartz window on an emitter, a metal plate. When the frequency of the electromagnetic radiation below a critical frequency no electrons are ejected. The maximum kinetic energy of the liberated electron, K_m depends on the frequency of the light ν and not its intensity and is given by

$$K_m = h\nu - W_{emitter} \tag{1}$$

where $W_{emitter}$ is the work function of the cathode from which the electrons are ejected/emitted. (It is the difference between the photon energy and the work done in liberating the highest occupied energy level.)

Clearly for frequencies less than a critical frequency no electrons are liberated, however strong the intensity of the incident light. The quantum explanation depends on the assumption that each photon has an energy $h\nu$ and this is absorbed by an individual electron.

If the frequency of the light is held fixed and the electrons that are emitted are collected by an electrode with some voltage applied to it there is a current even when the potential is zero. As V is made more positive the number of electrons gathered by the electrode increases and saturates since all the electrons liberated by the photons are collected. When the voltage is made negative electrons are repelled by the electrode and only those electrons with sufficient velocity to reach the electrode contribute to the current. Beyond the stopping voltage none of the electrons reach the other electrode. At this voltage $eV_s = K_m$ is the maximum kinetic energy of the electrons. Thus we

obtain

$$eV_s = h\nu - \phi$$

and V_s is linear in ν with a slope proportional to h/e. I have used ϕ to denote the work function. Knowing the electric charge we can determine Planck's constant.

Consider a standard experiment for observing the photoelectric effect which uses light of wavelength $250nm$ and a cathode made of nickel whose work function is $5.2eV$. *Fill in the blanks*: When the potential difference applied across the electrodes is zero, it is found that the current flow is ——— (zero/non-zero).

Preliminary aside: Recall

$$E = h\nu = \frac{hc}{\lambda}.$$

Note that the units of h and $\hbar \equiv \frac{h}{2\pi}$ is Energy $\times$ time or angular momentum. The value of h is $6.6262 \times 10^{-34}\,J.s$ or $4.16 \times 10^{-15}\,eV.s$ and hc is $1240\,eV.nm$. This is worth remembering. Visible light has wavelengths in the range $400nm$ (violet) to $750nm$ (red) with maximum sensitivity around $550nm$. Thus the energy is roughly between $3.1eV$ to $1.65eV$.

The given wavelength corresponds to $4.96eV$ and is less than the work function and so, the net current vanishes.

Would the ammeter read zero current or non-zero current if you

(i) doubled the intensity of the light source ? Explain your reasoning and why you might think of doing this in the first place?

(ii) increased the potential difference between the cathode and the anode to $5.5V$? Explain your reasoning.

(iii) replaced the nickel cathode with one of aluminum with a work function of $4.2eV$? Explain your reasoning.

Einstein was recommended to membership in the Prussian Academy of Science by Planck and others. Their early negative attitude toward the photon hypothesis is revealed in their signed affidavit, praising Einstein, in which they wrote: "Summing up, we may say that there is hardly one among the great problems, in which modern

physics is so rich, to which Einstein has not made an important contribution. That he may have sometimes missed the target in his speculations, as, for example, in his hypothesis of light quanta (photons), cannot really be held too much against him, for it is not possible to introduce fundamentally new ideas, even in the most exact sciences, without occasionally taking a risk."

The Compton effect

Context: The interaction of light with matter (its charged constituents like the electron) is fundamental to our understanding of nature.

First consider scattering of electromagnetic waves by free electrons, called *Thomson scattering.* Associated with the electromagnetic wave (assume a monochromatic wave with frequency ω) is a transverse (to the direction of propagation) electric field that oscillates at the frequency ω. The free electron feels a force $-e\vec{E}(t)$ and is hence accelerated. *Accelerated charges radiate.* The accelerated electron radiates at the same frequency. The scattered wave has the same wavelength/frequency as the incident wave. The intensity of the scattered wave turns out to be independent of the frequency.

Now consider scattering from a bound electron, an electron in an atom. This is treated in Griffiths' book on Electrodynamics. The result is worth remembering. The scattered intensity depends on the frequency of the incident wave and increases as the fourth power, ω^4. This shows that air molecules scatter the shorter wavelengths(blue) preferentially thus leading to blueness of the sky. Explain redness of the sunset by scattering of the blue light away from the line of sight. The wavelength of red light is $650nm$ while that of blue light is $450nm$ and the ratio of the frequencies is 1.44. Thus blue light is scattered $1.44^4 \approx 4.3$ times as effectively as red light.

When the incident energy of the photon $\hbar\omega$ becomes comparable to the rest mass of the electron quantum mechanical effects become important. In Compton scattering, the scattering event can be studied theoretically by treating light as a particle (photon) and using energy and momentum conservation.

The change in wavelength of light due to scattering by a free electron given by

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

where $\hat{k} \cdot \hat{k}' = \cos\theta$ yields the angle θ between the incident photon (along $\hat{k}$) and the scattered photon along $\hat{k}'$. The quantity $\frac{h}{m_e c}$ is the so-called electron *Compton wavelength* and is $2.43\,pm$. So would you use infrared photons or x-rays to do a Compton scattering experiment ?

This was verified by Arthur Holly Compton [1] and was crucial in convincing the physics community of the validity of the concept of the photon.

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[1] b. Sept. 10, 1892, Wooster, Ohio, U.S. d. March 15, 1962, Berkeley, Calif. American physicist and joint winner, with C.T.R. Wilson of England, of the Nobel Prize for Physics in 1927 for his discovery and explanation of the change in the wavelength of X rays when they collide with electrons in metals. Compton, a younger brother of the physicist Karl T. Compton, who was President of MIT, received his doctorate from Princeton University in 1916 and became head of the department of physics at Washington University, St. Louis, in 1920.

Some algebraic details:

Recall that the energy of a photon of wavelength λ and frequency $\nu = c/\lambda$ is given by $\boxed{E = \hbar\omega = h\nu = hc/\lambda}$. Its momentum is given by $\boxed{\hbar\vec{k} = (h/\lambda)\hat{k}}$ where $\hat{k}$ is the unit vector in the direction of the wave vector $\vec{k}$. The so-called dispersion relation relating the frequency and wavelength of a photon can be written as $\boxed{\omega = ck}$ where k is the magnitude of the wave vector $k = |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$ or equivalently $\boxed{\nu = c/\lambda}$.

For an electron with (rest) mass m_e and momentum $\vec{p}$ the energy E is given by $\boxed{E = \sqrt{\vec{p}\cdot\vec{p}\,c^2 + m_e^2 c^4}}$.

Consider the scattering of light of wavelength λ and wave vector $\vec{k} = (2\pi/\lambda)\,\hat{k}$ from a stationary electron. Let λ' denote the wavelength of the scattered photon and $\vec{k}'$ its wavevector. We will denote the energy and momentum of the electron after collision by E' and $\vec{p}'$ respectively. Energy conservation yields

$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E' \tag{2}$$

while momentum conservation reads

$$\frac{h}{\lambda}\hat{k} + \vec{0} = \frac{h}{\lambda'}\hat{k}' + \vec{p}' \tag{3}$$

where we have explicitly displayed the zero initial momentum of the electron.

To derive something useful it is important to be clear about the question one wishes to answer. Compton measured the wavelength of the photon scattered in different directions. Let us denote the angle between the initial and scattered electron by θ, i.e., $\hat{k}\cdot\hat{k}' = \cos\theta$. We wish to find a relation between λ' and θ which describe the scattered photon. This means that we have to eliminate all the unknown variables which describe the scattered electron, i.e., E' and $\vec{p}'$ and this can be done if we remember that

$$E'^2 - \vec{p}'\cdot\vec{p}'\,c^2 = m_e^2 c^4 \ . \tag{4}$$

So rewrite the conservation equations by separating out the variables we wish to eliminate:

$$E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2 \tag{5}$$

$$\vec{p}' = \frac{h}{\lambda}\hat{k} - \frac{h}{\lambda'}\hat{k}' \ . \tag{6}$$

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Substituting the expressions for E' and $\vec{p'}$ in equation (4) we eliminate all reference to the scattered electron. We obtain

$$[\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2]^2 - c^2 [\frac{h}{\lambda}\hat{k} - \frac{h}{\lambda'}\hat{k'}] \cdot [\frac{h}{\lambda}\hat{k} - \frac{h}{\lambda'}\hat{k'}] = m_e^2 c^4 \qquad (7)$$

Now it is a matter of straightforward algebra to expand the terms, collect and cancel various expressions and obtain $\boxed{\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)}$. *Please do the algebra!*

This is the famous Compton formula. Note that $\frac{h}{m_e c}$ has units/dimensions of length and is called the *Compton wavelength*; its magnitude is approximately $2.43 \times 10^{-12} m$.