Multi-Unit Demand Auctions with Synergies:
Behavior in Sealed-Bid versus Ascending-Bid Uniform-Price Auctions*

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Abstract

We construct a model of bidding with synergies and solve it for both open outcry and sealed-bid uniform price auctions. The model is simple enough to allow for direct interpretations of the experimental data, while still maintaining the essential behavioral forces involved in auctions with synergies: (1) A demand reduction force resulting from the monopsony power that bidders with multiple-unit demands have when synergies are relatively inconsequential and (2) Bidding above standalone values in order to capture significant complementarities between units. The latter creates a potentially important behavioral force - the "exposure problem" - as bidders may win only parts of a package and earn negative profits. Bidding outcomes are closer to equilibrium in clock compared to sealed-bid auctions. However, there are substantial and systematic deviations from equilibrium, with patterns of out-of-equilibrium play differing systematically between the two auction formats. These patterns of out-of-equilibrium play are analyzed, along with their effects on revenue and efficiency.

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The FCC spectrum auctions have reinvigorated theoretical and empirical research on auctions in efforts to better understand the effects of different auction institutions when individual bidders demand multiple units of a given commodity. One line of research has focused on the performance of auctions with uniform-price rules, where all winning bids pay the same highest rejected bid.[1] It is well known by now that in such auctions, when valuations are non-increasing, bidders have an incentive to reduce demand on some of their units in order to exploit the monopsony power they have when demanding multiple units. This strategy may result in winning fewer units, but when it does, it also reduces the price on units earned. (See, for example, Ausubel and Cramton, 1996 and Englebrecht-Wiggans and Kahn, 1998.) Demand reduction reduces economic efficiency and revenue relative to a full demand revelation. Experimental, and quasi-experimental research confirms that the demand reduction incentives are reasonably transparent and practiced even by relatively naive bidders (Kagel and Levin, 2001; List and Lucking-Reily, 2000). Further, experiments comparing sealed-bid auctions with ascending-price clock auctions reveal that although both auctions have the same normal form game representation, bidding is significantly closer to equilibrium in the clock auction, suggesting there are behavioral elements not fully captured in the theory (Kagel and Levin, 2001).

Uniform-price auctions which involve synergies, or complementarities, provide additional incentives and generate radically different bidding strategies than the same auctions without synergies. Synergies create an opposite incentive to the demand reduction force: aggressive bidding in order to acquire desired packages with their super additive value. Further, in sealed-bid uniform-price auctions, that do not permit package bids, the existence of synergies commonly dictates submitting bids above the standalone values for individual units in order to increase the probability of winning a package with its super additive value. However, this strategy is risky since if a bidder fails to acquire the whole package and wins only parts instead, she is likely to earn negative profits. Thus, in addition to the competing equilibrium incentives, an important "behavioral" force may affect bidding as well: Depending on the size of the potential loss, and risk preferences, bidders may refrain from such aggressive bidding in order to avoid exposure to such losses, despite the benefits of doing so (Bykowsky et al., 1995; Ausubel et al., 1997; Rothkopf et al., 1998). This avoidance has been referred to as the "exposure problem," a serious concern in some quarters at least, in designing auctions in the absence of that package bids.[2]

The present paper reports the results of an experiment in a highly simplified auction environment designed to maintain the essential richness of the economic and behavioral forces present in multi-unit demand auctions with synergies. We first construct a tractable model of bidding with synergies and solve it for both open outcry and sealed-bid uniform price auctions (highest losing bid determines the price paid). The model is different from any existing model of auctions with synergies (being closest in structure to the model developed by Krishna and Rosenthal, 1996; see below): It is simple enough to allow for direct interpretations of the experimental data, while still maintaining the essential behavioral forces involved in auctions with synergies. We compare outcomes of sealed-bid and ascending-bid (English-clock) uniform-price auctions. Under our design, the net effect of the demand reduction force and the synergy force is that in equilibrium: (1) at lower valuations, the demand reduction force dominates so that bidders shave their bids on marginal units, (2) at the highest valuations the synergy force dominates so that bidders "go for it," bidding high enough to insure winning the items, and (3) at middle valuations the two forces are at peak tension and are counterbalancing each other, with bidding above value (but short of "going for it") in the sealed-bid auctions and "going for it," conditional on rivals' observed dropout prices, in the clock auctions. The exposure problem works against the synergy effect, being most prominent at middle valuations when bidding above standalone values but short of going for it, and when valuations are such that going for it does not insure earning a positive profit.

Under our experimental design a human subject demanding two units of a commodity competes against different numbers of rivals demanding a single unit of the commodity in a uniform-price auction. Single-unit buyers have a dominant strategy, bidding their value, and are played by computers.[3] The standalone values for both items are the same for the human bidder, v_h, but earning both units generates *three* times the standalone value ($3v_h$). With independent private values drawn from a uniform distribution and with supply of two units, the equilibrium predictions for the “large” bidder correspond to the three regions characterized above. Thus, the experimental design is simple enough to yield equilibrium predictions while still maintaining the tension between the demand reduction and synergy forces. The design also abstracts away from the strategic uncertainties inherent in interactions between human bidders (e.g., problems of learning best responses given rivals’ out-of-equilibrium bids). Finally, the experimental design allows us to employ a limited number of values for the human bidders without distorting the equilibrium predictions. We exploit this by limiting the number of standalone values in each experimental session to three, with a number of replications at each value, thereby providing bidders with more systematic, and easier to process, feedback in this relatively complicated bidding environment. The standalone values employed span the strategy space and induce maximum differences in strategic behavior between the sealed-bid and clock auctions, while providing a number of replications at each value against which to evaluate behavior.

We do not expect bidders to be able to calculate and respond precisely to the fine cut-off points associated with such a complex auction environment. Thus, we focus on the following questions: Are bidders sensitive to the tradeoffs inherent in uniform-price auctions with synergies? Do they behave differently in cases where the demand reduction force dominates compared to cases where the synergy bonus is strong enough to dominate? What role, if any, does the exposure problem play in bidding? What are the nature of deviations from optimal bidding strategies, and are there systematic differences in the patterns of deviations between the two types of auctions studied? Are there public policy implications resulting from any systematic deviations from optimal bidding?

There has been some experimental work on multi-unit demand auctions with synergies. The work falls roughly into two major categories: First, "test bed" experiments designed to explore the effects of different auction rules for public policy purposes; in particular, to provide data and insights into urgent problems arising in the design of the FCC spectrum auctions, with an emphasis on the efficiency of alternative auction mechanisms (see, for example, Ledyard et al., 1997 and Plott, 1997). In most cases these experiments explore situations for which theory has little to say, with a strong focus on environments where package bidding might play a role in achieving efficient resource allocations (Ledyard et al., 1997). Our experimental environment represents a tremendous simplification relative to these experiments as we reduce the complexity of the game for multi-unit demand bidders to a well-defined decision problem. This clearly represents "backtracking" relative to the effort in these experiments to capture the complexity of the demand structure underlying the FCC auctions. However, our research strategy has several key advantages resulting from this "backtracking": (1) We are able to solve for the optimal bidding strategy which provides us with a clean theoretical benchmark against which to evaluate behavior, while still preserving many of the essential economic tradeoffs that multi-unit demand bidders face in more complicated settings with synergies, and (2) We have a large number of observations against which to evaluate behavior. As a result of these advantages we are able to examine in detail, against a clean theoretical benchmark, many of the important behavioral issues involved in auctions with synergies. In contrast, in the test bed experiments, as Ledyard et al. note, “No careful theoretical analysis or experimental design was followed, nor could one be, given the urgency of the situation.” (Ledyard et al., 1997, p. 641) Unfortunately, the large differences in underlying economic structure between our experiment and these test bed experiments makes direct comparisons of results problematic.

The second major category of experimental work on auctions with synergies are experiments designed to explore the feasibility of Vickrey auctions with package bids (Isaac and James, 2000; Brenner and Morgan, 1997). Although the emphasis in both these experiments is on the Vickrey auctions, they employ control treatments that are reasonably close in spirit to our experimental design: Brenner and Morgan employ a simultaneous ascending price auction designed to mimic FCC procedures. Isaac and James employ a simultaneous sealed bid second price auction similar to the one described in Krishna and Rosenthal (1996). In both cases the role of these control treatments is to provide a reference point against which to evaluate the potential efficiency gains associated with the Vickrey auction. There is virtually no analysis of behavior within the control treatments against which to compare our results: Brenner and Morgan limit their analysis to a single paragraph pointing out that they can reject a hypothesis of demand revelation in the simultaneous ascending price auction in favor of bid shaving. Isaac and James (p. 52) limit their analysis to an endnote pointing out that they saw cases in which high value bidders lost the auction because they bid too low on individual items and other cases where they bid over their valuations. Thus, there is little basis for comparing our experimental results to these control conditions, and we do not address the issue of package bids or the question of Vickrey auctions here.

Ausubel et al. (1997) and Morten and Spiller (1996) examine license interdependencies in some of the early FCC spectrum auctions. The FCC auctions involve heterogenous goods auctioned off simultaneously in a number of separate markets, in contrast to the single market with homogenous goods and synergies explored here (see below).

The auction model underlying our experiment is similar to one developed in Krishna and Rosenthal (1996) to explore simultaneous sealed-bid auctions with synergies. In both cases there is a single bidder demanding two units competing against a number of rivals demanding a single unit. The primary difference between the two models is that in the Krishna and Rosenthal model the bidder demanding multiple units competes in two separate second-price auctions against *n* single-unit demand bidders in each market.[4] In other words, in our model the two goods are perfect substitutes and sold together in a single uniform-price auction. In Krishna and Rosenthal the two goods are imperfect substitutes and sold in two separate second-price auctions. As a result, there is no demand reduction force present in their model as there is in ours. However, in regions of our experimental design where the synergy force dominates the demand reduction force, the two models make remarkably similar predictions: Single-unit bidders always bid their value. When bidding above value, the bidder with increasing returns always bids the same on both units, with bids increasing in the valuation drawn.[5] Once the valuation is high enough, there is a discontinuous jump in the bid function, so that the bidder with increasing demand "goes for it." Further, bids of the multi-unit demand bidder are weakly decreasing with more competition, as they are in our model. Finally, Krishna and Rosenthal do not extend their analysis to ascending-bid clock auctions as we do here.

Our experiment yields a number of basic insights: Bidders are always closer to optimal bidding strategies in a clock compared to sealed-bid auctions. Bidders are responsive to the underlying economic forces present in the auction even though there is considerable out-of-equilibrium play. Further, out-of-equilibrium play differs substantially and systematically between the two institutions. In the sealed-bid auctions there is a clear tendency for bidders to overbid at low values and underbid at high values. In contrast, in the clock auctions, absent secure, positive

expected profits, there is a general reluctance to bid above value when optimality requires it, consistent with the "exposure problem." At least part of this differential sensitivity to the exposure problem in clock compared to sealed-bid auctions results from the obvious ability to stop the bidding and assure a positive profit in the clock auctions. This is indicative of a clear presentation format effect, an outcome observed in other auction settings as well (Kagel et al., 1987; Kagel and Levin, 2001). As a result, the clock auction *fails* to improve efficiency relative to the sealed-bid auctions where the theory predicts it should, and the sealed-bid auctions generate uniformly higher revenue.

The plan of the paper is as follows: Section 1 develops the theoretical predictions for both ascending-bid and sealed-bid auctions. The experimental design is outlined in Section 2 along with the theoretical predictions specific to the experimental design. Results of the experiment are reported in Section 3. We close with a brief summary and discussion of our major results.

1. Theoretical Predictions

We investigate bidding in IPV auctions with (n+1) bidders and *m indivisible,* identical objects for sale, where $n \geq m$. Each bidder i (i = 1, ... , n) demands a single unit of the good, placing a value v_i on the good. These bidders are indexed by their values so that $v_1 \geq v_2 \geq, ..., \geq v_n$. Bidder h, the (n+1)th bidder, demands two units of the good, with the value of each unit *by itself* equal to v_h. However, earning two units generates synergies so that the value of winning both units = $2v_h + \alpha v_h$. Bidders' values were independent and identical draws (*iid)* from a *uniform* distribution on the interval [0,V] for all bidders. In what follows we work with $m = 2$, $\alpha = 1$, as these are the values employed in the experiment, and analyze behavior within the unit interval (V = 1).[6] For both sealed-bid and clock auctions there are three bidding regions, with distinctly different bidding strategies for the human bidder in each region, which are discussed in detail below.

A. Sealed-bid auctions: In the sealed-bid auction, each bidder simultaneously submits sealed bids for each of the units demanded. These are ranked from highest to lowest, with the *m* highest bids each winning an item and paying a price equal to the *m*+1 highest bid.

For bidders $i = 1, \ldots, n$, demanding a single unit of the commodity, there is a dominant strategy (the same as in the familiar single-unit Vickrey auction) to bid their value, v_i. Bidder *h* demands two units of the commodity and submits two bids, b_1 and b_2 for unit one and two respectively. Without loss of generality assume that $b_1 \geq b_2$. The optimal bidding strategy varies dramatically with v_h, since this directly affects the tradeoffs between incentives promoting demand reduction and those promoting bidding above v_h in order to benefit from the synergy bonus as a result of winning both units. There are three regions of interest. (See the top panel of Figure 1 which provides a characterization of the bid patterns over the unit interval for $n = 3$ and 5, the number of single unit bidders employed in our experimental design. Appendix A provides the derivation of the equilibrium bidding strategy for the sealed-bid auction.)

[Insert Figure 1 here]

Region 1: For $v_h < ½$, $b_1 = v_h$ and $b_2 = 0$. (For $v_h = ½$, $b_1 = b_2 = ½$.) The synergy value is not large enough to overcome the demand reduction incentive. Here, bidders are better off exercising drastic demand reduction, giving up any chance of winning two units.

Region 2: For $½ < v_h < v(n)$, $v_h < b_1 = b_2 < 1$, where $v(n)$ is defined as the upper bound of the interval for region 2, and is a function of the number of single unit bidders *h* is competing against (see Appendix A). That is, the optimal bidding strategy calls for submitting two equal bids above v_h, but not high enough to assure winning both units. Both the size of this interval and how much to bid above v_h varies with *n*, with a wider interval and more aggressive bidding the smaller *n* is.

Region 3: For $v_h \geq v(n)$, $b_1 = b_2 \geq 1$. That is, the optimal bidding strategy is to "go for it," bidding high enough to insure winning both units as the expected value of winning both units, even at the maximal bid price of V (=1) yields positive expected profit given the synergy bonus. In contrast, in region 2, values are not high enough to insure positive expected profit at the maximum bid price, so that bidders do not go for it in region 2.

Risk aversion has no effect on bids in region 1: $b_1 = v_h$ is based on a dominance argument, and risk aversion will lower b_2 in region 1, unless $b_2 = 0$ as in our design (see Appendix A). For v_h high enough in region 3, *h* is assured of earning nonnegative profits even paying the maximum possible price ($p = 1$), so that risk preferences play no role here as well. For region 2 and the remainder of region 3, the requirement that $b_1 = b_2$ is satisfied regardless of risk preferences, as it is based on a dominance argument as well. ($b_1 = b_2 \geq v_h$ dominates $b_1^* > b_2 \geq v_h$ since the higher b_1^* only matters when winning one unit and regretting it as the price must be above v_h.) However, the level of bids in this interval will, in general, be affected by risk aversion, as risk averse bidders require a risk premium to try for both units unless they are assured of earning positive profits. Further, to the extent that bidders are loss averse (have strong disutility for negative profits; Kahneman and Tversky, 1979) this will definitely reduce bids as in region 2 as bidders face the possibility of earning a single unit at a price greater than v_h. Loss aversion will also reduce bids in part of region 3 where bidders are not assured of earning positive profits, as in going for it may earn both units but at a price that is less than their valuation, even accounting for the synergy bonus.

B. Ascending-bid (Clock) Auctions: The ascending-bid version of the uniform-price auction (also referred to as a clock auction or an English-clock auction) starts with the price being zero and increasing rapidly thereafter. Bidders start out actively bidding on all units demanded, choosing the price to drop out of the bidding. Dropping out is irrevocable so that a bidder can no longer bid on a unit she has dropped out on.[7] The dropout price which equates the number of *remaining* active bids to the number of items for sale sets the market price. Winning bidders earn a profit equal to the value of their winning unit less the market price. All other units earn zero profit. Posted on each bidder's screen at all times is the current price of the item, the number of items for sale, and the number of units actively bid on, so that *h* can tell at exactly what price a rival has dropped out. Further, there is a brief pause in the progress of the price clock following a drop out during which *h* can drop out as well. Dropouts during the pause are recorded as having dropped out at the same price, but are indexed as having dropped later than the dropout that initiated the pause.[8]

Bidders i = 1, ..., *n* demanding a single unit have a dominant strategy to remain active until the clock price reaches their value, v_i. As in the sealed-bid version of the auction there are three regions of interest (see the bottom panel of Figure 1):

Region 1: For $v_h < ½$, the optimal bidding strategy for *h* is comparable to the sealed-bid auction strategy in the sense that *h* earns greater expected profit by winning a single unit and reducing the price paid by not winning a second unit. The strategy below assures winning at most one unit. However, there is considerably more flexibility in carrying out the optimal policy than in the sealed-bid auction:

If $v_2 \le v_h$, $d_1 = v_h$ and $0 \le d_2 \le v_2$, and

If $v_2 > v_h$, then $d_1 \in [v_h, \max(v_h, v_3)]$ and $d_2 \in [0, \max(v_h, v_3)]$

where d_1 and d_2 are *h*'s dropout price on units 1 and 2, respectively, and v_2 and v_3 are, respectively, the observed drop-out prices of the single-unit bidders with the *n-1* (second) and *n-2* (third)) highest valuations.

Region 2: For $v \in [½, 2/3)$ the optimal bidding strategy uses the information revealed by rivals' dropout prices. In particular there is a cutoff point, $\mathbf{P}^* = [3v_h - 1]$ such that:

If $v_2 \leq$ **P***, $d_1 = d_2 \geq 1$ and

If $v_2 >$ **P***, $d_1 = d_2 \in$ [**P***, max {**P***, v_3}].

In contrast to the sealed-bid auction, the bidding rule in this region does not depend on the number of rivals as the information revealed in the second highest computer's dropout price is sufficient to determine the optimal bidding strategy.

Region 3: For $v_h \geq 2/3$, $d_2 \geq 1$, so *h* "goes for it," winning both units for sure. Note that "going for it" in this interval assures positive profits, as *h* earns at least 2 and pays at most 2, so that the exposure problem is not an issue here. This strategy yields higher expected profits for any realization of v_2 compared to stopping the auction and earning a single unit. Further, the size of this interval is smaller than the interval in which bidders "go for it" in the sealed-bid auctions.

Risk aversion has no affect on bidding in region 1, since like in the sealed bid auction, $b_1 = v_h$ is based on a dominance argument, and the effect of risk aversion is to reduce *h*'s bid on unit 2, unless $b_2 = 0$. Risk aversion has no affect throughout region 3 as bidders are certain of winning and earning nonnegative profits throughout region 3. But, as in the sealed-bid auctions, it does impact on bidding in region 2. Here, bidders in going for it, earn both units but suffer losses as a consequence. Thus, to the extent that agents are loss averse, for which there is substantial empirical evidence in individual choice studies, this would definitely suppress the aggressive bidding called for when $v_2 \leq$ **P*** and may even get bidders to drop out before **P*** when $v_h <$ **P***.

Clearly both auction formats will produce inefficiencies. In region 1 this is the result of demand reduction by the multi-unit demand bidder. There will also be some *ex post* inefficiencies in region 2 as *h* bids based on the expected values of the single-unit bidders. However, for those valuations that fall in region 2 in the clock auctions efficiency will tend to be higher in the clock auctions as bidders condition their actions on observed dropout prices.

Revenues will be the same under both auction formats in region 1 and in the part of region

3 where going for it assures nonnegative profits. For those valuations that fall in region 2 in the clock auctions, but region 3 in the sealed-bid auctions, revenues are bound to be higher in the sealed-bid auctions as bidders go for it in all cases, whereas in the clock auctions they drop out if $v_2 \leq \mathbf{P^*}$. In the remainder of region 2 revenues will tend to be higher in the clock auctions as conditioning on observed dropout prices often requires bidders to go for it, as opposed to bidding somewhat above their valuation in the sealed-bid auctions.

2. Experimental Design and Theoretical Implications for Treatments Employed

A. Procedures. Valuations were i.i.d. from a uniform distribution with support [0, $7.50].[9] Bidders with single-unit demands were represented by computers programmed to submit bids equal to their value in the sealed-bid auctions or to stay active until the price reached their value in the clock auctions. Bidder *h* was played by subjects drawn from a wide cross-section of undergraduate and graduate students at the University of Pittsburgh and Carnegie-Mellon University. Students were recruited through fliers posted throughout both campuses, advertisements in student newspapers, electronic bulletin board postings, and classroom visits. Each *h* operated in her own market with her own set of computer rivals, knew they were bidding against computer rivals and the number of computer rivals, as well as the computers' bidding strategy.

The use of computer rivals has a number of advantages in a first foray into this area: *h*s face all of the essential strategic tradeoffs involved in auctions of this sort but in a very "clean" environment. The latter include no strategic uncertainty regarding other bidders' behavior and no issues of whether or not "common knowledge" assumptions are satisfied.

All clock auctions employ a "digital" clock with price increments of $0.01 every 0.1 second. Following each computer drop out there was a brief pause of 3 seconds. *h*'s dropping out during a pause counted as dropping at the same price, but later than, the computer's dropout.

h could drop out on a single unit by hitting any key other than the number 2 key. The first stroke of the key pad dropped out unit 2. Hitting the number 2 key, or a second key during the pause, permitted *h* to drop out on both units at the same price.

In the sealed-bid auctions the sequencing required subjects to submit bids on unit 1 followed by unit 2. Any nonnegative bid was accepted for unit 1, with the bid for unit 2 required to be the same or lower than the bid on unit 1.[10] There was no opportunity to submit a single bid for both units combined or a bid contingent on winning only one or winning both units.

In all sessions, instructions were read out loud, with copies for subjects to read as well. The instructions included examples of how the auctions worked as well as indicating some of the basic strategic considerations inherent in the auctions. For example, the instructions pointed out that the higher *h*'s value, the more valuable the synergy bonus was, hence the greater value of earning two rather than one unit, and that when bidding above v_h winning a single unit would necessarily involve losses (see the instructions for full details). Finally, it was emphasized to subjects that "...in thinking about bidding, earning an item is of no intrinsic value. Your sole objective should be to maximize your earnings."

Subjects were told that in each auction period the computers would (and did) receive fresh values. At the conclusion of each auction, bids were ranked from highest to lowest and posted along with the corresponding values. Winning bids were identified, prices were posted, profits were calculated, and cash balances were updated. Bidders only observed the outcomes for their own market. Sessions began with three dry runs to familiarize subjects with the auction procedures, followed by thirty-three auctions played for cash.

Bidders were given starting capital balances of $5. Positive profits were added to this and negative profits subtracted from it. End-of-experiment balances were paid in cash. Expected profits were sufficiently high that we did not provide any participation fee.[11] Sessions lasted between 1.5 and 2 hours.

B. Treatments and Their Theoretical Implications. Since single-unit bidders have a dominant strategy independent of *h*'s valuation, this permits us to employ a limited number of values for *h* without distorting the equilibrium predictions. Given the complexity of the environment, we limited ourselves to four values designed to fully represent/span the strategy space, and to induce maximum differences in strategic behavior between the sealed-bid and clock auctions. By repeating the use of the same valuations within an experimental session we provide bidders with considerable experience at each value, which might be expected to ease decision making in such a complex environment, while providing us with multiple observations against which to evaluate behavior.

Sessions employed either three or five computer rivals (n = 3 or 5), with the number of computer rivals remaining constant within each session. In each session v_h varied randomly over three of the four values employed using a block random design. All three values occurred in each consecutive series of three auctions, but in random sequence. The lowest v_h, \$3.00, calls for complete demand reduction in both auctions, and is employed exclusively with n = 3. (The expected cost of deviating from the optimal strategy with v_h = \$3.00 and n = 5 is quite small, and involves a rather large opportunity cost in terms of foregone observations at more salient values.) The highest v_h, \$5.10, requires "going for it," and insures a *secure* (minimum) profit of 30¢ per auction. It is employed exclusively with n =5 out of cost considerations and the fact that there is little 'bang for the buck' at this value in auctions with n = 3.[12] It provides a check on the first order rationality of bidders (and/or their sensitivity to the underlying economic contingencies).

The middle v_h' s employed make different predictions between sealed-bid and clock auctions and were employed with both n = 3 and 5. These are clearly the most difficult values for bidders to get right as well as the most critical values for distinguishing the extent to which they are response to the underlying economic forces at work.

v_h = \$4.00: In the sealed-bid auctions $b_1 = b_2$ = \$4.34 with $n = 3$ and $b_1 = b_2$ = \$4.16 with $n = 5$. The clock auction also requires bidding above value on both units: If $v_2 \leq$ **P*** = \$4.50, h goes for it as the expected value of winning two units is positive and greater than the value of stopping the auction at p = v_2 and winning a single unit. If $v_2 >$ **P***, h drops on both units at the cutoff point **P*.**

v_h = \$4.40: It pays to "go for it" in the sealed-bid auctions regardless of whether $n = 3$ or 5. The clock auction also calls for bidding above value, with the cutoff point **P*** = \$5.70.

Finally, for both intermediate values, if h mistakenly stays beyond max{**P***, v_3} when $v_2 >$ **P***, the certain loss associated with stopping the auction and winning one unit is greater than the expected loss associated with remaining active and winning both units. Thus in equilibrium, or outside equilibrium, at these intermediate values, optimal bidding requires that h should never win only one unit in the clock auction.

The top part of Table 1 summarizes these predictions regarding h's bids for each of the computer values employed. While these predictions are of primary interest, there are also clear implications for h's profits, seller revenue, and auction efficiency. Profits, revenue, and efficiency all predicted to be the same between auction formats within regions 1 and 3 (v_h = \$3.00 and \$5.10), as bid outcomes are predicted to be the same in these regions. Profits, revenue, and efficiency are all predicted to be substantially higher in the clock auction with v_h = \$4.00, as h decides to go for it contingent on the observed drop-out prices of the single-unit bidders. This makes for much finer distinctions than under h's relatively conservative, fixed bidding strategy in the sealed-bid auctions. Note that profits and revenue can *both* increase in this treatment due to the increased efficiency. At v_h = \$4.40, revenue is predicted to be higher in the sealed-bid auctions as bidders go for it, while in the clock auctions going for it is contingent on the observed drop-out prices of the single-unit bidders. As a consequence profits are predicted to be higher under the clock auctions. However, there is very little to choose between the two auction formats on efficiency grounds. The intuition for this last result is that ex post, at times bidders should have gone for it in the clock auction, whereas they always go for it in the sealed bid auctions and these two effects are essentially offsetting for v_h = \$4.40. These predictions regarding revenue, profits and efficiency are summarized in the bottom half of Table 1.

[Insert Table 1 around here]

3. Experimental Results

Our focus will be on bidding behavior, with some attention to revenue and efficiency as well. Throughout the analysis we will concentrate on bidding in the last 6 auctions under each v_h, when bidders would have become reasonably familiar with the environment (recall, that there were 11 auctions played for cash at each $v_{h,}$ and 1 dry run).[13]

A. Bid Patterns: Table 2 compares the frequency of optimal play between the two auction institutions. This table employs rather strict definitions for optimal play, with the exception of providing 5¢ "allowances" for "trembles" throughout. (For example, with v_h = \$3.00, in the clock auctions we count as equilibrium $d_2 \le v_2 + 0.05$ when $v_2 \le v_h$. Our results are robust to either eliminating these allowances or increasing them a bit.) Conclusion 1 is based on these results.

[Insert Table 2 around here]

Conclusion 1*: Bids are closer to optimal outcomes in the clock auction for all valuations.*

Bids being closer to equilibrium/optimal play in clock versus sealed-bid auctions are consistent with results from a large number of auction experiments: single-unit private value auctions (Kagel, Harstad, and Levin, 1987), single-unit common value auctions (Levin, Kagel, and Richard, 1996), and multi-unit demand, uniform-price auctions without synergies (Kagel and Levin, 2001; also see Kagel (1995) and Kagel and Levin (2002) for reviews of these results.) The experimental manipulations reported in Kagel and Levin (2001) suggest that it is the clock auction format, in conjunction with the information revealed to *h* by dropout prices, that is responsible for its superior performance.[14]

What's missing from Table 2, and will provide the focus of the detailed analysis that follows, is the pattern of deviations from equilibrium, which is quite different between the two auction institutions.

Conclusion 2*: In the sealed-bid auctions the data show a clear pattern of increasing bids at higher values as the theory predicts. However, bidders are imperfectly calibrated, as bids are substantially higher than they should be at lower values, and are lower than they should be at higher values, with the possible exception of* v_h *= \$5.10, where bids are close to optimal.*

[Insert Table 3 around here]

Support for this conclusion is reported in Table 3, where we have fit random effect Tobits to the bid data. We employ Tobits as there is a mass point at \$7.50, and we truncate all bids greater than \$7.50. An error components specification is employed with the error term $\epsilon_{it} = \delta_i + \zeta_{it}$, where δ_i is a subject-specific error term assumed to be constant between auctions within an experimental session, and ζ_{it} is an auction period error term. Standard assumptions regarding the error terms are employed; i.e., $\delta_i \sim (0, \sigma_\delta)$ and $\zeta_{it} \sim (0, \sigma_\zeta)$ where δ_i and ζ_{it} are independent among each other and among themselves. (Note, in our experimental design, the use of computer rivals, clearly supports the assumption that the δ_i are independent among each other.) With $n = 3$ we use $v_h = \$3.00$ as the intercept of the bid function and create a dummy variable (DV4) for $v_h = \$4.00$ (DV4 = 1 if $v_h \geq \$4.00$, 0 otherwise), and a second dummy, (DV440) for $v_h = \$4.40$ (DV440 = 1 if $v_h = \$4.40$, 0 otherwise). For $n = 5$ we use $v_h = \$4.00$ for the intercept of the bid function and create separate dummy variables $v_h = \$4.40$ (DV440 = 1 if $v_h \geq \$4.40$, 0 otherwise) and for $v_h = \$5.10$ (DV510 = 1 if $v_h = \$5.10$, 0 otherwise). Also reported, for the reader's convenience, are the

95% confidence intervals for bids predicted by the regression equations.

For the n = 3 both dummy variables are significant at the 5% level or better, indicating that bids were increasing in v_h, as they should be, for both b_1 and b_2. However, bids are substantially higher than they should be at lower values, and are lower than they should be at higher values, with the possible exception of v_h = \$5.10, where bids are close to optimal. For v_h = \$3.00 and \$4.00 the lower bound of the 95% confidence interval for unit 1 bids is well above the optimal bid, with unit 2 bids showing a similar pattern. For v_h = \$4.40 the upper bound of the 95% confidence interval for unit 1 bids is \$7.50, as it should be, the upper bound for unit 2 bids falls well short of the optimal bid of \$7.50. Finally, for v_h = \$4.00 the upper bound for unit 2 bids falls short of the lower bound for unit 1 bids. This is indicative of a relatively high frequency with which $b_1 \neq b_2$ when bidding above value, in violation of dominance arguments (but a dominance argument that would be far from transparent to most subjects).[15]

Similar results are reported for the n = 5 case. Both dummy variables are significant at the 5% level or better, indicating that bids were increasing in v_h, as they should be, for both b_1 and b_2. For v_h = \$4.00, the lower bound of the 95% confidence interval for unit 1 bids is well above the optimal bid of \$4.16, as is the lower bound for unit 2 bids. For v_h = \$4.40, the upper bound of the 95% confidence interval for unit 1 bids includes \$7.50, but is short of the optimal bid of \$7.50 for unit 2 bids. What is different from the n = 3 case is that there is overlap between unit 1 and unit 2 bids in all cases, indicating that we cannot reject a null hypothesis that $b_1 = b_2$, at least on the aggregate level.[16] No doubt these differences result from the fact that bidding less on unit 2 compared to unit 1 (conditional on $b_1 > v_h$) is more costly with n = 5 than n = 3.[17] Finally, the regression shows that for v_h = \$5.10, the lower bound of the 95% confidence interval for unit 1 bids is \$7.50, and the 95% confidence interval for unit 2 bids includes the optimal bid of \$7.50 as well. As such, even though the equilibrium requirement of $b_1 \geq b_2 \geq$\$7.50 is only satisfied 40.6%

of the time, bidders win both units 71.9% of the time. The net result is that deviations from optimality in payoff space are substantially smaller than deviations from optimality in the choice space.

Conclusion 3*: There is directional consistency in the clock auctions in the sense that two units are won more often at higher valuations (when they should be won) than at lower valuations (when they should not win two units). Further, at v_h = \$5.10 bidders "go for it" most of the time, as they should. However, at the intermediate valuations of region 2, there are large deviations from optimal bid patterns that are best explained by the exposure problem.*

Support for Conclusion 3 can be found in Table 4. First note that for $v_2 \leq v_h$ the frequency of winning two units is increasing in v_h. Further, consistent with optimal bidding, two units are won close to 100% of the time (in 83.3% of all auctions) with v_h = \$5.10. However, substantial deviations from optimal bid patterns are reported in region 2, at the intermediate values of \$4.00

[Insert Table 4 around here]

and \$4.40, where bidders face an exposure problem. This exposure problem expresses itself in three distinct ways for these two valuations:

(1) For $v_2 \leq v_h$ the primary deviation from equilibrium bidding involves demand reduction (winning 1 unit with positive profits). Further, in most cases this involves complete demand reduction; i.e., dropping out at the same time (or prior to) v_2, thereby not affecting the market price (87.5% of all cases with v_h = \$4.00, 74.7% of all cases with v_h = \$4.40) and never having to bid above v_h.

(2) In cases where $v_h < v_2 \leq \mathbf{P^*}$, where optimality requires "going for it," and necessarily involves bidding above v_h, there is a high frequency of winning zero units. The combined frequency of winning zero units and winning one unit (which involves "going for it" but then reconsidering when bidding above v_h) is consistently higher than the frequency of winning two

units. Further, random effect probits show that bidders win two units significantly less often than when $v_2 \leq v_h$. In contrast, the theory predicts no difference in the frequency of winning two units between these two cases.

(3) In cases where $v_2 > \max(\mathbf{P^*}, v_h)$ equilibrium play calls for bidding up to **P*** and dropping on both units at this point. This necessarily involves bidding above v_h, but rarely happens. Rather the modal response in three out of four of these cases is to drop out *prior to* **P***, typically dropping out very close to v_h (and dropping too soon is within a whisker of the modal response for the fourth case - $n = 3$ and $v_h = \$4.40$).

In contrast to bidding at these intermediate valuations ($4.00 and $4.40), bidders have little problem bidding above their valuation for $v_h = \$5.10$, when they are assured of a minimum profit of 30¢ by going for it. This response to the exposure problem in the clock auction is in marked contrast to the sealed-bid auction where bidders show no reluctance to bid above v_h on both units. This suggests that it is both the fear of losses, in conjunction with the auction format, that is responsible for the greater response to the exposure problem in the clock auction. What is it about the clock auction that accounts for this heightened fear of losses? In single-unit private value auctions when it is a dominant strategy to bid your valuation, there is close conformity to the dominant bidding strategy in clock auctions (with feedback on bidders' dropout prices), as opposed to substantial overbidding in sealed-bid auctions (Kagel et al., 1986). The evidence suggests that this results from the fact that it is much more transparent to bidders in the clock auctions that losses can and do occur strictly as a consequence of bidding above value (Kagel et al., 1986; Kagel, 1995; Kagel and Levin, 2001). In single-unit private value auctions, and in multi-unit demand auctions without synergies, this heightened awareness of the perils of bidding above one's valuation helps to improve bidder profits and to move play closer to the equilibrium outcome. Here it holds bidders back from achieving maximum profit and promotes deviations from the equilibrium outcome.

One final thing to note in Table 4 is the sharp (and consistent) difference between the frequency of winning two units for n = 3 versus n = 5 when $v_2 \leq v_n$, and when $v_2 \leq$ **P*** for the region 2 valuations of $4.00 and $4.40.[18] We suspect that this has little to do with the different number of rivals in the two treatments, but is a hysteresis effect brought on by the different behavior patterns rewarded under the remaining valuation in each case: v_h = $3.00 calls for complete demand reduction with n = 3 and v_h = $5.10 calls for "going for it" with n = 5. This result is summarized in Conclusion 4.

Conclusion 4*: There appears to be a strong hysteresis effect in the data, as with resale values of $4.00 and $4.40, the exposure problem is much more severe with n = 3 than with n = 5.*

The next conclusion takes a closer look at bidding with v_h = $5.10. In almost all respects this should be (and is) the valuation for which play is closest to optimal in both auction formats as it only takes a little arithmetic to realize that "going for it" yield a secure, minimum profit of 30¢ per auction. As a result, with repeated exposure to the problem one would expect more subjects to "get it." And they do, as the data show a clear learning effect, converging toward optimal play.

Conclusion 5*: With v_h = $5.10, we observe clear adjustments over time toward optimal play in both sealed-bid and clock auctions. However, winning two units is still more pronounced in the clock auctions.*

Support is provided by the random effect probits reported in Table 5, where we have pooled the data for both clock and sealed-bid auctions for v_h = $5.10. The dependent variable takes on a value of 1 in cases where two units were won (as optimal bidding requires) and 0 otherwise. We use the data for all auctions, excluding the dry runs. Model 1 includes a single dummy variable, Dclock = 1 if a clock auction, 0 if sealed bid. The coefficient for the dummy is positive and significant at the 10% level, indicating that play is closer to the optimal outcome in the clock auctions. Model 2 introduces a second dummy variable, Dearly = 1 for the first 5 auctions with v_h = \$5.10, 0 for the last 6, in an effort to identify possible learning/adjustment effects. These are clearly present as indicated by the relatively large, statistically significant negative coefficient value for the Dearly dummy. Finally, the introduction of the Dearly dummy has virtually no effect on the magnitude of the Dclock dummy or its standard error.[19]

[Insert Table 5 around here]

This is the one case where we observe clear learning/adjustments toward optimal play in the data. Specifications searching for learning/adjustment effects for other valuations reveal considerably more noise in early play (higher variances and less stable coefficient values), as opposed to any clear adjustments toward optimal play.[20]

B. Profits, Efficiency and Revenue: Bidders' profits provide a measure of success in terms of payoffs, and provide a convenient way of characterizing performance in terms of a single outcome measure. Employing the standard measure of efficiency for auctions of this sort (see Ledyard et al., 1997 or Plott, 1997), efficiency is defined as the sum of the values of two units sold in each auction period (including the synergy bonus, if relevant) as a percentage of the *highest* total value for two units. Note that 100% efficiency is not always attainable at the Nash equilibrium for our game. Revenue is what the seller would have earned in each period. In computing revenue and efficiency we are keenly aware that the same results might not emerge in auctions where all bidders are human. As noted, computer rivals were employed to minimize possible complications associated with learning against human rivals who may be playing out-of-equilibrium strategies. Out-of-equilibrium play may affect different institutions differently. (See, for example, Katok and Roth, 2004, who demonstrate that the ascending bid auction is much more sensitive to out-of-equilibrium play by small bidders than the descending bid auction.) On the other hand, there is very limited experimental data on efficiency and revenue, measures of central importance to economists, in environments such as this, and we believe that the present data are at least suggestive of what will be observed in interactive settings. Finally, in reporting revenue and efficiency we provide a benchmark against which future experiments with all human bidders can compare results on these important issues.

Conclusion 6*: Profits are consistently and significantly less than would have been achieved with optimal bidding in all but one case (v_h = \$4.00, n = 3, sealed-bid auction). Realized profits are always higher in the clock auctions, but in a number of cases the significantly higher profits the theory predicts fail to be realized.*

Table 6 reports profits -- actual, predicted and the difference between the two -- for both auctions, along with the difference between actual profits in the two auctions (sealed-bid less clock). At v_h = \$3.00, bidders earned negative profits averaging -60¢ and -15¢ in the sealed-bid and clock auctions respectively. Using subject averages as the unit of observation, profits in the sealed-bid auctions were significantly below zero ($t = 2.80$, $p < .01$, 2-tailed test), and significantly less than in the clock auctions ($t = -1.93$, $p < .10$, 2-tailed test), reflective of the large differences in equilibrium play between the two auctions. For $n = 3$, with v_h = \$4.00 and \$4.40 optimal bidding predicts 20–24% higher profits in clock compared to sealed-bid auctions. These higher predicted profits result from the greater flexibility afforded by the information revelation in the clock auctions. However, these better profit opportunities do not result in significantly higher realized profits as (i) the exposure problem serves to promote the strong demand reduction found in the clock auctions, which wipes out most of the advantages resulting from information revelation in this case, and (ii) the deviations from equilibrium in the sealed-bid auctions are not severe enough to impact significantly on earnings. With $n = 5$, there are even larger percentage differences between predicted profits in clock versus sealed-bid auctions for v_h = \$4.00 and \$4.40. These are realized in one case (v_h = \$4.00), with reasonably large, but statistically insignificant, differences in the predicted direction in the other

[Insert Table 6 around here]

case (v_h = $4.40). The theory comes off better here compared to n = 3 case because (i) the demand reduction effect is weaker in the clock auctions with n = 5 (recall Conclusion 4) and (ii) there are larger reductions in realized profits compared to predicted profits in the sealed-bid auctions with n = 5. Finally, what differences there are in bidding with v_h = $5.10 do *not* translate into significant differences in earnings between the two auction formats (t = -1.29).[21]

Conclusion 7*: Optimal bidding predicts either the same or higher efficiency in clock compared to sealed-bid auctions. In contrast to these predictions, actual efficiency differences are quite mixed, with the only significant difference recorded in favor of the sealed-bid auction.*

Table 7 reports efficiency outcomes. At the $3.00 value actual efficiency is very close to predicted levels in both auctions. This is not surprising for the clock auctions where bidding is relatively close to equilibrium, but is somewhat unexpected in the sealed-bid auctions with its large deviations from equilibrium outcomes. These deviations apparently have minimal impact on efficiency since the overbidding occasionally produces large efficiency gains (relative to the predicted outcome) as a result of the synergy bonus. At the $4.00 value, the clock auction should yield large efficiency gains compared to the sealed-bid auction. However, these gains go almost entirely unrealized as (i) the exposure problem serves to promote demand reduction in the clock auctions, which wipes out most of the efficiency gains predicted, and (ii) deviations from equilibrium bidding in the sealed-bid auctions have essentially no effect relative to predicted efficiency. This last result is due to the fact that with the synergy bonus-overbidding occasionally produces large efficiency gains relative to the predicted outcome.[22] Finally, actual efficiency is well below predicted efficiency at higher values in both auctions as bidders do not “go for it” often enough to take full advantage of the synergy bonus.

[Insert Table 7 around here]

Conclusion 8*: Revenue is consistently higher in the sealed-bid auctions, and is significantly higher in four out of six cases. The higher revenues do not come at the expense of any significant efficiency loss relative to the clock auctions.*

Revenues are reported in Table 8. Revenues are predicted to be the same at v_h = \$3.00 and \$5.10, to be higher in the clock auctions with v_h = \$4.00, and to be higher in the sealed-bid auctions with v_h = \$4.40. The actual data, however, show uniformly higher revenues in the sealed-bid auction: 4.0% to 18.8% higher with n = 3, 3.0% to 12.7% higher with n = 5 (calculated as a percentage of realized revenue in the clock auctions). Note that these increased revenues do not come at the expense of reduced efficiencies, as Table 7 reports no significant decreases in efficiency in sealed-bid compared to clock auctions, and one case with significantly higher efficiency in the sealed-bid auction.

[Insert Table 8 around here]

4. Summary and Conclusions

We develop a model of multi-unit demand auctions with synergies, and explore behavior experimentally, comparing sealed-bid and ascending-bid uniform-price auctions (winning bidders pay the highest rejected bid). We employ a simple, tractable demand structure: Several single-unit demand bidders and one bidder demanding up to two units. We further simplify the structure by having computers play the dominant strategy that single-unit bidders have of always bidding their value. In spite of its simplicity, the key economic incentives present in uniform-price auctions with synergies are all captured: the synergy effect, which promotes bidding above standalone values; the exposure problem which may deter bidders from pursing this aggressive biding strategy, thereby reducing economic efficiency; and the monopsony power that multi-unit demand bidders can exploit to reduce prices in a uniform-price auction.

The experiment shows that bidding is closer to optimal outcomes in the clock auctions, consistent with evidence from a number of other auction environments (Kagel, et al., 1987; Levin, et al., 1996; Kagel and Levin, 2001). Further, although they do not bid optimally, in most cases bidders behave sensibly: Bidding under the highest valuation, where the optimal play is most transparent, generates levels of optimal play that are comparable to the highest levels reported for subjects in any auction experiment. Demand functions estimated for the sealed-bid auctions are monotonically increasing in bidders' valuations. In the clock auctions, there is a higher frequency of "going for it" at higher valuations, when multi-unit demand bidders should attempt to obtain both units.

Nevertheless, there is considerable out-of-equilibrium play under both institutions, with systematic differences in the pattern of out-of-equilibrium play. The most interesting and dramatic differences occur at intermediate valuations where the theory requires balancing demand reduction incentives against the synergy bonus, while exposing bidders to possible losses. At these values the exposure problem promotes relatively strong demand reduction in the clock auctions, whereas optimal bidding requires that bidders "go for it." In contrast, there is barely any response to the exposure problem in the sealed-bid auctions, with bidders consistently bidding above value on both units. This suggests that it is both the fact that bidders are exposed to losses, in conjunction with the auction format, that is responsible for the greater response to the exposure problem in the clock auction. The clock auction format (with feedback on bidders dropout prices) makes it much more transparent to bidders that losses can and do occur strictly as a consequence of bidding above value (Kagel et al., 1987; Kagel, 1995; Kagel and Levin, 2001). This heightened awareness of the perils of bidding above ones' valuation helps to improve bidder profits and to move play closer to the equilibrium outcome in single-unit private value auctions and in multi-unit demand auctions without synergies. With synergies it holds bidders back from achieving maximum profit and generates deviations from the equilibrium outcome.[23]

The element of the exposure problem at play in our experiment stems from loss aversion.[24] One might argue that this is strictly a psychological phenomena as fully rational bidders should not be deterred by the prospect of losses from bidding aggressively to maximize expected profits. Be that as it may, this does not make the problem any less real.

How relevant are these findings of out-of-equilibrium play to "real world" auctions? We argue that for low stakes field settings, or those that for one reason or another, bidders do not employ high-powered consultants (as occurred in at least one U. S. spectrum auction), our observations are directly relevant. (See, for example Naik (1996) who discusses problems small firms had in PCS auctions, and Mills (1997) who discusses problems a number of bidders had in the Interactive Video and Data Services (IVDS) auction.)

But what about high-stakes settings with high-powered consultants on all sides, as is typically the case in auctions involving many millions or even billions of dollars? Extrapolating laboratory results to such cases is clearly more speculative and one must be much more careful in assessing the applicability of the results. However, we do know that the exposure problem has been a major concern in designing auctions with synergies. What our experiment highlights is that the problem is likely to be more prevalent in an ascending price format than in a sealed-bid auction format.

There are a number of obvious and interesting extensions to the experimental results reported here. One would be to conduct these auctions with all human bidders to see what differences possible out-of-equilibrium play by single-unit bidders would have on multi-unit demand bidders. Another would be to permit the use of package bidding, either under the present set-up or with all human bidders, to see how well this serves to overcome the exposure problem and to identify what, if any, "complexity problems" this might pose for bidders. In addition, a natural next step would be to extend the analysis to environments with several multi-unit demand bidders and/or to environments in which items are imperfect substitutes and sold in separate auction markets. And to consider some of the other tradeoffs between sealed-bid and clock auctions such as the increased costs of clock auctions (both in terms of configuring bidders and the longer time required to conduct such auctions), the clock's potential for information aggregation in common value auctions, and differences in collusive possibilities between the two auction formats.[25]

1. Particular attention has been given to the effects of uniform-price auction rules because they are relatively easy to characterize and to implement, and are reasonably close in format to the one employed in the spectrum auctions (see Cramton, 1995).

2. Bykowsky et al. discuss two types of exposure problems that may exist in complex environments with synergies. In our simple environment only the first of these potential problems exists, namely exposure to bidding above the stand-alone value and not obtaining the desired package, or obtaining it but at higher prices than anticipated. Bidder responsiveness to the exposure problem in this case is akin to *loss aversion* (Kahneman and Tversky, 1979). Package bidding has its own problems. These include the free rider/threshold problem and the computational complexity problem (Charles River Associates and Market Design Inc., 1998).

3. That is, we announce the fact that subjects are playing against computers and that the computers will always bid their value, but do not discuss the basis for the computers' bidding strategy.

4. Krishna and Rosenthal extend their analysis to auctions in which there is more than one bidder with increasing returns. The Krishna-Rosenthal model is (arguably) closer, in some respects, to the U. S. spectrum auctions than our experimental design. Their model does not, however, deal with possible synergies *within* a given market as ours does. It is these underlying economic and behavioral forces that our experiment is designed to investigate rather than any effort to faithfully replicate any particular spectrum sale design.

5. This result comes about for different reasons in the two models: In our case, bidding the same on both items follows from a dominance argument (see Appendix A). In Krishna and Rosenthal it follows from the assumption of equal numbers of single-unit bidders in each market in conjunction with their focusing on symmetric Nash bidding strategies.

6. We are keenly aware that the way synergies have been modeled here is not without loss of generality. However, we need to implement a design that is both theoretically tractable and easy to implement and explain. Setting $\alpha = 1$ and $m = 2$ greatly simplifies bidders' calculations while still yielding the rich behavioral space characterized in Figure 1 below.

7. Given that the computers follow the dominant strategy, h has no incentive to drop out and re-enter the auction. However, we plan to conduct additional experiments where the irrevocable exit rule may become relevant.

8.The auction is formally modeled as a continuous-time game. However, we want to take into account the possibility that bidder *i*'s strategy may be to reduce her quantity at a given time, while bidder *j*'s strategy may be to reduce his quantity at the soonest possible instant after bidder *i* does. This requires allowing "moves that occur consecutively at the same moment in time" (Simon and Stinchcombe, 1989; also see Ausubel, 1997).

9.The underlying support for valuations, along with the number of computer rivals, were chosen with an eye towards comparing behavior here with earlier multi-unit demand auctions with flat demands (no synergies) which call for demand reduction at all values (Kagel and Levin, 2001).

10.This restriction was built into the software. We have tested for the impact of this restriction in other multi-unit demand auctions (without synergies), with these tests showing that this restriction has no effect on bidding strategies (Kagel and Levin, 2001).

11.In those few cases where end-of-experiment earnings were below \$2.00, a token \$2.00 payment was provided. This was not announced in advance and only applied to two or three subjects.

12.In the clock auctions with $n = 3$, v_1 is often below \$5.10, so there is very little information to be gained about whether or not *h*'s recognize that "going for it" is the optimal strategy in these cases. This problem is reduced substantially with $n = 5$. Further, rather substantial deviations from the optimal bidding strategy of $b_1 = b_2 \geq \$7.50$ (for example, bidding above \$5.10 but below \$7.50) frequently incur no penalty in sealed-bid auctions with $n = 3$, but such deviations are much more likely to be punished with $n = 5$.

13.The choice of the last 6 auctions is somewhat arbitrary, but it does distinguish more from less experienced play, and the results are robust to adding or dropping an auction or two to either side of 6. We will occasionally make reference to obvious learning/adjustment patterns in bids, but forgo any kind of detailed analysis as the paper is already quite long.

14.Kagel and Levin (2001) show that in a multi-unit demand, uniform-price auction, with flat demand (i.e., with no synergies so that only the demand reduction effect is in present), a clock auction *with no feedback* on rivals' drop-out prices looks no different from the sealed-bid version of the auction. Further, a sealed-bid auction with the crucial dropout information employed in the clock auction *provided by the experimenter*, improves performance, but still comes up short compared to a clock auction with rivals' drop-out prices announced.

15. The raw data on this score is as follows: Conditional on $b_1 > v_h + 0.05$, the frequency of $b_1 - b_2 > .25$ or $b_2 \leq v_h$ is 51.0% with v_h = \$3.00, 35.6% with v_h = \$4.00 and 31.9% with v_h = \$4.40. (Note, our calculations provide small allowances for deviations from optimal play to leave some room for "trembles.")

16. This is clear from the raw data as well, which shows that conditional on $b_1 \geq v_h$, the relative frequency of $b_1 > b_2$ has been cut by 50% compared to n =3: for n = 5, conditional on $b_1 > v_h + 0.05$, the frequency of $b_1 - b_2 > .25$ or $b_2 \leq v_h$ is 17.2% with v_h = \$4.00, 17.7% with v_h = \$4.40 and 12.0% with v_h = \$5.10.

17. For example, with v_h = \$4.00, numerical evaluation of outcomes for the rule of thumb, $b_1 = 1.5v_h$ and $b_2 = v_h$, yields positive profits, but profits that are lower by approximately 33¢ per auction, than the optimal strategy with n = 3. With n = 5, this rule yields negative profits which are lower by approximately 53¢ per auction than profits generated by the optimal strategy.

18. These differences are statistically significant at conventional levels. Tests for statistical significance consisted of the following: Take all cases where $v_2 \leq v_h$. Run a random effects probit (with subject as the random component) and dependent variable = 1 when a bidder wins 1 unit with positive profits), 0 otherwise. Let v_h = \$3.00 serve as the baseline and define dummy variables DV4 = 1 if $v_h \geq$ \$4.00, 0 otherwise; DV440 = 1 if $v_h \geq$ \$4.40, 0 otherwise; and DV510 = 1 if v_h = \$5.10, 0 otherwise; and DN5 = 1 if n = 5, 0 otherwise. This yields:

$$\underset{(0.292)^{+}}{\text{Win1} = 0.502} - \underset{(0.222)^{**}}{0.601\text{DV4}} - \underset{(0.159)}{0.187\text{DV440}} - \underset{(0.258)^{**}}{0.630\text{DV510}} - \underset{(0.358)^{**}}{0.932\text{DN5}}$$

19. A third specification introducing an interaction effect between the Dearly and Dclock dummies fails to reduce the log likelihood function by a significant amount.

20. We have made limited inquires into the effect of bringing back experienced bidders. These sessions show that experienced subjects in sealed-bid auctions do not perform materially better than inexperienced ones. There are, however, significant differences depending on bidders' past experience: Those with clock experience bid less aggressively at lower valuations compared to those with sealed-bid experience. These differences constitute a direct carryover of the differences observed between auctions for inexperienced bidders (see www.econ.ohio-state.edu/kagel/final.pdf).

21.The differences we do find are aided by the higher predicted profits in the clock auctions. The latter results from sampling variability in terms of the computer values drawn, as optimal bidding yields the same equilibrium outcome in both cases.

22.Actual efficiency is greater than predicted efficiency in the sealed-bid auctions in these two cases. There is no inconsistency here since predicted efficiency is less than 100%.

23.Katok and Roth (2004) compare descending price "Dutch" auctions and ascending price "English" in an environment similar to ours (homogeneous goods sold in a single market with one large and several small bidders). One of their treatment conditions in the ascending price auctions presents an exposure problem for the large bidder in that the equilibrium calls for bidding above his/her stand alone value for a single unit. In 38% (49/129) of all such auctions the large bidder is *not* willing to bid above this stand alone value, which represents an exposure problem similar to the one reported here.

24.In auctions with synergies one can readily establish distributions of values for which, absent package bidding, the only auction outcome possibilities are inefficiency (as the synergies fail to be realized) or individual losses (see, for example, Bykowsky, et al., 1995). This too is referred to as the exposure problem. However, there is a dramatic difference between this exposure problem, where rational bidders' bid passively in equilibrium, and the exposure problem identified here which is based on the behavioral phenomena of loss aversion in conjunction with the clock auction format.

25.Alternatives to the clock auction, with much of the same potential benefits and fewer drawbacks (in theory at least) on these dimensions are "survivor" auctions (Fujishima et al., 1999) or two-stage sealed-bid auctions (Perry et al., 2000). We are currently conducting experiments comparing survivor auctions with sealed-bid and clock auctions.

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Table 1

Summary of Theoretical Predictions for Multi-Unit Bidder Values Employed

Equilibrium Bids		
Bidder value (v_h)	Sealed Bid Auctions	Clock Auctions
\$3.00 (with 3 computer rivals only)	$b_1 = v_h; b_2 = 0$	If $v_2 \le v_h, d_1 = \$3.00$ and $d_2 \le v_2$. If $v_2 > v_h$, $d_1 \in$ [\$3.00, max (\$3.00, v_3)] and $d_2 \in$ [0, max (\$3.00, v_3)].
\$4.00 (with 3 and 5 computer rivals)	with 3 computers: $b_1 = b_2 = \$4.34$ with 5 computers: $b_1 = b_2 = \$4.16$	If $v_2 \le = \$4.50$, $d_1 = d_2 = \$7.50$ If $v_2 > = \$4.50$, $d_1 = d_2 \in$ [\$4.50, max (\$4.50, v_3)].
\$4.40 (with 3 and 5 computer rivals)	$b_1 = b_2 \ge \$7.50$ (earn two units)	If $v_2 \le$ \$5.70, $d_1 = d_2 = \$7.50$. If $v_2 >$ \$5.70, $d_1 = d_2 \in$ [\$5.70, max (\$5.70, v_3)]
\$5.10 (with 5 computer rivals only)	$b_1 = b_2 \ge \$7.50$ (earn two units)	$d_1 = d_2 = \$7.50$.
Revenue, Profits and Efficiency		
\$3.00 & \$5.10	Revenue, profits and efficiency are the same in both auction formats.	
\$4.00	Revenue, profits and efficiency are higher in the clock auctions than in the sealed bid auctions	
\$4.40	Revenue is higher and profits are lower in the sealed-bid auctions. Efficiency is essentially the same in the two auctions.	

Table 2
Comparing Frequency of Equilibrium Play Under Different Auction Institutions
(raw data in parentheses)

v_h	No. Computers	Clock	Sealed Bid*
\$3.00	3	46.3% (111/240)	2.6% (5/189)
\$4.00	3	23.7% (57/240)	1.6% (3/188)
	5	22.3% (54/240)	3.1% (6/192)
\$4.40	3	38.8% (93/240)	27.7% (52/188)
	5	35.8% (86/240)	27.1% (52/192)
\$5.10	5	79.2% (190/240)	40.6% (78/192)

* For n = 3, one subject left before her session ended resulting in fewer than 6 auctions at each v_h value for that subject.

Table 3

Sealed Bid Auctions: Random Effect Tobit Estimates of Bid Function

No. of Computers		95% confidence interval for bids [predicted bids in brackets]				No. Subjects	No. Observations
		v_h=$3.00	v_h=$4.00	v_h=$4.40	V_h=$5.10		
3	b_1= 6.19 V3 + 2.08 DV4 + 1.62 DV440 (0.782)** (0.948)* (0.989)	4.65-7.50 [3.00]	6.68-7.50 [4.34]	7.50 [7.50]	NA	32	565
	b_2 = 4.19V3 + 1.33 DV4 + 0.67 DV440 (0.491)** (0.513)** (0.524)	3.23-5.16 [0.0]	4.54-6.50 [4.34]	5.20-7.18 [7.50]	NA		
5	b_1 = 6.99 V4 + 1.04 DV440 + 1.69 DV510 (0.515)** (0.452)* (0.482)**	NA	5.98-7.50 [4.16]	6.99-7.50 [7.50]	7.50 [7.50]	32	576
	b_2 = 6.09 V4 + 0.70 DV440 + 0.88 DV510 (0.155)** (0.154)** (0.160)**	NA	5.78-6.39 [4.16]	6.48-7.10 [7.50]	7.34-7.50 [7.50]		

* significantly different from 0 at the .05 level, 2-tailed test

** significantly different from 0 at the .01 level, 2-tailed test

TABLE 4

Bid Patterns in Clock Auctions
(raw data in parentheses)

$v_2 \le v_h$					$P^* \ge v_2 > v_h$			$v_2 > \text{Max}\{v_h, \mathbf{P}^+\}$				
No. Computers	Value	Win 2 Units	Demand Reduction	Other	Win 2 Units	Win 0 Units	Win 1 Unit	Equilibrium	Drop Too Soon & Win 0	Drop Too Late & Win 0	Win 1	Win 2
3	\$3.00	23.7% (18/76)	**63.2% (48/76)**	13.2% (10/76)	NA	NA	NA	**44.5% (73/164)**	13.4% (22/164)	14.6% (24/164)	20.1% (33/164)	7.3% (12/164)
	\$4.00	**33.8% (44/130)**	49.2% (64/130)	16.9% (22/130)	**25.0% (6/24)**	50.0% (12/24)	25.0% (6/24)	**8.1% (7/86)**	53.5% (46/86)	8.1% (7/86)	17.4% (15/86)	12.2% (11/86)
	\$4.40	**47.0% (70/149)**	43.0% (64/149)	10.1% (15/149)	**38.0% (19/50)**	38.0% (19/50)	24.0% (12/50)	**9.8% (4/41)**	39.0% (16/41)	4.9% (2/41)	2.4% (1/41)	43.9% (18/41)
5	\$4.00	**45.2% (28/62)**	25.8% (16/62)	29.0% (18/62)	**39.1% (9/23)**	39.1% (9/23)	21.7% (5/23)	**11.0% (17/155)**	47.7% (74/155)	14.8% (23/155)	12.9% (20/155)	13.5% (21/155)
	\$4.40	**60.8% (45/74)**	25.7% (19/74)	13.5% (10/74)	**47.2% (34/72)**	31.9% (23/72)	20.8% (15/72)	**7.4% (7/94)**	38.3% (36/94)	12.8% (12/94)	19.1% (18/94)	22.3% (21/94)
	\$5.10	**83.3% (95/114)**	11.4% (13/114)	5.3% (6/114)	**75.4% (95/126)**	14.3% (18/126)	10.3% (13/126)	NA	NA	NA	NA	NA

Cells with outcomes in bold constitute equilibrium predictions; i.e., they should be 100% in all cases.
NA: Not applicable

Table 5

Probits Comparing Winning 2 Units in Clock vs Sealed Bid Auctions: v_h = $5.10

Variable	Model 1	Model 2
Constant	0.706 (0.211)**	0.981 (0.222)**
DClock	0.439 (0.252)+	0.458 (0.255)+
DEarly	-----	-0.540 (0.122)**
Log Likelihood	-362.9	-352.8
No. Observations	792	792
No. Subjects	72	72

+ Significantly different from 0 at the 10% level, 2-tailed test

** Significantly different from 0 at the 1% level, 2-tailed test

Table 6
Profits (in dollars)
(standard errors of mean in parentheses)

No. Computers	v_h value	Sealed Bid Auctions Actual	Sealed Bid Auctions Predicted	Sealed Bid Auctions Difference	Clock Auctions Actual	Clock Auctions Predicted	Clock Auctions Difference	Difference (actual): Sealed Bid less Clock (t-statistics)
3	$3.00	-0.60 (0.214)	0.35 (0.056)	-0.959** (0.187)	-0.15 (0.127)	0.36 (0.032)	-0.495** (0.124)	-0.45 (-1.93)+
	$4.00	0.72 (0.176)	0.88 (0.176)	-0.168 (0.111)	0.73 (0.120)	1.09 (0.144)	-0.365** (0.104)	-0.01 (-0.02)
	$4.40	1.24 (0.207)	1.74 (0.240)	-0.497** (1.33)	1.29 (0.150)	2.10 (0.183)	-0.805** (0.146)	-0.05 (-0.20)
5	$4.00	-0.25 (0.123)	0.21 (0.073)	-0.459** (0.092)	0.18 (0.093)	0.44 (0.104)	-0.260** (0.074)	-0.43 (-2.80)**
	$4.40	0.25 (0.144)	0.54 (0.260)	-0.286** (0.077)	0.46 (0.145)	0.84 (0.140)	-0.385** (0.074)	-0.21 (-0.89)
	$5.10	2.03 (0.263)	2.69 (0.149)	-0.661** (0162)	2.45 (0.202)	2.93 (0.138)	-0.490** (0.129)	-0.42 (-1.29)

+ Significantly different from 0 at the 10% level, 2 tailed t-test
** Significantly different from 0 at the 1% level, 2 tailed t-test

TABLE 7

EFFICIENCY
(standard errors in parentheses)

No. Computers	v_h Value	Sealed Bid Auctions			Clock Auctions			Difference (actual): Sealed Bid less Clock (t-statistic)
		Actual	Predicted	Difference	Actual	Predicted	Difference	
3	$3.00	91.9% (0.83)	92.7% (0.96)	-0.8% (1.18)	92.9% (0.72)	93.5% (0.56)	-0.6% (0.81)	-1.00 (-0.93)
	$4.00	91.9% (1.04)	90.8% (0.78)	1.2% (1.17)	89.0% (1.05)	98.9% (0.19)	-9.8%** (1.08)	2.90 (1.93)+
	$4.40	90.6% (1.29)	99.8% (0.08)	-9.2%** (1.29)	88.4% (1.34)	99.7% (0.07)	-11.4%** (1.35)	2.20 (1.17)
5	$4.00	93.8% (0.63)	92.6% (0.70)	1.2% (1.00)	94.0% (0.62)	97.9% (0.28)	-3.9%** (0.70)	-0.20 (-0.15)
	$4.40	94.0% (0.78)	99.0% (0.17)	-5.0%** (0.84)	93.6% (0.80)	99.2% (0.15)	-5.7%** (0.86)	0.40 (0.39)
	$5.10	94.4% (1.39)	100% (0.00)	-5.6%** (1.39)	95.4% (1.20)	100% (0.00)	-6.1%** (1.20)	-1.00 (-0.52)

** Significantly different from 0 at the 1% level, 2 tailed t-test

TABLE 8

Revenue (in dollars)
(standard error in parentheses)

No. Computers	v_h Value	Sealed Bid Auctions			Clock Auctions			Difference (actual): Sealed Bid less Clock (t-statistic)
		Actual	Predicted	Difference	Actual	Predicted	Difference	
3	\$3.00	8.48 (0.33)	5.80 (0.12)	2.68** (0.29)	7.14 (0.24)	5.84 (0.12)	1.30** (0.24)	1.34 (3.38)**
	\$4.00	9.33 (0.24)	8.42 (0.09)	0.91** (0.23)	8.31 (0.23)	9.93 (0.13)	-1.62** (0.24)	1.02 (3.01)**
	\$4.40	9.60 (0.26)	11.45 (0.24)	-1.85** (0.30)	9.23 (0.31)	10.81 (0.18)	-1.58** (0.26)	0.37 (0.88)
5	\$4.00	11.10 (0.17)	9.01 (0.12)	2.08** (0.19)	9.85 (0.16)	10.16 (0.13)	-0.31 (0.16)	1.25 (5.24)**
	\$4.40	11.66 (0.17)	12.64 (0.14)	-1.00** (0.16)	10.88 (0.21)	11.72 (0.13)	-0.84** (0.19)	0.78 (2.78)**
	\$5.10	12.13 (0.16)	12.60 (0.15)	-0.47** (0.15)	11.78 (1.27)	12.37 (0.14)	-0.59 (0.16)	0.35 (1.43)

** Significantly different from 0 at the 1% level, 2 tailed t-test.

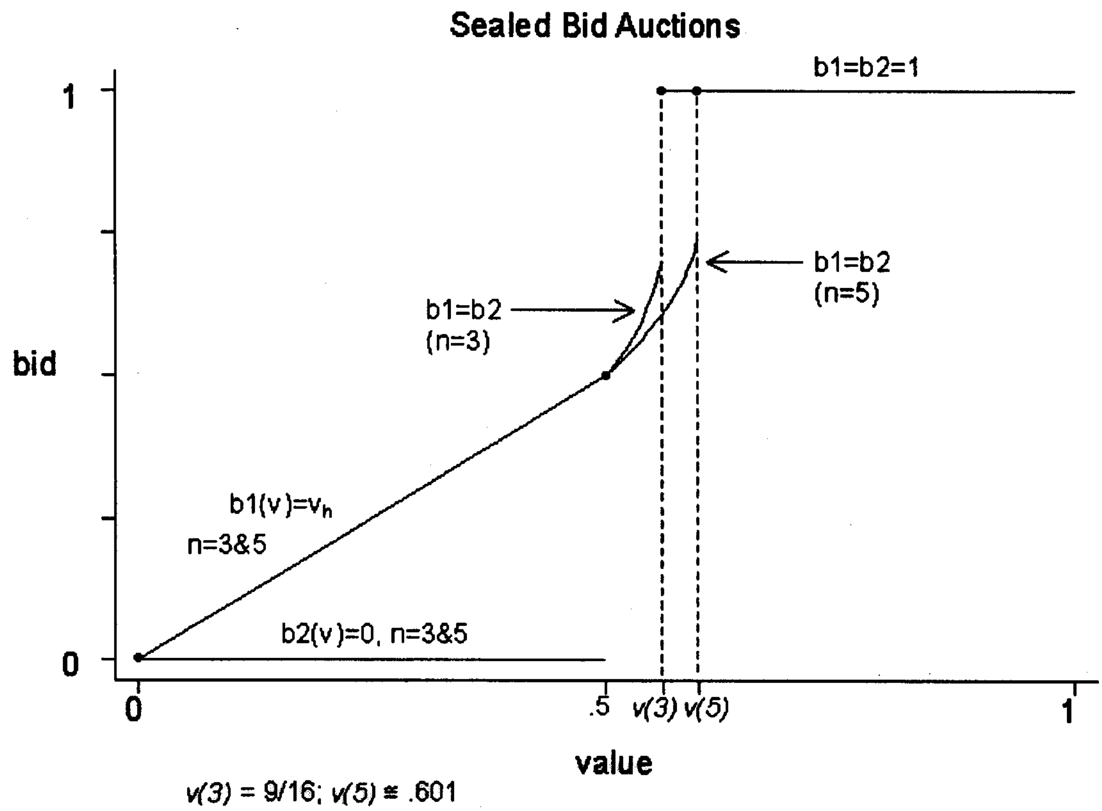
Figure Caption

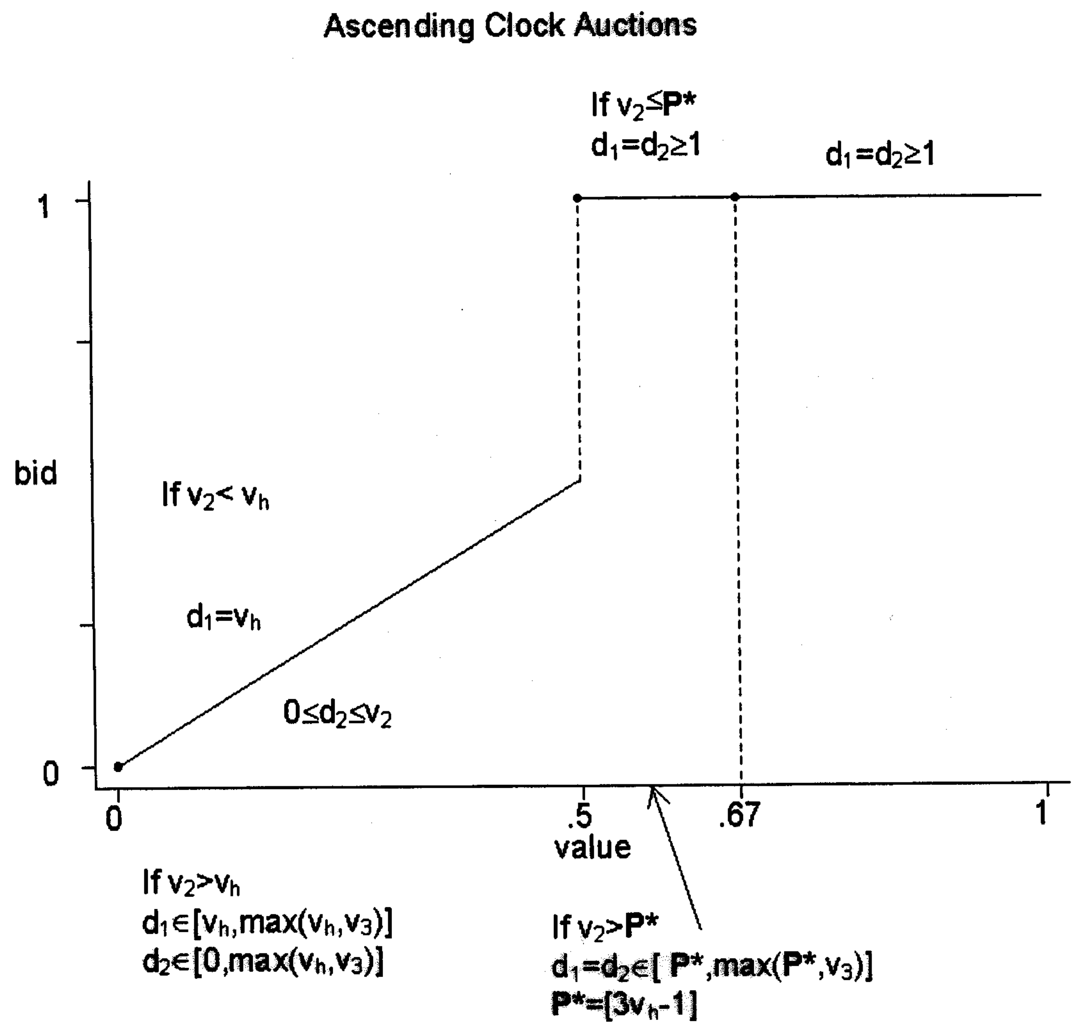
Figure 1: Bid patterns for the multi-unit demand bidder (*h*) in the sealed-bid auctions (top panel) and the ascending price auctions (bottom panel). There are three bid regions:

Region 1: Allocations and prices are the same in both auctions as bidder practice demand reduction.

Region 2: Prices and allocations differ as *h* never wins one unit in the ascending price auctions, but does so at times in the sealed bid auctions. There is a potential exposure problem in region 2 in both cases.

Region 3: Bidders "go for it," winning both units for sure. The size of region 3 is smaller in the ascending price auctions and, unlike the sealed-bid auctions, there are no potential exposure problems.


Sealed Bid Auctions
b1=b2=1
1
b1=b2
(n=5)
b1=b2
(n=3)
bid
b1(v)=v_h
n=3&5
b2(v)=0, n=3&5
0
0
.5
v(3)
v(5)
1
value
v(3) = 9/16; v(5) ≅ .601


Ascending Clock Auctions
If $v_2 \leq P^*$
$d_1=d_2\geq 1$
$d_1=d_2\geq 1$
1
bid
If $v_2 < v_h$
$d_1=v_h$
$0\leq d_2\leq v_2$
0
0
.5
.67
1
value
If $v_2>v_h$
$d_1\in[v_h,\max(v_h,v_3)]$
$d_2\in[0,\max(v_h,v_3)]$
If $v_2>P^*$
$d_1=d_2\in[P^*,\max(P^*,v_3)]$
$P^*=[3v_h-1]$

Appendix A: **Sealed-Bid Uniform Price Auction with Synergies.**

We derive theoretical predictions provided in Sections I and II of the text..

There are $(n+1) > 2$ bidders and $m = 2$ units auctioned. The $(n+1)^{th}$ bidder, denoted by h, has a concave utility function $u(\pi)$ that is normalized so that $u(0) = 0$ and $u'(0) = 1$ and where π represents earnings net of cost of purchasing units. h demands two units valuing each, by itself, at $u(V)$. Bidders $1, 2, ..., n$, demand only one unit valuing it at $V_1, V_2, ..., V_n$, respectively. $V_1, V_2, ..., V_n$ and V are independent random variables from $F(\cdot)$ and $F_h(\cdot)$, respectively on the common support $[0, 1]$. $V_{(k)}$ denotes the k^{th} *order statistic* of $V_1, V_2, ..., V_n$, and $F_{(k)}$ its distribution function. Let $v_1, v_2, ..., v_n$ and v, be the realizations of $V_1, V_2, ..., V_n$ and V and, without loss of generality, assume that $v_1 \geq v_2 \geq, ..., \geq v_n$. The good is only available in integer units. We are interested in a *sealed-bid uniform-price* (highest losing bid) auction (SBUPA). Bidders $1, ..., n$, who demand a single unit have a *dominant strategy* to bid their true value. Denote by p the price per unit h pays. Although the value of winning a single unit for h is $u(v-p)$, there is a supper addictive value for winning both units. If h wins both units her utility is $u(2v + g(v) - 2p)$, i.e., she is getting an extra $g(v)$, where $g(0) = 0$ and $g'(v) > 0$. In the experiment $g(v) = v$, which we assume throughout the appendix.

Without loss of generality assume that $b_1(v) \geq b_2(v)$ represents h's two (optimal) bids.

Lemma 1 ***(a)*** $b_1(v) \geq v$. ***(b)*** *If* $b_1(v) > v$, *then* $b_1(v) = b_2(v)$.

Proof. (**a**) Suppose (a) does not hold. This implies that there exists v^* such that $v^* > b_1(v^*) \geq b_2(v^*)$. But then, raising $b_1(v^*)$ from $b_1(v^*) < v^*$ to $b_1^{\#}(v^*) = v^*$ makes h, better off when it matters since in such events h will win one unit rather than zero, which results in strictly positive expected utility. (**b**) Suppose (b) does not hold. This implies that there exists v^* such that $b_1(v^*) > v^*$ and $b_1(v^*) > b_2(v^*)$. Case 1. $b_2(v^*) \geq v^*$. In this case the pair $\{b_1^{\#}(v^*) = b_2(v^*), b_2(v^*)\}$ dominates the alternative $\{b_1(v^*) > b_2(v^*), b_2(v^*)\}$, i.e., reducing $b_1(v^*) > b_2(v^*)$ to $b_1^{\#}(v^*) = b_2(v^*)$ dominates. Here is the reason: If h wins two or zero units, the proposed reduction in $b_1(v^*)$ does not matter. However, if h wins one unit then the price is at least v^* and with a positive probability strictly higher than v^*, implying that $E[u(v^* - p)] < 0$. Therefor, by reducing b_1 to b_2 h wins no units, which generates strictly positive expected utility realtive to $b_1 > b_2$ and losing money on the single unit earned. Case 2. $b_2(v^*) < v^*$. Using similar arguments we can show that the pair of bids $\{b_1^{\#}(v^*) = v^*, b_2(v^*) < v^*\}$ dominate $\{b_1(v^*) > v^*, b_2(v^*) < v^*\}$. ■

Part 1: We start the analysis by assuming first that $b_1(v) = b_2(v)$ and thus by Lemma 1-(a), $b_1(v) = b_2(v) \geq v$. In this case h's maximization problem becomes:

(A1) $\underset{b \geq v}{Max}\{\int_0^b nf(t)[F(t)]^{n-1}u(3v-2t)dt + n(1-F(b))[F(b)]^{n-1}u(v-b)\}.$

The integral component of (A1) represents h's expected utility from winning two units, in which case n bids are below b. The second part of (A1) represents h's expected utility from winning one unit, an event where v_1, the highest rivals' bid, is higher than b but all other bids are below b. In all other events h earns $u(0) = 0$. The first order condition (FOC) for maximization of (A1) after rearranging becomes:

(A2) $u(3v-2b) - u(v-b) + (n-1)u(v-b)\frac{1-F(b)}{F(b)} - u'(v-b)\frac{1-F(b)}{f(b)} = 0.$

The *left hand side* (LHS) of (A2) evaluated at $b = v$ is:

(A3) $u(v) - \frac{1-F(v)}{f(v)} =: H(v).$

We assume that $H(v)$ is strictly monotonic in v.[1]

Lemma 2 *There exists a unique $v = v_c$, satisfying: (**a**) $v_c = b_1(v_c) = b_2(v_c)$ which solves the FOC (A2). (**b**) $\forall v > v_c$, $b_1(v) = b_2(v) > v$. (**c**) $\forall v < v_c$, $b_1(v) = v > b_2(v)$.*

Proof. (**a**) $H(0) < 0$, $H(1) > 0$ and by assumption $H'(v) > 0$.Thus, there exists a unique v_c, such that $v_c = b_1(v_c) = b_2(v_c)$ that solves the FOC (A2). (**b**) $\forall b = v > v_c$, the LHS of (A2) is strictly positive and the optimal bids are $b_1(v) = b_2(v) > v$. (**c**) $\forall b = v < v_c$, the LHS of (A2) is strictly negative. But Lemma 1 implies that we cannot have $b_1(v) = b_2(v) < v$, and thus $b_1(v) > b_2(v)$. Further, from Lemma 1, $b_1(v) \geq v$ but a strict inequality implies (by the second part of Lemma 1) that $b_1(v) = b_2(v) > v$ a contradiction. We conclude that $\forall v < v_c$, $b_1(v) = v > b_2(v)$. ■

With a *risk neutral* (RN) h and after rearranging, equation (A2) becomes:

(A4) $(v-b)[1 + (n-1)\frac{1-F(b)}{F(b)}] + v - \frac{1-F(b)}{f(b)} = 0$

In our work $F(\cdot)$ is a *uniform distribution*, $H(v_c) = 0$ implies $v_c = 1/2$, and equation (A4) becomes: $(v-b)[1 + (n-1)\frac{1-b}{b}] + v - 1 + b = 0$, implying that, $(v-b)[b+(n-1)(1-b)] + vb - b + b^2 = (n-1)b^2 - \{n + (n-3)v\}b + (n-1)v = 0$. By denoting $\Phi(n, v) =: \frac{n+(n-3)v}{n-1}$ we can write (A4) as a quadratic equation:

[1] McAfee and McMillan (1987, p. 708, fn.11) define $J(v) =: v - \frac{1-F(v)}{f(v)}$ and showed that it must be increasing in v in order to have a well behaved bidding function in equilibrium. $J(v)$ is also referred to as *virtual valuation* by Myerson (1981), and as *marginal valuation* by Bulow and Roberts (1989). The role that the monotonicity of $J(v)$ plays for optimality and efficiency of *Independent-Private-Value* auctions is well understood. Our $H(v)$ coincides with $J(v)$ for a *risk neutal* h that we assume in the text but is "slightly" different otherwise.

(A5) $b^2 - \Phi(n,v)b + v = 0.$

Differentiating the LHS of (A5) with respect to b yields:

(A6) $\frac{\partial}{\partial b}\{b^2 - \Phi(n,v)b + v\} = 2b - \Phi(n,v).$

The second order condition (SOC) for maximization requires that (A6), evaluated at the optimal b, is negative. Thus, $2b - \Phi(n,v) < 0$, implying that $\frac{2b^2-\Phi(n,v)b+(v-v)}{b} < 0$, or finally, after using (A5):

(A7) $\frac{b^2-v}{b} < 0.$

We can write the solution to the quadratic FOC (A5) as:

(A8) $b_{1,2} = \{\Phi(n,v) \pm [(\Phi(n,v))^2 - 4v]^{1/2}\}/2.$

Note, that once $[(\Phi(n,v))^2 - 4v] < 0$, there is no solution to equation (A5). It is easy to verify that since $v \leq 1$, $[(\Phi(n,v))^2 - 4v]$ is strictly decreasing in v for all $n \geq 1$. Let v_{cn}, be the value of v that solves:

(A9) $[(\Phi(n,v_{cn}))^2 - 4v_{cn}] = 0.$

Thus, $\forall v > v_{cn}$, $[(\Phi(n,v))^2 - 4v] < 0$, and the LHS of (A5) is strictly positive implying that the optimal bid is $b(v) = 1$. Namely, for such (high) v's, $h's$ optimal strategy is to "*go for it,*" bidding 1, winning two units for sure and enjoying the synergy bonus, v. In what follows we restrict attention to v's that satisfy $v \in (v_c, v_{cn}] = (\frac{1}{2}, v_{cn}]$.

The positive root of (A8) yields, $b^2 > (\Phi(n,v))^2/2 - v = [b + \frac{v}{b}]^2/2 - v$, where the last equality is obtain by using (A5). Thus, $b^2 - v > [b^2 + 2v + (v/b)^2 - 4v]/2 = [b^2 - 2v + (v/b)^2]/2 = [b - (v/b)]^2/2 > 0$, which violates (A7). On the other hand, by using (A6) and (A8) it is easy to verify that the negative root of (A8) yields, $2b - \Phi(n,v) < 0$, so that the SOC is satisfied. With some additional tedious algebra one can verify that for $\forall v \in (\frac{1}{2}, v_{cn}]$, the negative root also yields $b > v$ as required. Thus, the negative root of (A8) satisfies the FOC, SOC and $b > v$, and is rewritten as:

(A10) $b = \{\Phi(n,v) - [(\Phi(n,v))^2 - 4v]^{1/2}\}/2.$

Although the solution proposed in (A10) satisfies the FOC and the SOC, it assures only a local maximization since the objective function may not be *quasi-concave*. Let $E[\pi(b(v))]$ denote the expected payoffs for h who has $V = v \in (\frac{1}{2}, v_{cn}]$ and is using $b(v)$ as defined by (A10). $E[\pi(b(v))]$ ={expected net gain of winning two units}{probability of winning two units} + {expected net gain of winning one unit}{probability of winning one unit}. Or formally:

(A11) $E[\pi(b(v))] = \{3v - \frac{2n}{n+1} b(v)\}\{[b(v)]^n\} + \{v - b(v)\}\{n[1-b(v)][b(v)]^{n-1}\}$

Let $E[\pi(b \geq 1)]$ denote the expected payoffs for h who has $V = v \in (\frac{1}{2}, v_{cn}]$ and bids $b \geq 1$ on both units which assures winning both of them:

(A12) $E[\pi(b \geq 1)] = [3v - \frac{2n}{n+1}]$,

as $3v$ is the value of winning two units and $\frac{2n}{n+1}$ is the expected payment in this case. Let v_n^*, be the v that equates equation (A11) and (A12). It turns out that,

(A13) **a**) $v_n^* \in (\frac{1}{2}, v_{cn}]$, **b**) $\forall v \in (\frac{1}{2}, v_{cn}]$, the optimal bids are, $b_1(v) = b_2(v) = \{\Phi(n,v) - [(\Phi(n,v))^2 - 4v]^{1/2}\}/2$ and **c**) $\forall v \in [v_n^*, 1]$ the optimal bids are $b_1(v) \geq 1$ and $b_2(v) \geq 1$.

Part 2. Here we solve for the region where $v < v_c$, implying by Lemma 1 and Lemma 2 part (c) that $b_1(v) = v > b_2(v)$. Simplify by concentrating only on $b_2(v)$ and denoting it by $b(v)$. Everything is the same as in part 1 with the exception, that we first derive the results for any number of units auctioned, $m < (n+1)$, and summarize them for our current experiment where $m = 2$ at the end.

There are three regions (events) to consider here.[2]

Region 1: Here, $V_{(m-1)} \leq b$, thus, $E[u(v,b)] = \int_0^b u(3v - 2p) dF_{(m-1)}(p)$.

Region 2: Here, $V_{(m)} \leq b < V_{(m-1)}$, thus, $E[u(v,b)] = u(v-b)[F_{(m)}(b) - F_{(m-1)}(b)]$.

Region 3: Here, $b < V_{(m)} < v$, thus, $E[u(v,b)] = \int_b^v u(v-b) dF_{(m)}(p)$.

Region 1, is the event that h wins both units and earns $u(2v+v-2p)$; region 2 is the event that h wins only one unit, and her bid, b, sets the price (which affects her gains on the unit won); region 3 is the event that h wins only one unit and does not set the price. We differentiate with respect to b and collect terms from the three region to obtain the following FOC for maximization:

$\partial E[u(v,b)]/\partial b = [u(3v - 2b)) f_{(m-1)}(b)] - \{u'(v-b)[F_{(m)}(b) - F_{(m-1)}(b)] + u(v-b)[f_{(m)}(b) - f_{(m-1)}(b)]\} - [u(v-b) f_{(m)}(b)] = [u(2(vb)) - u(v-b)] f_{(m-1)}(b) - u'(v-b)[F_{(m)}(b) - F_{(m-1)}(b)]$,

where, $f_{(k)} \geq 0$ is the derivative of $F_{(k)}$. Finally, using the fact that $[F_{(m)}(b) - F_{(m-1)}(b)] = \binom{n}{n-1}[1 - F(b)]^{m-1}[F(b)]^{n+1-m}$, and that $f_{(m-1)}(b) = \binom{n-1}{m-2}[1 - F(b)]^{m-2}[F(b)]^{n+1-m} f(b)$, we obtain:

[2] The derivation here is similar to the derivation of the FOC for the multi-unit demand SBUPA auctions with flat or decreasing demands developed in Ausubel and Cramton (1996), and Kagel and Levin (2001).

(A14) $[u(3v-2b^*)-u(v-b^*)]-\frac{u'(v-b^*)(1-F(b^*))}{(m-1)f(b^*)}\leq 0,$

where b^* is the solution to the problem with a *risk averse* (RA) h, and where strict inequality holds only if $b^*=0$.

Fact 1. When $m=2$, as in our experimental design, when $v<v_c=\frac{1}{2}$, $b^*<v$, so that even with synergies there is *demand reduction* on the second unit. To see why, consider the LHS of (A14). At $b=v$ it is equal to $[u(v)-\frac{1-F(v)}{f(v)})]<[v-\frac{1-F(v)}{f(v)})]$, since $u(v)$ is concave, $u(0)=0$ and $u'(0)=1$. However, since $v<v_c=\frac{1}{2}$, $[v-\frac{1-F(v)}{f(v)})]<[v_c-\frac{1-F(v_c)}{f(v_c)})]=0$ (see the proof to Lemma 2). Thus, the LHS of (A14) evaluated at $b=v$ is strictly negative. Note from (A14) that b^* is independent of n, the number of single unit demanders, for all (concave) u's. This surprising result is reminiscent of the optimal reservation price result in a single unit, *independent-private-value* auction. For the RN case (A14) becomes:

(A15) $(2v-b)-\frac{1-F(b)}{(m-1)f(b)}\leq 0,$

with strict inequality *only if* the optimal $b=0$. It is easy to verify that a sufficient condition to assure quasi-concavity of the objective function for the RN case is:

(A16) $(1-F(b))f'(b)-(m-2)[f(b)]^2\leq 0.$

With a *uniform* distribution, $f'(b)=0$ so that (A16) is satisfied for all $m\geq 2$.

We turn now to the effect of RA on bidding for the case of $v<v_c$. Let $v>b^*>0$. Thus, $0=[u(3v-2b^*)-u(v-b^*)]-\frac{u'(v-b^*)(1-F(b^*))}{(m-1)f(b^*)}<[u'(v-b^*)(2v-b)]-\frac{u'(v-b^*)(1-F(b^*))}{(m-1)f(b^*)}=u'(v-b^*)[(2v-b)-\frac{1-F(b^*)}{(m-1)f(b^*)}]$, where the first equality is just (A14) and the strict inequality is due to the concavity of u. We conclude that $[(2v-b)-\frac{1-F(b^*)}{(m-1)f(b^*)}]>0$, which we use to establish the next fact.

Fact 2: The effect of RA is to reduce the bid of h on the second unit, $b_{RA}(v)<b_{RN}(v)$, unless $b_{RN}(v)$ is already zero. That is, under condition (A16) a RA h bids no more on the second unit than a RN h when $b_{RN}(v)=0$, and strictly less when $b_{RN}(v)>0$. Quasi-concavity of the RN case and the fact that the FOC for the RN h evaluated at b^*_{RA} is strictly positive is sufficient to establish fact 2.

In the RN case it is convenient to rewrite the optimal b (from A15) as:

(A17) $b(v)=\left\{\begin{matrix} 0, & b\notin[0,v], \\ 2v-\frac{1-F(b)}{(m-1)f(b)}, & b\in[0,v]\end{matrix}\right\},$

which implicitly solves for the (optimal) b. In our design $F(\cdot)$ is *uniform* on $[0,1]$ so that (A17) reduces when $m > 2$ to,

$$\text{(A18)} \quad b(v) = \left\{ \begin{array}{ll} 0, & v \in [0, \frac{1}{2(m-1)}], \\ \frac{(m-1)2v-1}{m-2}, & v \in [\frac{1}{2(m-1)}, \frac{1}{m}] \end{array} \right\}.$$

When $m = 2$, and $v < v_c = 1/2$, the LHS of (A15) becomes $[2v - 1]$ which is strictly negative. Thus, establishing for a RN h, a *uniform* distribution and $m = 2$ (as in our experimental design):

$$\text{(A19)} \quad b(v) = 0, \forall v \in [0, \tfrac{1}{2}).$$

Summary of Appendix A for our Experimental Design.

The n bidders with a single unit demand have a dominant strategy $b(v_i) = v_i$ and are replaced by computers who employ this strategy and the human knows this fact. In our experiment the number of units auctioned is always two, n is either three or five, and $g(v) = v$ (so that $g(0) = 0$ and $g'(v) > 0$ and $g'(0) = 1$ are satisfied). The distribution function $F(\cdot)$ is *uniform* on $[\$0, \$7.5]$.

Transforming the analysis above from the domain of $F(\cdot)$ on $[0,1]$ to $[\$0, \$7.5]$ is trivial. We end up with a partition of $[\$0, \$7.5]$ to three regions.

Region 1 is $[\$0, \$3.75)$. In this region the optimal bid is $b_1(v) = v$, $b_2(v) = 0$.

Region 2 is $[\$3.75, \$7.5v_n^*)$, where v_n^* equates equations (A11) and (A12) in the appendix. In this region the optimal bid is $b_1(v) = b_2(v) = \$7.5\{\Phi(n,v) - [(\Phi(n,v))^2 - 4v]^{1/2}\}/2$.

Region 3 is $[\$7.5v_n^*, \$7.5]$. In this region the optimal bids are $b_1(v) \geq \$7.5$, $b_2(v) \geq \$7.5$.

Appendix B: **English-Clock Auction (ECA) with Synergies.**

In what follows we simplify notation by using v for v_h. Also recall that the synegy bonous is modeled as earning an extra $g(v) = \alpha v$ if h obtains both units.

The optimal strategy for h in the ECA can be nicely described by patitioning the domain of values to three regions:

(B1) $\mathbf{A} =: [0, \frac{1}{1+\alpha})$, $\mathbf{B} =: [\frac{1}{1+\alpha}, \frac{2}{2+\alpha})$, $\mathbf{C} =: [\frac{2}{2+\alpha}, 1]$.

A. *Optimal Behavior for h when $V = v \in \mathbf{A} =: [0, \frac{1}{1+\alpha})$.*

A.1 If $v_3 \geq v$, drop the first unit at any price, $p \in [0, v_3]$ If $v_3 < v$, drop the first unit at any price, $p \in [0, \min\{v, v_2\}]$.

A.2 Drop the second unit at price, $p \in [v, max\{v, v_3\}]$

Proofs and observations:

Observation 1. *In region A it is never optimal for h to win both units.*

To win two units h must stay IN with both beyond clock price, $p = v_2$. By dropping a unit at $p = v_2$, h stops the auction, "wins one unit" (WOU) and earns, $\pi(WOU) = [v - v_2]$. Suppose that h decides to stay IN with both units an extra δ beyond $p = v_2$, (as long as $v_2 + \delta \leq 1$) and drop out at $p = v_2 + \delta$ if $V_1 = v_1$ does not drop by then. Recall that given that $V_2 = v_2$, $V_1 | V_1 \geq v_2$ is distributed *uniformly* on $[v_2, 1]$. With a probability $\frac{\delta}{1-v_2}$, v_1 drops within the next δ, h wins two units and earns: $(2+\alpha)v - 2E[V_1 | v_2 + \delta \geq V_1 \geq v_2] = [(2+\alpha)v - 2v_2 - \delta]$. With a probability of $\frac{1-v_2-\delta}{1-v_2}$ v_1 does not drop in that interval and h stops the clock at $p = v_2 + \delta$ to win one unit and earn $(v - (v_2 + \delta))$. (Note that since the possibility $p = v_2 + \delta = 1$ is allowed, we are also including the strategy that assures winning two units.) Thus, expected profits for staying IN beyond v_2 and "possibly winning two units" are: $\pi(WTU) = \frac{\delta}{1-v_2}[(2+\alpha)v - 2v_2 - \delta] + \frac{1-v_2-\delta}{1-v_2}[v - v_2 - \delta] = \frac{\delta}{1-v_2}[(1+\alpha)v - v_2] + [v - v_2 - \delta]$. However, since $v < \frac{1}{1+\alpha}$, $\{\pi(WTU) - \pi(WOU)\} = \frac{\delta}{1-v_2}[(1+\alpha)v - v_2 - (1 - v_2)] = \frac{\delta}{1-v_2}[(1+\alpha)v - 1] < 0$.

Observation 2. *As a result of Observation1, h never wants to win even one unit at a clock price, $p > v$,* as it earns *negative profits rather than zero with no units won.*

Rules A.1 and A.2 are the most general rules that implement these conclusions. Note that $max\{v, v_3\} \geq v \geq min\{v, v_2\}$. Also note that the requirement $p \in [0, min\{v, v_2\}]$, rather than $p \in [0, v_2]$, both of which assure winning no more than one unit, is to avoid staying IN with two units beyond $p = v$, in which case it is not desirable to win even one unit.

Note that in region A h's optimal strategy yields the same prices and allocation as the strategy: "Drop unit 1 at clock price, $p = 0$ and stay IN until the clock price, p, reaches your value v." Thus, in region A bidding yields the same allocations and prices as in a sealed bid uniform price auction (see Appendix A). Further, prices and allocations in region A are the same as when h has flat or decreasing demand for the the second unit (Kagel and Levin, 2001).

Behavior outside equilibrium in region A: Dropping both units too early leaves no further action. If h errs, staying active on the first unit longer than is optimal (i.e., $p > v$) and then realizes his mistake, h ought to drop out right away.

B. *Optimal Behavior for h when* $V = v \in \mathbf{B} =: [\frac{1}{1+\alpha}, \frac{2}{2+\alpha})$.

Let the clock price $p^* =: [(2+\alpha)v - 1]$. Note that since $v \in [\frac{1}{1+\alpha}, \frac{2}{2+\alpha})$ in region B, $p^* - v = (1+\alpha) - 1 \geq 0 > (2+\alpha)v - 2 = p^* - 1$. Thus, $v \leq p^* < 1$.

B.1 If $v_2 < p^*$, "Go All The Way," (ATW).

B.2 If $v_2 \geq p^*$, drop both units at clock price $p \in [p^*, max\{p, v_3\}]$.

Proofs and observations:

B.1 For any given realization $V_2 = v_2$, this strategy yields profits, $\pi(ATW) = (2+\alpha)v - 2E[V_1|V_2 = v_2] = (2+\alpha)v - 2\frac{1+v_2}{2} = (2+\alpha)v - 1 - v_2 \geq (2+\alpha)v - 1 - p^* = 0$. Winning one unit earns (at best) profits of, $\pi(WOU) = v - v_2$. $[\pi(ATW) - \pi(WOU)] = (1+\alpha)v - 1 \geq 0$, since $v \geq \frac{1}{(1+\alpha)}$ and is strictly positive with any $v > \frac{1}{(1+\alpha)}$. Thus, ATW, dominates winning one unit or winning none.

B.2 Following the strategy prescribed in B.2 yields zero units and zero profits. Winning one unit earns $v - v_2 < 0$ and staying IN beyond the clock price, $p = p^*$, to win two units earns $(2+\alpha)v - 2E[V_1|v_2 \geq p^*] < (2+\alpha)v - 2E[V_1|v_2 = p^*] = (2+\alpha)v - 1 - p^* = 0$.

It is easy to show that staying longer, $\delta > 0$, beyond $p = max\{p, v_3\}$, (as long as $p^* + \delta < 1$) and dropping out if no one else drops out once the price reaches $max\{p, v_3\} + \delta$, yields negative expected profits.

Behavior outside equilibrium in region B. Case **B.1** Dropping two units too early leaves no further action. If h dropped the first unit too early h needs to stay with the second unit no longer than clock price $p = max\{v, v_3\}$. Case **B.2** (This is the most interesting "out of equilibrium" behavior.) If h realizes that she stayed IN with both units too late i.e., although $p > max\{p^*, v_3\}$ and $v_2 > p^*$, then if v_2 is still IN ($v_2 \geq p$), h must drop immediately and nothing happens relative to the optimal policy. However, if v_2 has dropped OUT already ($v_2 < p$), then h should "go all the way" in order to win both units.

C. *Optimal Behavior for h when* $V = v \in \mathbf{C} =: [\frac{2}{2+\alpha}, 1]$.

Following the strategy prescribed in C yields two units and earns positive expected profits of: $(2+\alpha)v - 2E[V_1] > 0$, since $v \geq \frac{2}{(2+\alpha)}$ and $E[V_1] < 1$. For any given realization $V_2 = v_2$, this strategy yields profits of $\pi(ATW) = (2+\alpha)v - 2E[V_1|V_2 = v_2] = (2+\alpha)v - (1+v_2)$. On the other hand, winning one unit yields profits of $\pi(WOU) = v - v_2$. Thus, $\{\pi(ATW) - \pi(WOU)\} = \{[(2+\alpha)v - (1+v_2)] - [v - v_2]\} = (1+\alpha)v - 1 > (1+\alpha)\frac{2}{2+\alpha} - 1 = \frac{2+2\alpha-2-\alpha}{2+\alpha} = \frac{\alpha}{2+\alpha} > 0$.

Dropping early and winning zero units earn zero profits. Thus, the prescribed strategy is optimal.

Behavior outside equilibrium in region C: If h dropped both units there is nothing to do. If h erred and dropped one unit she ought to stay IN with the second unit as long as the clock price, $p < \max\{v, v_3\}$, and drop the second unit immediately when $p \geq \max\{v, v_3\}$.