Almost Common Values Auctions Revisited*

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Abstract

In *almost common value* auctions one bidder (the *advantaged* bidder) has a valuation advantage over all other (*regular*) bidders. It is well known that in second-price auctions with two bidders, even a slight private value advantage can have an explosive effect on auction outcomes as the advantaged bidder wins all the time and auction revenue is substantially lower than in a pure second-price common-value auction. We explore the robustness of these results to the addition of more regular bidders in second-price auctions, and the extent to which these results generalize to ascending-price English auctions in an effort to provide insight into when and why one ought to be concerned about such slight asymmetries.

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1 Introduction

Common-value auctions are auctions where the *ex-post* value of the item is the same for all bidders. What makes such auctions interesting is that bidders do not know the value prior to bidding, but instead receive affiliated private information signals that are related to the underlying value.[1] *Almost common-value* auctions differ from pure common value auctions by having an *advantaged* bidder whose *ex-post* valuation is (slightly) higher than that of all other, n *regular*, ($n \geq 1$), bidders. Although it does not seem obvious that a small private-value advantage can have a dramatic impact on auction outcomes, that is precisely what happens in second-price sealed-bid auctions and in ascending-price English auctions. Bikhchandani (1987) shows that in a sealed-bid, second-price auction with only two bidders, even the tiniest such asymmetry causes the advantaged bidder to win all the time and drastically reduces seller's revenue compared to the corresponding pure common value auction setting (also see Avery and Kagel, 1997; Klemperer, 1998).[2] Perfect symmetry is a convenient modeling assumption but in many circumstances firms are known to have some idiosyncratic, private-value advantage in an otherwise common-value auction.[3] Thus, these findings raise several important questions that we address in this

[1] In dynamic auctions, such as ascending-bid English auctions, bidders also observe the prices at which other bidders drop out and may use this additional information to reformulate their bidding strategy. A survey of common value auctions can be found in Kagel and Levin, 2001.

[2] However, Avery and Kagel (1997) show that this "explosive" effect does not carry over to first-price sealed bid auctions.

[3] For example, in the U. S. governments's spectrum (air wave rights) regional auctions, it was common knowledge that PacTell had a private value advantage in the Los Angeles and San Francisco markets (Cramton, 1997).

paper.

First, we explore whether the explosive impact on auction outcomes resulting from a small private-value advantage in second-price auctions extends to auctions with more than one regular bidder. Using the "wallet game" as the benchmark model (Avery and Kagel, 1997; Klemperer, 1998) we find that the explosive effect reported in the two bidder case does not generalize to the addition of more regular bidders. In fact, increasing the number of regular bidders generates a continuous dampening of the explosive effects found in the two bidder case. However, somewhat surprisingly, the private value advantage remains, and is substantial, even as the number of regular bidders grows without bound.

In addition, we argue that, other things equal, the seller actually *benefits* from the additional aggressiveness of an advantaged bidder. What hurts the seller, and often dominates this outcome, is that regular bidders tend to bid less aggressively in most circumstances, which suppresses revenue. Thus, in cases where the regular bidders response to the aggressiveness of the advantaged bidder is small, or nonexistent, revenues may be higher than in the pure common value auction case.

A second example deals with English auctions which always have just two bidders in the last stage. As such, English auctions would seem to be most vulnerable to the explosive effects of small asymmetries. This example shows that even when the private value advantage is such that there exist explosive effects with two bidders, this explosive effect evaporates with the addition of a second regular bidder. That is, in this example, the addition of a second regular bidder is sufficient to eliminate the explosive effect found in the two bidder case, even though the auction inevitably reduces to two bidders in the last stage of the game.

The structure of the paper is as follows: Section 2 introduces the basic model and then goes on to provide the examples yielding the results outline above. Section 3 summarizes our analytical results and discusses them in relationship to the limited experimental data on auctions in which one bidder has a private value advantage. Throughout we use very simple models to shed light on these issues and to answer these important questions.

2 The Model

Preliminary: Let there be $n+1$ bidders, n *regular* bidders denoted by i, $i = 1, 2, ..., n$, and one *advantaged* bidder denoted by $n+1$. Each of the $n+1$ bidders privately observes a signal $X_i \in [0,1]$, $i = 1, 2, ..., n, n+1$, *i.i.d.* from a distribution function $F(t)$, on $[0,1]$, with $F'(t) = f(t) > 0$, on $(0,1)$. Denote the valuation of each bidder by V_i and the vector of all $n+1$ signals by $x = (X_1, ..., X_n, X_{n+1})$. A pure common-value auction in such an environment usually assumes that $V_i = V(x)$ for all i. In an *almost common-value* auction $V_i = V(x)$ for only the regular bidders, $i = 1, ..., n$, while the advantaged bidder, the $(n+1)^{th}$ bidder, has an (*ex-post*) valuation $V_{n+1}(x) \geq V(x)$, $\forall x$, with a strict inequality for some x. The idea of a small private value advantage is captured by allowing the possibility that $\forall \epsilon > 0$, although $V_{n+1}(x) \geq V(x)$,

$V_{n+1}(x) \leq V(x) + \epsilon$.

For example, in the "wallet game" there is one regular bidder ($n = 1$) and one advantaged bidder with $V_1(x) = X_1 + X_2$ and $V_2(x) = V_1(x) + \epsilon X_2$. (Avery and Kagel, 1997; Klemperer, 1997, 1998). In a second-price, sealed-bid common-value auction with $\epsilon = 0$, the symmetric equilibrium bidding strategy is, $b(X_i) = 2X_i$. With $\epsilon = 0$, the bidder with the highest signal, X_H, wins the auction and pays the equilibrium bid of the rival, the holder of the lower signal, which is $2X_L$. There is no ex post regret for both the winner and the loser since the winner earns $(X_H + X_L) - (2X_L) = X_H - X_L \geq 0$ and the loser, should she have won, would have earned $(X_H + X_L) - (2X_H) = X_L - X_H \leq 0$. Clearly, each bidder's *ex-ante* probability of winning is $1/2$ and seller's revenue is $2E_F[X_L|X_L \leq X_H]$. However, once $\epsilon > 0$, no matter how small ϵ is, the results change drastically: (1) The advantaged bidder wins all the time and (2) Since the equilibrium strategy of the regular bidder is now $b_1(X_1) = X_1$, seller's revenue becomes $E_F[X]$, which is substantially lower than revenue in the symmetric case.[4],[5]

The first example shows that the explosive effects of the second-price wallet game do not generalize to auctions more than two bidders.

Example 1. Consider the wallet auction with $n \geq 2$ regular bidders

[4]Avery and Kagel (1997) report a class of asymmetric equilibria in the private value advantage case, and like here, analyze the most aggresive equilibrium in the class.

[5]Note that under the standard assumption of increasing hazard rates, $\partial[\frac{f(t)}{1-F(t)}]/\partial t > 0$, $2E_F[X_L|X_L \leq X_H] - E_F[X] = \int_0^1 \frac{1-F(t)}{f(t)}[1 - 2F(t)]f(t)dt > \frac{1-F(\bar{t})}{f(\bar{t})} \int_0^1 [1 - 2F(t)]f(t)dt = \frac{1-F(\bar{t})}{f(\bar{t})}[1 - \int_0^1 2F(t)]f(t)dt] = 0$, where $\bar{t}$ is the unique solution to $[1 - 2F(\bar{t})] = 0$. The reduction in expected revenue can be quite large. With a *uniform* distribution it goes down with any $\epsilon > 0$, by 25%, from $\frac{2}{3}$ to $\frac{1}{2}$.

where, without loss of generality, we use averages, rather than sum of valuations. The common value for each of the n regular bidders is the average of all the $n+1$, *i.i.d.* signals and the value for the advantaged bidder is larger by having a slightly "extra" value from her private signal. Formally: $\forall i = 1, ..., n$, $V_i = V(x) = \frac{1}{n+1} \sum_{j=1}^{n+1} X_j$, $V_{n+1} = V(x) + \frac{\epsilon X_{n+1}}{n+1}$.

Proposition 1 *$b_{n+1}(X) = \alpha(n, \varepsilon)X$, and $b_i(X) = [\frac{(n-1)-2\epsilon}{(n-1)(1+2\epsilon)}]\alpha(n, \varepsilon)X$, $i = 1, ..., n$ is a Nash equilibrium for the second-price auction with $n \geq 2$, where, $\alpha(n, \varepsilon) = \frac{[(n-1)(n+3)-2\epsilon][1+2\epsilon]-2n\epsilon}{2(n+1)[(n-1)-2\epsilon]}$*[6].

Proof. See appendix. ■

Note that with $\epsilon = 0$ we have $\forall i = 1, ..., n$, $b_i(X) = b_{n+1}(X) = \frac{n+3}{2(n+1)}X$, which is the symmetric equilibrium for the pure common value case. With an advantaged bidder, $\epsilon > 0$, $\frac{b_{n+1}(X)}{b_i(X)} = \frac{(n-1)(1+2\epsilon)}{(n-1)-2\epsilon}$, $\forall i = 1, ..., n$. Here with a tiny $\epsilon > 0$ and $n > 1$, the advantaged bidder is only slightly more aggressive than a regular bidder even with just two regular bidders.[7] Next we note that even with just two regular bidders it is obvious that for any small $\delta > 0$, $\exists$ small enough $\varepsilon(\delta) > 0$, such that the *ex-ante* probability of winning the auction does not exceed $\frac{1}{3} + \delta$ for the advantaged bidder and is not less than $\frac{1}{3} - \frac{\delta}{2}$ for the regular bidders. Thus, the advantaged bidder wins only slightly more often than

[6] *There is a whole class of Nash equilibria but ours is the "closest" to the symmetric one. There are asymmetric, bully-sucker, equilibria even in the pure CV case where one bidder is very aggressive and others are bidding very law. In such equilibria the bully may win all the time and revenue may be very low even in the symmetric structure.*

[7] With only two bidders, $n = 1$, the expression for the advantaged bidder, $b_2(X_2)$ is "divided by zero," suggesting that in this case the advantaged bidder ought to bid "very aggressively." Bikhchandani's (1987) paper explicitly poses the question of the stability of the symmetric equilibirum with $\varepsilon > 0$ and $n > 1$, with no solution to this question identified prior to this.

the regular bidders.

The question is, why do these explosive effects disappear in the wallet auction with the addition of just one more regular bidder? In any realization of private signals in such an auction the expected utility of the advantaged bidder, $E[u^a]$, is strictly higher than that of the regular bidder, $E[u^r]$. Bidding in a second-price auction reflects the (maximum) willingness to pay conditional on the information inferred by the winning event. In other words, each bidder is just indifferent between winning or losing, if she has to pay her own bid. It is impossible to have such a common bid price with only one regular bidder. This is because in equilibrium it implies that the willingness to pay of the advantaged and disadvantaged bidders are identical, conditional on the two signals that give rise to such a common bid. But, this contradicts the assumption that the advantage bidder's expected utility is strictly higher in such an event, which implies that she prefers to bid higher and break the tie. This rules out the possibility that the range of the possible bids for the two types overlap, and is the basis for the explosive effect of the private value advantage with just one regular bidder.

However, such a contradiction cannot be established with more than one regular bidder. To see why, take the case of two regular bidders and let $b(z)$, $b_1(y) = b_2(y)$ represent the bid functions of the advantaged and the two regular bidders. Assume that there are z_0 and y_0 such that $b(z_0) = b_1(y_0) = p$. The event used to calculate the willingness to pay of the advantaged bidder is: $A =:$ $\{z = z_0,\ y^H = y_0 \geq y^L\}$, where y^H and y^L, represent the highest and lowest signals of the regular bidders. In contrast, a regular bidder who holds y_0, must consider an additional event in calculating her willingness to pay, namely: $B =: \{z \leq z_0,\, y^H = y_0 = y^L\}$. B is the event that the tieing price, p, is coming from *another* regular bidder, a possibility that does not exist with only *one* regular bidder. Is event B more favorable than event A for a regular bidder? In other words, is $E[u^r|B] > E[u^r|A]$? A positive answer is entirely possible and very likely. When the advantaged bidder is more aggressive in equilibrium, as in the auction considered here, it follows that $z_0 < y_0$. Thus, the more favorable conditioning of moving from $y^L \leq y_0$ in event A to $y^L = y_0$ in event B, more than compensates for the worsening conditioning of moving $z = z_0$ in A to $z \leq z_0$ in B. As a result, it is even possible that $E[u^r|B] > E[u^a|A] > E[u^r|A]$. Nevertheless, in equilibrium, as in our proposition, the overall expected utility of a regular bidder weighted by the probabilities of events A and B can be (is) the same as the expected utility of the advantaged bidder conditional on event A. An additional complication that must be kept in mind (accounted for in our derivation) is to calculate the correct posterior probabilities of a tie coming from the advantaged bidder or from another regular bidder.

A more intuitive, though less revealing, answer as to why the explosive effect disappears is that with more than one regular bidder, the regular bidder needs to shave her bid by less to guard against the *winner's curse* coming from the aggressiveness of the advantaged bidder, since the tieing bid may be with another regular bidder who is not as aggressive as the advantaged bidder. The result is less *bad news* as a result of winning for a regular bidder, hence less need to shave their bids in response to the winner's curse.

The impact of additional competition on bidding, although continuing to mitigate the asymmetry, does not eliminate it altogether. With one more regular bidder, $\frac{b_{n+2}(X)}{b_i(X)}$ represents the new bidding ratio and it is simple to show that $[\frac{b_{n+2}(X)}{b_k(X)}|_{k=1,\ldots,n+1} - \frac{b_{n+1}(X)}{b_i(X)}|_{i=1,..,n}] = \frac{[n(n-1-2\epsilon)-(n-1)(n-2\epsilon)](1+2\epsilon)}{(n-2\epsilon)(n-1-2\epsilon)} = [\frac{-2\epsilon(1+2\epsilon)}{(n-2\epsilon)(n-1-2\epsilon)}] < 0$. Thus, as competition grows this ratio is getting smaller, starting with $\frac{1+2\epsilon}{1-2\epsilon}$ with only two regular bidders. However, the equilibrium does not converge to that of the symmetric second-price auction equilibrium as n grows: As $\lim_{n\to\infty} b_{n+1}(X) = \frac{1+2\epsilon}{2}X$, and $\lim_{n\to\infty} b_i(X) = \frac{1}{2}X$ so that $\lim_{n\to\infty} \frac{b_{n+1}(X)}{b_i(X)} = [1+2\epsilon] > 1$. Further, as n grows without bound, $\lim_{n\to\infty} b_i(X) = \frac{1}{2}$, so that the probability of the advantaged bidder winning is $\frac{2\epsilon}{1+2\epsilon}$.

So far we have addressed the robustness of the explosive effects in the wallet auction to a bidder having a private value advantage. Our results show that once there is more than one regular bidder: (1) The advantaged bidder does not win all the time in a second-price auction, (2) The change in bidding strategies corresponds to the size of the advantage in a continuous and non-drastic fashion so that a tiny ϵ corresponds to only a tiny reduction in revenue, and (3) Although the effects of a private value advantage on bidding strategies are smaller with larger numbers of regular bidders, they remain proportional to the size of the private value advantage even as competition grows without bound.

Our next observation corrects for a possible misconception in the literature on almost common-value auctions with just two bidders namely, that the existence of an advantaged bidders always reduces seller's revenue (see Bikhchandani, 1987; Avery and Kagel, 1997; and Klemperer 1998). *Ceteris paribus*, a seller in a second-price auction necessarily benefits from higher bidding. However, in equilibrium we expect regular bidders to accommodate the more aggressive advantaged bidder by lowering their bids. And, since the second-highest bid sets the price, we might expect a lower price than in the absence of the aggressive advantaged bidder. Indeed such results are reported in previous papers analyzing the "wallet game." However, it is obvious that there is a genuine trade off here. In cases where regular bidders do not accommodate the advantaged bidder, or their adjustments are mild relative to the symmetric equilibrium, we can expect revenue to increase. One example of this is the "maximum game" studied in Bulow and Klemperer (2002) and Cambell and Levin (2001). In this game the common value for each of the n regular bidder is the highest signal among the $n+1$, *i.i.d.* signals and the value for the advantaged bidder is "slightly" higher. Formally: $V_i = V(x) = Max\{X_i\}_1^{n+1}, \forall i = 1, ..., n,\ \ V_{n+1} = (1+\epsilon)V(x)$. In this game it is easy to show that $\forall n \geq 1:$ A) $b_i(X_i) = X_i\ \forall i = 1, ..., n$, is the dominant solvable bidding strategy for all regular bidders. B) $b_{n+1}(X_{n+1}) \geq 1$, is the dominant solvable bidding strategy for the advantaged bidder. C) The advantaged bidder wins all the time. D) $\forall \epsilon > 0$, seller's expected revenue is (substantially) **higher** than in the pure common value auction with $\epsilon = 0$. Thus, with such valuations a seller is necessarily better off having an advantaged bidder. Further, even within the often studied "wallet game" with just one regular bidder the seller benefits from an advantaged bidder once we use a generalized uniform distribution, $F(t) = t^{\alpha}$, with $\alpha \in (0, [1+\sqrt{5}]/4)$, rather than the uniform case where $\alpha = 1$[8]

Our second example addresses our final question: Does a demand structure in which a private value advantage produces an explosive effect in the two bidder case always perpetuate that effect in an English auction with more than one regular bidder? A negative answer would be quite alarming since an English auction reduces to two bidders in the end. An affirmative answer, on the other hand, while not assuring the absence of such an explosive effect, at least demonstrates that such an effect is not inevitable.

Example 2. Consider the following informational structure: There are three signals, $x := (X_1, X_2, X_3)$, each is *i.i.d*, from a well behaved distribution function $F(\cdot)$ defined on $[0, 1]$. Denote by $Y_1 > Y_2 > Y_3$, the highest, the middle and the lowest signal. Let the common value of a regular bidder and the valuation of the advantaged bidder be defined by $V_{reg}(x) = \frac{Y_1+Y_2+Y_3}{3}$ and $V_{adv}(x) = V_{reg}(x) + \frac{\varepsilon}{3}(Y_2 - Y_3) > V_{reg}(x)$, where $\varepsilon \in (0, 1)$. Consider first a SPA with only two bidders. The SNE bidding function for the pure CV case ($\varepsilon = 0$), is given by $B_1(X) = B_2(X) = \frac{2}{3}X + \frac{1}{3}\int_0^1 tdF(t)$. Although not exactly the *wallet game, both* models have the same implications: a) $\forall x$, $V_{adv}(x) > V_{reg}(x)$, b) By mimicing the arguments provided earlier it can be shown that the introduction of even a slight private value advantage has the same explosive effects as in the original wallet game*, i.e.,* the advantaged bidder wins all the time and seller revenue is substantially lower than in the pure common value auction game. Consider now an *English* auction version of this game with

[8] Details are ommited but will be provided by the authors upon request.

three bidders, one advantaged and two regular bidders. We Let $d^1_{adv}(X)$ and $d^1_{reg}(X)$ denote the dropping price rule of the advantaged and the regular bidder given that their signal is X and that no one has dropped yet. We assume in this example that when the first bidder drops the two remaining bidders can tell whether a regular bidder or an advantaged bidder has dropped.[9] Thus, let $d^2_{adv}(Z, d^1)$ denote the dropping price rule for the advantaged bidder who has a signal Z and who observes the first drop-out price. We denote by $d^2_{reg}(Z, d^1_{adv})$ and $d^2_{reg}(Z, d^1_{reg})$ the dropping price rule for a regular bidder who has a signal Z and who observes the first drop-out price from an advantaged bidder or a regular bidder, respectively.

Proposition 2 *The profile of strategies:* $d^1_{adv}(X) = d^1_{reg}(X) = X$, $d^2_{adv}(Z, d^1) = 1$, $d^2_{reg}(Z, d^1_{adv}) = (2Z + d^1_{adv})/3$, $d^2_{reg}(Z, d^1_{reg}) = (Z + 2d^1_{reg})/3$. *is a Bayesian Nash Equilibrium of this English Auction.*

Proof. We show first that there are strictly positive expected profits for all bidders in the proposed equilibrium. If all bidders follow the proposed strategies then the holder of Y_3, regardless of her type, would be the first to drop out so that $d^1 = Y_3$. Case 1: The advantaged bidder drops first. In this case the regular bidder holding Y_2, drops next and sets a price of $d^2_{reg}(Z, d^1_{adv}) = \frac{2Z+d^1_{adv}}{3} = \frac{2Y_2+Y_3}{3}$. The winner is the regular bidder holding the highest signal, Y_1 and her

[9] We also have an example of an English auction with three bidders where only the drop-out price but not the identity of the bidder is revealed. This example makes the same point as the one here. In it, the advantaged bidder adopts in equilibrium the same strategy as a regular bidder regardless of her signal value and there is no explosiveness. However, the example provided here is more realistic (and challenging) as one could argue that often bidders know the identity of those who drop out.

payoffs are: $V_{reg}(x) - d^2_{reg}(Z, d^1_{adv}) = \frac{Y_1+Y_2+Y_3}{3} - \frac{2Y_2+Y_3}{3} = \frac{Y_1-Y_2}{3} > 0$. Case 2: A regular bidder drops first at $d^1 = Y_3$. In this case the price is set by the other regular bidder $d^2_{reg}(Z, d^1_{reg}) = (Z + 2d^1_{reg})/3 = \frac{Z+2Y_3}{3}$. The winner is the advantaged bidder and her payoffs are $[\frac{Y_1+Y_2+Y_3}{3} + \frac{\varepsilon}{3}(Y_2 - Y_3)] - d^2_{reg}(Z, d^1_{reg}) = \frac{(Y_1-Z)+(Y_2-Y_3)}{3} + \frac{\varepsilon}{3}(Y_2 - Y_3) \geq \frac{(1+\epsilon)(Y_2-Y_3)}{3} > 0$.

Next, we show that an advantaged bidder has no incentive to deviate from the proposed equilibrium when all others follow it. Case 1: The advantaged bidder is holding the lowest signal and ought to drop first at $d^1 = Y_3$. Dropping even earlier does not matter. Dropping later matters only if the other two regular bidders drop first. But, in this case the holder of Y_2 drops first at $d^1 = Y_2$ and the holder of Y_1 drops next and sets the price at $d^2_{reg}(Z, d^1_{reg}) = (Z + 2d^1_{reg})/3 = \frac{Z+2Y_2}{3} = \frac{Y_1+2Y_2}{3}$. By winning the advantaged bidder earns: $[\frac{Y_1+Y_2+Y_3}{3} + \frac{\varepsilon}{3}(Y_2 - Y_3)] - [\frac{Y_1+2Y_2}{3}] = \frac{\varepsilon}{3}(Y_2 - Y_3) - \frac{Y_2-Y_3}{3} < -\frac{(Y_2-Y_3)(1-\varepsilon)}{3} < 0$. Case 2: The advantaged bidder is holding one of the two highest signals. In this case she wins for sure and earns positive payoffs. Raising her bid would not matter and dropping out would only eliminate her positive payoffs of winning.

Finally we show that a regular bidder has no incentive to deviate from the proposed equilibrium when all others follow it.. Case 1: The advantaged bidder is holding the lowest signal. In this case the advantaged bidder drops first at $d^1 = Y_3$ and the specified equilibrium of the second stage is the standard one so proof is omitted. Case 2: The advantaged bidder is holding one of the two highest signals. In this case one of the regulars must be holding the lowest signal. If that regular bidder (who is holding the lowest signal) drops earlier than $d^1 = Y_3$, it does not matter. If she stays longer it maters only if she wins. In this event, if the other regular drops first then the advantage bidder stays active until the price reaches 1 and winning by a regular bidder at a price of 1 assures losses. If on the other hand, as a result of not dropping at $d^1 = Y_3$ the advantaged bidder drops first it implies that her signal must be Y_2. In this case the price is set by the other regular who holds Y_1 at $d^2_{reg}(Z, d^1_{adv}) = (2Z + d^1_{adv})/3 = \frac{2Y_1+Y_2}{3}$. Thus by deviating and winning such a regular bidder earns: $\frac{Y_1+Y_2+Y_3}{3} - \frac{2Y_1+Y_2}{3} = -\frac{Y_1-Y_3}{3} < 0$. Thus, in case 2, the regular bidder who holds the lowest signal does not wish to deviate and drops first at $d^1 = Y_3$. Given this, the other regular bidder who holds one of the two highest signals has no reason to deviate, as winning against the advantaged bidder (who bids 1 in this case) assures losses.

It is worth noting that in cases where a regular bidder drops first, and the advantaged bidder bids aggressively enough to assure winning, the remaining regular bidder is not using dominated bids. That is, once a regular bidder drops at $d^1 = Y_3$, the remaining regular bidder who holds $Z \geq d^1$infers that the value of the item is at least $\frac{2d^1+Z}{3}$, as the signal of the advantaged bidder must be at least d^1,and in equilibrium does not use dominated lower bids. ■

In equilibrium the bidder with the lowest signal drops out first regardless of her identity. If the advantaged bidder drops first then the two remaining regular bidders proceed as if in a pure common-value auction. However, if, as equilibrium dictates, a regular bidder drops first then the advantaged bidder bids aggressively enough to assure winning. Thus, although in a two bidders auction the advantaged bidder wins all the time, here in equilibrium her *ex-ante* probability of winning is only 2/3. And, of course, there is positive incentive for regular bidders to enter the auction in the first place.

In any realizations where the advantaged bidder holds the lowest signal the seller's revenue is the same as in the pure common-value auction. In realizations where the advantaged bidders has one of the two highest signals and wins the differences between the seller's revenue in the almost common-value auction and the pure common-value auction is $E_F[(Y_1+2Y_3-3Y_2)/6]$. It worth nothing that this last expression may be positive for certain distribution functions.[10]

The reason that a third bidder "stabilizes" the *English* auction is quite different here than in our first example. The *English* auction is a dynamic auction where bidders update their beliefs and thus their assessment of the value of the item as the auction progresses. An advantaged bidder who wishes to exploit her advantage while holding the lowest signal must refrain from exiting the auction. However, defection by such inaction necessarily raises the price to a level that such defection is unprofitable. This is the case in spite of the fact that the remaining regular bidder would adopt a less aggressive strategy after observing that a regular bidder had dropped out first.

[10]For $F(x) = x^\lambda$, it is easy to show that $sign E_F[(Y_1 + 2Y_3 - 3Y_2)/6] = sign(1 - 3\lambda)$.

3 Summary and Conclusion

Bikhchandani (1987) was the first to establish that in a second-price common value auction the existence of a small private value advantage can have an explosive effect on auction outcomes, with the advantaged bidder winning all the time and a sharp reduction in seller's revenue. His analysis was confined to the case of two bidders, leaving open the question of the extent to which the results would generalize to more than two bidders. Klemperer (1998) extends the analysis to a simple "wallet game" auction game that can serve as a useful teaching device, to takeover battles with "toeholds" (also see Bulow, Huang, and Klemperer, 1999), and relates the theoretical results to outcomes from US Airwaves Auctions and to a notable merger case in the UK. Klemperer (2002) also relates the results to English auctions, pointing out that since such auctions always end with just two active bidders, that the explosive effects on seller's revenue are a key consideration in sensible auction design. Avery and Kagel (1997) experimentally investigate the wallet game, comparing a pure second-price common value auction to one in which there is an advantaged bidder. They extend the theoretical analysis showing that the explosive effect does not emerge in a first-price sealed-bid auction. Further, Avery and Kagel's experimental results suggest a proportionate response to the private value advantage in the second-price auctions, rather than the explosive effect the theory predicts. This suggests a possible behavioral constraint on the theory's predictions.

The present paper extends the analysis in several directions. First, we show that the explosive effect in the wallet game does not extend to a second-price auction with more than one regular bidder. In this case the advantaged bidder does not win all the time and the change in bidding strategies (compared to the pure common value auction case) corresponds to the size of the private value advantage in a continuous fashion, so that a tiny ϵ corresponds to only a tiny reduction in seller revenue. However, somewhat surprisingly, the private value advantage remains as the number of regular bidders grows without bounds. In addition, we correct the impression that in such auctions revenue necessarily decreases. Our second example provides an information structure in which there is an explosive effect on revenue and winning in the two bidder case, but this explosive effect does not carry over to an English auction with more than one regular bidder. This is important since one can legitimately argue that an English auction reduces to a two bidder auction in the end. While far from proving that one need not worry about such explosive effects in English auctions, it does demonstrate that these explosive effects are not inevitable in English auctions with more than one regular bidder even when they are present in the two bidder case.

The available empirical evidence also leaves ample scope for experimental investigation of almost common value auctions. As noted, Avery and Kagel (1997) found a proportionate rather than explosive response to one bidder having a private value advantage in the wallet game, contrary to the theory's prediction. This might be explained by the fact that both inexperienced and once experienced bidders suffered from a winner's curse (failed to account for the winner's curse) in the symmetric second-price common value auctions used as a control condition. Recall that within the theory the explosive effect of the private value advantage results from the regular bidder fully accounting for the heightened adverse selection effect associated with beating the advantaged bidder. However, to the extent that bidders fail to fully account for this adverse selection effect (they suffer from a winner's curse) this explosive effect on revenue will not be realized.[11] The Avery-Kagel experiment begs the question of whether more experienced bidders who have learned to largely avoid the winner's curse would respond appropriately to the presence of an advantaged bidder. While one might presume this to be the case, to the extent that learning tends to be situation specific, rather than involving some deeper understanding of the economic forces at work in the environment (for which there is some evidence, at least with repsect to the winner's curse; see Kagel and Levin, 1986), the fact that bidders have learned to avoid the winner's curse in the symmetric case might not prepare them for the heightened adverse selection effect associated with the private value advantage case. Further, and with an eye to situations outside the lab, one must consider the extent to which bidders can be taught to understand these adverse selection effects, as advantaged bidders have an obvious incentive to have their disadvantaged rivals understand their disadvantageous position in order to induce them to bid more passively.[12]

[11] There are several other examples where bidders suffering from a winner's curse fail to obey the comparative static predictions of the theory: the effects of public information on seller revenue, the revenue raising possibilities inherent in ascending price English auctions compared to sealed-bid auctions, and the effect of a bidder with inside information on seller revenue. See Kagel and Levin (in press) for a review of these cases.

[12] Klemperer (2002) reports that in the US spectrum auctions that one firm with a private

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value advantage hired a prominent economist to alert rivals to the possibilites of the winner's curse, just in case they had not figured it out on their own.

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Appendix: Proof To Proposition 1

Proof. First we derive the equilibrium maximum willingness to pay, w, for the advantaged bidder and the n regular bidders. Denote by z the signal of the advantaged bidder and by $y = Y_n^1$ the highest signal of the n regulars. As in the symmetric case, the maximum willingness to pay is derived as that price where a bidder is indifferent between winning and paying w or losing, accounting for the information in such an event. This is, in equilibrium, there is a tie w between that bidder and another bidder which makes that bidder indifferent between winning or losing at that price. For the advantaged bidder such an event implies a tie with a regular's bid (as there is only one advantaged bidder). Thus, (A) $\frac{(1+\varepsilon)z(w)+y(w)+(n-1)y(w)/2}{n+1} = w$, the left hand side is the value to the advantaged bidder who ties at w with the highest regular bidder, where $y/2$, is the expected value of each one of the other $(n-1)$ regular bidders given our assumption that signals are i.i.d. from a *uniform* distribution. For the regular bidder the willingness to pay equation is complicated by the fact that a tie may also be with one of the other $(n-1)$ regular bidders and that given a tie the probability of a tie with the advantaged bidder is different (larger) than a tie with one regular. Thus, (B) $\frac{z(w)+y(w)+(n-1)y(w)/2}{n+1}[\frac{y(w)}{z(w)+(n-1)y(w)}] + \frac{2y(w)+(z(w)+(n-2)y(w))/2}{n+1}[\frac{(n-1)z(w)}{z(w)+(n-1)y(w)}] = w$, where, $\frac{z(w)+y(w)+(n-1)y(w)/2}{n+1}$ and $\frac{2y(w)+(z(w)+(n-2)y(w))/2}{n+1}$ are the values of the item to the regular given a tie at w with the advantaged bidder and a regular bidder respectively and $[\frac{y(w)}{z(w)+(n-1)y(w)}]$ and $[\frac{(n-1)z(w)}{z(w)+(n-1)y(w)}]$, are the probabilities that, given a tie at w, the tie is with the advantaged bidder and one of the $(n-1)$ other regulars respectively. The two bidding functions proposed in the proposition simultaneously solve equations (A) and (B), with $w(z) = \alpha(n,\varepsilon)z$, and $w(y) = [\frac{(n-1)-2\epsilon}{(n-1)(1+2\epsilon)}]\alpha(n,\varepsilon)y$. As in the symmetric (pure) second-price, common-value case it is easy to verify (resulting from the way we construct the maximum willingness to pay functions) that: i) in equilibrium, the winner's expected earning is positive; ii) neither the advantaged bidder, nor any of the n regular bidders wish to deviate from the proposed bidding functions.

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