

# Indicative Bidding: An Experimental Analysis\*

John Kagel,<sup>†</sup> Svetlana Pevnitskaya,<sup>‡</sup> and Lixin Ye<sup>†</sup>

## Abstract

Indicative bidding is a practice commonly used in sales of complex and very expensive assets. Theoretical analysis shows that efficient entry is not guaranteed under indicative bidding, as there is no equilibrium in which more qualified bidders are more likely to be selected for the final sale. Furthermore, there exist alternative bid procedures that, in theory at least, guarantee 100% efficiency and higher revenue for the seller. We employ experiments to compare actual performance between indicative bidding and one of these alternative procedures. The data shows that indicative bidding performs as well as the alternative procedure in terms of entry efficiency, while having other characteristics that favor it over the alternative procedure. Our results provide an explanation for the widespread use of indicative bidding despite the potential problem identified in the equilibrium analysis.

Key words: indicative bidding, two-stage auctions, efficient entry, experiment.

JEL classification: D4, D44, C92

## 1 Introduction

Indicative bidding is a two-stage auction process commonly used in the sales of business assets with very high values. In the first stage, the auctioneer (usually an investment banker serving as the financial

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<sup>†</sup>Department of Economics, The Ohio State University, Columbus, OH 43210, USA

<sup>‡</sup>Department of Economics, Florida State University, Tallahassee, FL 32306, USA

adviser to the selling party) actively markets the assets through phone calls and mailings to a large group of potentially interested buyers who are asked to submit non-binding bids. These bids are meant to be indicative of bidders' interest in the asset for sale. The highest of these non-binding bids, in conjunction with decisions regarding bidders' qualifications, are then used to establish a short list of final (second-stage) bidders. These short-listed bidders then engage in extensive studies to acquire more information about the asset for sale. Finally, a standard first-price sealed-bid auction is conducted, in which the short-listed bidders submit firm and final bids.

The practice of indicative bidding is quite widespread. For example, it has been used in divesting billions of dollars of electrical generating assets in the U.S. in the last decade in response to state government restructuring of the electric power industry designed to separate power generation from transmission and distribution. From late 1997 through April 2000, 51 investor owned-utilities (IOUs; 32 percent of the 161 IOUs owning generation capacity) have either divested or are in the process of divesting approximately 156.5 gigawatts of power generation capacity, which represents approximately 22 percent of the total U.S. electric utility generating capacity. For the dozens of cases about which there is detailed information, indicative bidding was used in every case.<sup>1</sup> Indicative bidding is also commonly used in privatization, takeover, merger and acquisition contests. Leading examples include the privatization of the Italian Oil and Energy Corporation (ENI), the acquisition of Ireland's largest cable television provider Cablelink Limited, and the takeover contest for South Korea's second largest conglomerate Daewoo Motors. Finally, indicative bidding is commonly used in the institutional real estate market, which has an annual sales volume of something in the order of \$60 to \$100 billion (Foley, 2003).

A prominent feature of this type of high-value asset sales is that the cost of acquiring information about the assets can be quite substantial. In other words, entry into the auction can be quite costly. For example, in the sale of a billion-dollar asset, the second-stage bidders often spend millions of dollars to study the asset closely in order to prepare a bid. One rationalization for indicative bidding relates to these high bid preparation costs since bidders may be reluctant to enter an auction in which they must pay these high valuation costs while having little idea about their chances of winning the auction

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<sup>1</sup>An overview is available from the Energy Information Administration website:  
[http://www.eia.doe.gov/cneaf/electricity/chg\\_stru\\_update/chapter9.html](http://www.eia.doe.gov/cneaf/electricity/chg_stru_update/chapter9.html).

The big IOU's that have divested their generation assets through indicative bidding include, among others, Dominion Resources (Virginia Power), Unicom (Formerly Commonwealth Edison), Pacific Gas & Electric Co., Southern California Edison, Consolidated Edison, General Public Utilities System, Potomac Electric Power Co., Niagara Mohawk Power, Illinois Power, and Duquesne Light, etc.

against the final rival bidders. As a result, other things equal, these high bid preparation costs reduce seller's expected revenue.<sup>2</sup> Furthermore, it may be that bidders with the highest ultimate valuation for the asset do not enter the auction for these reasons.<sup>3</sup> In contrast, with indicative bidding, bidders can at least make some relatively inexpensive calculations as to their likely value of the asset to inform their initial bids, and only when selected for the short list conduct the expensive asset valuation process under full knowledge that they have a reasonable chance of winning the item.

Despite the importance of indicative bidding, it has received little attention from auction theorists. To our knowledge, Ye (2007) is the only attempt at analyzing indicative bidding theoretically.<sup>4</sup> One major result in Ye is that there does not exist a symmetric increasing equilibrium for indicative bidding. Absent a symmetric increasing equilibrium, efficient entry is not guaranteed. That is, the most qualified bidders will *not* be reliably selected to be on the short list competing in the second-stage bidding. Given the widespread use of indicative bidding and the billions of dollars involved, this efficiency loss could be substantial.

Ye (2007) identifies a number of alternative two-stage bidding procedures that, in theory at least, guarantee efficient bidding in the sense that the short list consists of those bidders with the highest preliminary (first-stage) valuations, while preserving the best properties of indicative bidding; namely, avoidance of the costly (thorough) asset valuation process for all but the short list of final stage bidders. Most prominent among these is a uniform-price procedure in which first-stage bids are binding and

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<sup>2</sup>There is a growing literature on auctions with costly entry (see, for example, French and McCormick (1984), Samuelson (1985), McAfee and McMillan (1987), Engelbrecht-Wiggans (1993), Levin and Smith (1994), Ye (2004), Pevnitskaya (2004), Landsberger (2005), and Gal et al. (2006)). Taking costly entry into account, the analysis differs from the traditional auction analysis which in general assumes that the set of bidders is fixed, and the bidders are costlessly endowed with information (see, for example, Vickrey (1961), Riley and Samuelson (1981), Myerson (1981), and Milgrom and Weber (1982)).

<sup>3</sup>For an overview on how important encouraging entry is, see, e.g., Klemperer (2002).

<sup>4</sup>A two-stage auction process similar to the one considered in our research is analyzed in Perry et al. (2000), and McAdams et al. (1999). However, there are two major differences between their approaches and ours. First, they do not consider entry cost. Second, the first-stage bids in their setup are binding, as they serve as the minimal bid (reserve price) for second-stage bidding. Compte and Jehiel (2006) suggest a model with costly information acquisition to justify the use of two-stage auctions, but they also model binding, instead of non-binding first-stage bidding in their analysis. Recent works on “qualifying auctions” by Boone and Goeree (2005) and Boone et al. (2006) are most closely related to indicative bidding modeled here. In their setting, bidders place non-binding bids and all but the lowest bidder are allowed to participate in stage two, which is a standard second-price auction augmented with a reserve price. Note that again, their model is different from ours mainly because they do not consider entry cost.

establish an entry fee (the highest rejected first-stage bid) for those bidding in the second stage. This alternative two-stage bid process is relatively simple and is, arguably, the “fairest” of the possible first-stage selection processes as all entrants pay the same second-stage entry fee, a very appealing characteristic.

The experiment reported here compares this uniform-price, two-stage bid process with indicative bidding. Experiments are a particularly appropriate tool here for two reasons. First, the non-existence of a symmetric increasing equilibrium for indicative bidding does not necessarily lead to inefficient entry; it merely says that efficient entry is not guaranteed. As such one can use laboratory experiments to see how far actual behavior deviates from the efficient outcome.<sup>5</sup> Second, the fact that the uniform-price, two-stage bid process yields fully efficient outcomes in theory is no assurance that it will do so in practice, as considerable prior experimental research has demonstrated.<sup>6</sup> As such it is relevant to compare outcomes from indicative bidding with some alternative, and presumably superior, bidding mechanism as opposed to some idealized outcome.

Our experiment shows that indicative bidding performs as well as the alternative bid process in terms of efficiency. This results from two factors: (1) there is sufficient heterogeneity (or random variation) in first-stage bids under the uniform-price process to destroy 100% entry efficiency and (2) under indicative bidding there is a clear tendency for first-stage bids to reflect first-stage (preliminary) valuations so as to induce fairly high entry efficiency. Further, indicative bidding does better on other dimensions; most importantly, indicative bidding yields higher average profits and fewer bankruptcies than the alternative two-stage process in the initial auction periods as there is systematic overbidding in stage one early on under the alternative bidding process. Although the higher revenue is good for sellers in the short run, the bankruptcies clearly indicate the greater difficulty bidders have early on with the uniform-price two-stage process. Should these problems carry over to field settings they could destroy the long run viability of the alternative mechanism.

From a broader perspective our results suggest a trade-off between types of mechanisms: One with clear equilibrium predictions insuring efficiency in theory, but which is relatively complex for bidders. The other with no clear equilibrium prediction but with relatively simple rules.<sup>7</sup> Game theorists would

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<sup>5</sup>This corresponds to using experiments as a “wind tunnel” against which to evaluate a proposed mechanism (see, for example, Plott, 1987).

<sup>6</sup>The considerable body of experimental research on auctions has yet to identify a mechanism that insures, in practice, 100% efficiency (see, Kagel, 1995, for a review of the experimental auction literature).

<sup>7</sup>The entry fee in the uniform-price auction is endogenously determined by the first-stage bids; while under indicative bidding it is given exogenously. Other things equal, a fixed entry fee makes planning easier than one that varies randomly.

naturally favor a mechanism with clear equilibrium predictions, assuming very sophisticated bidders. However, full rationality should only be expected in situations with a relatively easy-to-follow equilibrium. Play in the first stage is quite complicated under the alternative two-stage process considered here, as well as other possible alternatives to indicative bidding. In such contexts, a mechanism with relatively simple rules, though absent a clear equilibrium outcome, may be more desirable. This, in conjunction with the relatively high efficiency achieved under indicative bidding compared to these alternative processes, may provide an explanation for the widespread use of indicative bidding.<sup>8</sup>

This paper is organized as follows. Section 2 establishes the theoretical framework. Section 3 describes our experimental design and procedures. Section 4 analyzes the data and presents our main results. We compare indicative bidding with the uniform-price two-stage process in terms of entry efficiency, first- and second-stage bidding, and auction payoffs including entry fees, bidders' profit, and seller's revenue. Section 4 also provides a robustness check of efficiency under indicative bidding, comparing entry efficiency against a two-stage bid process with discriminatory bidding in stage one. Section 5 concludes. All equilibrium bid functions and equilibrium payoffs are derived in the appendix.

## 2 Theoretical Considerations

There is a single, indivisible asset for sale to  $N$  potentially interested buyers (firms). Values are revealed in two stages. In the first stage, each potential buyer obtains a preliminary estimate,  $X_i$ , of the value of the asset. The  $X_i$ 's are independent draws from a distribution with CDF  $F(\cdot)$ . Bidder  $i$  knows its own  $X_i$  but not its competitors' (denoted as  $X_{-i}$ ). In the second stage, by incurring an entry cost  $c$ , each second-stage bidder obtains an updated estimate of the value of the item,  $S_i = X_i + Y_i$ , where the  $Y_i$ 's are independent draws from a distribution with CDF  $G(\cdot)$ .<sup>9</sup> For ease of analysis, we assume that  $X$  and  $Y$  are independent, and the density function of  $X$ ,  $f(\cdot)$  is continuous and strictly positive on its compact support  $[\underline{x}, \bar{x}]$ .

The sale of the asset proceeds in two stages. Bids in the first stage can be binding or non-binding. With indicative bidding, the first-stage bids are non-binding. Note that indicative bids are not cheap talk, as bidders entering into the second stage incur costs to determine their updated values for the asset. Under the alternative bid process considered here, first-stage bids are binding, with all the bidders

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Further discussion of the greater simplicity of indicative bidding is reserved for the concluding section of the paper.

<sup>8</sup>In this same spirit also see Kagel and Levin (2006), whose experimental results suggest a trade-off between the simplicity and transparency of a mechanism and the strength of its solution concept for less than fully rational agents.

<sup>9</sup>This is the case of entry with private value updating. In Ye (2007) entry with common value updating is also considered.

selected to bid in stage-two paying an entry fee equal to the highest rejected stage-one bid, and those not making it into the second stage pay nothing.<sup>10</sup> Since this alternative process selects stage-two finalists based on a uniform-price procedure, we will often refer to it as the uniform-price scheme or procedure.

Based on the first-stage bids, the highest  $n$  bids are selected to bid in stage two, where  $n$  is announced by the auctioneer prior to bidding in the first stage. Upon being selected, the  $n + 1$ st highest first-stage bid (the highest rejected bid) is announced to the  $n$  second-stage bidders.<sup>11</sup> Each stage-two bidder incurs a cost  $c$  to learn  $Y_i$ . Then they engage in a final first-price sealed-bid auction to determine who wins the item.

Throughout we will focus on efficient entry in which  $n$  bidders with the  $n$  highest first-stage values are selected to bid in the second-stage auction, where  $n$  is pre-announced.<sup>12</sup> The importance of efficient entry can be seen from the value function  $S_i$ . Since the first-stage signal enters the total value function in an additive way, the higher the first-stage signal, the more likely that the bidder is to end up with a higher total value.<sup>13</sup>

To guarantee efficient entry, the bid function in the first stage must be symmetric and strictly increasing in  $x \in [\underline{x}, \bar{x}]$  (almost everywhere). Note that a weakly increasing bid function is not sufficient to guarantee efficient entry. For brevity of exposition, we refer to *the symmetric increasing equilibrium* as the pure symmetric equilibrium in which each bidder bids according to a strictly increasing first-stage bid function.

In what follows we use  $X_{j;n}$  to denote the  $j$ th highest value among all  $n$  draws of  $X$ , and  $X_{j;-i}$  to denote the  $j$ th highest value among all draws of  $X$  except  $X_i$ . In addition let  $\hat{X}_i$  be the first-stage signal (value) of bidder  $i$  who has succeeded in making it into the second stage, and  $\hat{S}_i = \hat{X}_i + Y_i$  be the total value for bidder  $i$ .

Since the highest rejected first-stage bid is revealed to all second-stage bidders, assuming efficient en-

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<sup>10</sup>That is, under the alternative process bidders bid for the entry rights first, then they bid for the asset after being selected for the final stage.

<sup>11</sup>There are variations regarding the revelation of the first-stage bids in practice. We consider revealing the highest rejected bid in our model so that each final bidder enters the second stage with identical beliefs about the first-stage observations.

<sup>12</sup>More formally, efficient entry involves an optimal number of stage-two bidders that maximizes the expected revenue. The optimal number of  $n$  is investigated in Ye (2007), but in the experiment the focus is on efficient entry conditional on  $n$ , which may not be the revenue-maximizing number of final bidders.

<sup>13</sup>Note that efficient entry does not imply ex post efficiency. In fact, inefficiency may arise when a bidder's initial value,  $X_i$ , is high but her final value  $X_i + Y_i$  is low compared to what other bidders' valuations would have been.

try, payoff equivalence holds among the second-stage standard auctions including first-price and second-price auctions. As a result, bidder  $i$ 's expected profit from the second-stage auction conditional on being the marginal entrant is given by:

$$E\Omega(x_i) = E[1_{\{s_i > \hat{s}_{1,-i}\}}(s_i - \hat{s}_{1,-i}) | X_{n,-i} = x_i] \quad (1)$$

The major results regarding indicative bidding and the uniform-price procedure can now be stated. The proofs can be found in Ye (2007).

**Proposition 1** *Under the indicative bidding mechanism, no symmetric increasing equilibrium exists such that each bidder bids according to a strictly increasing function in the first stage.*

The intuition for this non-existence result can be understood as follows. Suppose everyone but bidder  $i$  bid according to a strictly increasing bid function  $B(\cdot)$ . Consider the marginal effect of varying bidder  $i$ 's bid. This marginal effect is not zero only in the event that bidder  $i$  is the marginal entrant, i.e., when  $x_i = X_{n,-i}$ . In such an event, the expected gain from bidding in the second-stage auction is  $E\Omega(x_i)$ , while the loss is the entry cost  $c$ . So in equilibrium

$$E\Omega(x_i) f_{X_{n,-i}}(x_i) = c \quad (2)$$

Since this condition holds only for bidder “types” ( $X_i$ 's) with probability measure zero, no symmetric increasing equilibrium exists. Absent a symmetric increasing equilibrium, efficient entry is not guaranteed.<sup>14</sup>

However, when  $E\Omega(x)$  is strictly increasing, it can be shown that there exists a unique symmetric increasing equilibrium under the uniform-price scheme.

**Proposition 2** *Under a uniform-price scheme, there exists a unique symmetric increasing equilibrium if  $E\Omega(x)$  is strictly increasing in  $x$ . The unique symmetric increasing equilibrium bid function is given by:*

$$\varphi^U(x) = E\Omega(x) - c \quad (3)$$

The intuition is clear. The first order condition (FOC) (2) cannot be balanced under indicative bidding since first-stage bids are non-binding and entry cost  $c$  is positive. However, under the alternative

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<sup>14</sup>For the extreme case in which entry does not involve value updating, Milgrom (2004) provides an alternative argument for the non-existence result.

bid procedure, first-stage bids are binding, which opens up the possibility to restore the incentive compatibility condition. Specifically, under a uniform-price auction, in equilibrium bidders will submit a bid equal to the expected gain from entry conditional on being the marginal entrant, which gives rise to the equilibrium bid function (3).

Note that entry may be subsidized or even taxed under the uniform-price procedure. Let  $\hat{c}$  be the “adjusted” entry cost (for the bidders) after a subsidy is provided or a tax is imposed. In this case, the entry cost  $c$  in the equilibrium bid function (3) should be replaced by the “adjusted” entry cost  $\hat{c}$ .

### 3 Experimental Design and Procedures

Each experimental session started with two auction markets with six bidders each ( $N = 6$ ) operating simultaneously. Between 14 and 16 subjects were recruited for each session, with the extra subjects acting as standbys in any given auction.<sup>15</sup> In each auction subjects were randomly assigned to one of the two auction markets, with members of the stand-by group guaranteed to be assigned to active bidding status in the next auction. In case of bankruptcies, the number of subjects in the stand-by group was decreased, with only one auction market in operation (with  $N = 6$ ) in case where the number of bankruptcies resulted is fewer than 12 eligible bidders. Sessions lasted for two hours and, with one minor exception, each had 25 auctions played for cash.<sup>16</sup>

Table 1: Experimental Treatment Conditions

Session	Number of Subjects	Auctions Completed	Auctions with Two Markets	Number of Bankruptcies
Indicative Bidding				
IB1	14	25	23	4
IB2	16	25	25	3
IB3	14	25	25	3
Uniform-Price Procedure				
U1	14	23	2	9
U2	16	25	10	5
U3	16	25	13	7

<sup>15</sup>Our goal in all cases was to have 16 subjects to start each session. The vagaries of the recruiting process resulted in 14 subjects showing up in half the sessions.

<sup>16</sup>In one case there were 23 auctions played for cash as bankruptcies reduced the number of eligible bidders after period 23 (see Table 1 below).

Each bidder demanded a single unit, with first-stage values ( $X_i$ ) drawn iid from a uniform distribution with support  $[500, 1500]$  (with integer values only being drawn). In all auctions the number of bidders participating in the second-stage auction was 2 ( $n = 2$ ). Second-stage values ( $Y_i$ ) were randomly drawn, and were either 500 or 1500 with equal probability.<sup>17</sup> This particular information structure permits analytical solutions for the equilibrium bid functions and equilibrium payoffs (see the appendix for detailed derivations).

The essential difference between indicative bidding and the alternative bid procedure is that under indicative bidding stage-two bidders each pay a fixed entry cost ( $c$ ) to evaluate the asset but no entry fee based on first-stage bids. In contrast, under the alternative procedure, stage-two bidders must pay the asset valuation cost ( $c$ ) plus an endogenously determined entry fee defined on the basis of the stage-one auction rule. In order to simplify the experimental procedures, the only fee stage-two bidders pay under the alternative bid procedure is the endogenously determined entry fee. That is, the seller effectively subsidizes all the asset valuation costs ( $c$ ) under the alternative bid procedure. Although the primary reason for this is to simplify the experimental procedures, this simplification does not affect equilibrium outcomes. Ye (2007) shows that any level of subsidization for  $c$  (including the full subsidization employed in our experimental design) will not affect equilibrium outcomes (the expected revenue to the seller, and the expected profit to the bidders, etc.). Had we maintained the asset valuation cost for the alternative procedure we would have had to distinguish between two “costs” of entry – namely the entry fee coming from the highest rejected stage-one bid and the asset valuation cost. By eliminating the asset valuation cost for the alternative procedure, we can use exactly the same language under both procedures (by referring to “entry fees” in both cases).

Before submitting their first-stage bids, subjects could see on their computer screens their first-stage values ( $X_i$ ) and the values of  $N$  and  $n$ . Also shown on each bidder’s screen was the possible total value a bidder would have if they were bidding in stage two. Although this is easy enough to calculate, we wanted to make sure bidders were cognizant of their potential total value when formulating their stage-one bid. In indicative bidding sessions subjects also saw the fixed entry cost  $c$ . After the first-stage bids were collected, bidders were told if they were eligible to bid in stage two, and all bidders in all treatments were told the third-highest (the highest rejected) stage-one bid. Record sheets were provided for subjects to record this information along with their bids and the auction outcomes in an

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<sup>17</sup>So the second-stage value updating has a simple implication: bidders will learn an either “low” or “high” additional value component.

effort to provide *all* bidders with some idea of the likely entry fees they would have to pay for the right to bid in stage two under the uniform-price mechanism. Prior to submitting their stage-two bids subjects were informed of their stage-two values ( $Y_i$ ) and their total values (which were calculated by the computer automatically), and the stage-two entry fee/cost. At the end of the auction, the price of the item (the highest stage-two bid) was reported back to *all* active bidders in *that* market, with no information provided regarding the high bidder's valuation or the second-highest bid.<sup>18</sup>

First-stage bids were restricted to be non-negative, with second-stage bids restricted to be no less than 1,000, the lowest possible stage-two value ( $S_i$ ). This restriction on second-stage bids effectively imposes a reserve price at the minimal possible total value.

At the beginning of the experiment each subject was given an initial cash balance of 1,000, with auction earnings and entry fees/costs added to or subtracted from this starting cash balance. Subjects were paid their end of experiment balances in cash under the conversion rate of 1 experimental dollar equal to \$0.03.

Each session began with instructions which were read out loud to subjects, with copies for them to follow along with as well.<sup>19</sup> This was followed by three dry runs. Subjects who went bankrupt were no longer permitted to bid and were dismissed from the session. All subjects were paid a \$6 participation fee (including the bankrupt bidders) along with their end of experiment cash balance.

Subjects were recruited through e-mail solicitation from a database consisting of the several thousand students enrolled in economics classes at the Ohio State University for that quarter and the previous quarter. Subjects were only allowed to participate in a single experimental session, although they may have had experience with other experiments.

It is shown (in the appendix) that the equilibrium expected entry fee is 196.5 under the uniform-price procedure. To preserve the comparability, we choose a fixed entry fee of 200 for the indicative bidding treatment.

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<sup>18</sup>Stand-by subjects' screens displayed no information other than that they were on stand-by status for this auction, with a guarantee of the right to participate in the next auction.

<sup>19</sup>Instructions employed a meaningful context to motivate the rationale behind the two-stage bid process. We did this because we thought it would provide subjects with a better framework for thinking about the problem than using a generic context which would not provide any framework for motivating the novel two-stage auction procedure. Copies of the instructions can be found at <http://www.econ.ohio-state.edu/lixinye/Experiment/Indicative>. We ran several sessions of each type using the same procedures but without the meaningful/motivating context. The results of these sessions are essentially the same as those reported here.

## 4 Experimental Results

Table 1 shows the experimental treatment conditions along with the number of subjects who started each session. There were three indicative bidding (IB) sessions and three uniform-price (U) sessions. Also shown are the number of auctions completed with two markets running simultaneously and the number of bankruptcies in each session.

**Observation 1** *Bankruptcies are a more common problem under the uniform-price procedure than under indicative bidding.*

The most obvious manifestation of the higher bankruptcy rate under the uniform-price procedure is the raw numbers averaging 7 subjects per session compared to 3.3 under indicative bidding. These differences are statistically significant using subject as the unit of observation ( $Z = 2.29$ ,  $p < .05$ , two-tailed Mann-Whitney test) or using session values for the frequency with which subjects went bankrupt ( $p < .10$ , two-tailed Mann-Whitney test). This also shows up in terms of the number of auctions in which two markets operated simultaneously, which lasted for all 25 auctions under indicative bidding in almost all cases, but for a maximum of 13 auctions with the uniform-price procedure. Given that most of the bankruptcies occurred early in the uniform-price case, and in keeping with common practice in the experimental literature, in what follows we will distinguish between early auctions (1-12) where subjects are still learning and later auctions (13-25) where the most inept bidders have been eliminated and remaining bidders are well along in the learning process.

### 4.1 First-stage Bids and Entry Efficiency

The primary motivation for examining alternatives to indicative bidding is that the latter cannot guarantee, in theory, efficient entry into the second stage, whereas the alternative bidding procedure can. Thus, the question to be answered in this section is how actual efficiency compares across auction institutions.

Table 2: Entry Efficiency

Session	Percentage of $X_{1,6}$ & $X_{2,6}$ Enter Stage Two			Percentage of $X_{1,6}$ or $X_{2,6}$ Enter Stage Two			Ratio Efficiency		
	1-12	13-25	All	1-12	13-25	All	1-12	13-25	All
<b>Indicative Bidding</b>									
IB1	16.7 (7.8)	25.0 (9.0)	20.8 (5.9)	95.8 (4.2)	95.8 (4.2)	95.8 (2.9)	87.8 (2.2)	90.2 (1.9)	89.0 (1.4)
IB2	41.7 (10.3)	50.0 (10.0)	46.0 (7.1)	91.7 (5.8)	100 (0.0)	96.0 (2.8)	91.8 (2.2)	95.5 (1.2)	93.8 (1.2)
IB3	23.5 (10.6)	23.8 (9.5)	23.7 (7.0)	70.6 (11.4)	85.7 (7.8)	78.9 (6.7)	85.7 (3.1)	89.9 (1.8)	88.0 (1.7)
IB Average	29.2 (5.4)	31.6 (5.4)	30.4 (3.8)	86.1 (4.1)	94.7 (2.6)	90.5 (2.4)	88.6 (1.4)	91.6 (1.0)	90.2 (0.8)
<b>Uniform Procedure</b>									
U1	14.3 (9.7)	36.4 (15.2)	24.0 (8.7)	85.7 (9.7)	100 (0)	92.0 (5.5)	89.7 (2.3)	93.5 (2.0)	91.4 (1.6)
U2	31.8 (10.2)	23.1 (12.2)	28.6 (7.7)	90.9 (6.3)	84.6 (10.4)	88.6 (5.5)	92.4 (2.3)	93.8 (2.1)	92.9 (1.6)
U3	11.1 (7.6)	15.4 (10.4)	12.9 (6.1)	77.8 (10.1)	76.9 (12.2)	77.4 (7.6)	84.2 (3.0)	85.0 (3.9)	84.5 (2.4)
U Average	19.6 (5.4)	24.3 (7.2)	21.5 (4.3)	85.7 (4.7)	86.5 (5.7)	86.0 (3.6)	88.8 (1.5)	90.6 (1.8)	89.5 (1.1)

Table 2 reports entry efficiency under both auction institutions. Three efficiency measures are reported: the first is the frequency with which the bidders with the highest and second highest stage-one values ( $X_{1,6}$  and  $X_{2,6}$ ) entered the second stage. The second is the frequency with which the bidders with the highest or the second highest stage-one values ( $X_{1,6}$  or  $X_{2,6}$ ) entered the second stage. These frequency measures are “rough” since they account only for who gets into the second stage, but not for the loss of efficiency associated with lower valued bidders getting into the second stage. The third efficiency measure calculates the ratio of the sum of the first-stage values for those actually entering the second stage divided by the sum of the highest two first-stage values. This is the conventional efficiency measure employed in the experimental auction literature (referred to as the *ratio efficiency* in the table).

With the exception of the first two measures in the second uniform-price session (U2), all of the efficiency measures in all sessions are increasing with experience between auctions 1-12 and 13-25. And even in this exceptional case the conventional ratio efficiency measure increases between auctions 1-12 and 13-25. Furthermore, although the average frequency with which bidders with the highest and second-highest first-stage values entered stage two is relatively low in both treatments (30.4% and

21.5% overall for the indicative bidding and uniform-price treatments respectively), they are significantly higher than one would expect if stage-one bids were strictly random (6.7%). Most importantly, looking at overall averages between treatments, with the exception of the standard ratio efficiency measure for auctions 1-12, efficiency is higher under indicative bidding than under the uniform-price procedure. These differences are, however, quite small so that it comes as no surprise that for all three efficiency measures we are unable to reject a null hypothesis of no difference in efficiency between treatments at anything approaching conventional significance levels (using Mann-Whitney tests with auction outcomes as the unit of observation).

**Observation 2** *Efficiency is generally improving with experience and clearly beats random bidding under both procedures. Overall efficiency measures tend to be a bit higher under indicative bidding than the alternative procedure, but none of the differences approach statistical significance at conventional levels. Thus, we conclude that indicative bidding does as well as the alternative bidding procedure in generating entry efficiency.*

Table 3 checks for the monotonicity of the first-stage bid functions by looking at Spearman rank order correlations between first-stage bids and values averaged by auction period from the two auction institutions.<sup>20</sup> Looking at auction periods 1-12, rank order correlations are higher under indicative bidding than the uniform-price procedure in 10 of the 12 periods, which is statistically significant at the 10% level using a two-tailed sign test. However, in periods 13-25 average rank order correlations by auction period are higher as often under indicative bidding as under the uniform-price institution (7 out of 13 periods). Thus, in terms of the monotonicity of first-stage bidding, indicative bidding does better than the uniform-price scheme early on in the bidding process, with the two essentially tied after that.<sup>21</sup>

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<sup>20</sup>That is, for each auction market we calculate the relevant correlation coefficient first. We then average across markets for a given auction period. Thus, for example, the data reported for period 4 under IB represents the average of six auction markets (2 for each session times 3 sessions).

<sup>21</sup>Under both procedures, we occasionally got negative rank order correlations in a particular auction. These are usually swamped by positive correlations in other auctions for the same auction period. This did not happen in period 25 in the uniform-price case where there were only two auctions – one with a negative rank order correlation of -0.580 and the other with a positive rank order correlation of .314. Larger negative rank order correlations than this can be found in other auctions.

Table 3: Correlation coefficients for first-stage bids relative to values, by period\*

Period	SPEARMAN RANK ORDER CORRELATIONS	
	Indicative	Uniform
1	0.539	0.344
2	0.373	0.349
3	0.500	0.299
4	0.292	0.194
5	0.300	0.383
6	0.299	0.237
7	0.432	0.337
8	0.521	0.443
9	0.342	0.682
10	0.788	0.539
11	0.491	0.448
12	0.664	0.345
13	0.373	0.180
14	0.493	0.790
15	0.297	0.686
16	0.486	0.286
17	0.390	0.372
18	0.630	0.670
19	0.283	0.540
20	0.403	0.467
21	0.522	0.537
22	0.775	0.324
23	0.606	0.048
24	0.720	0.600
25	0.269	-0.133

\* Averaged across auction markets in each period

**Observation 3** *Spearman rank order correlations are used to check the consistency with which stage-one bids increase with stage-one values. Average Spearman rank order correlations by auction period are consistently higher early on under indicative bidding than under the uniform-price procedure, but are essentially the same after that.*

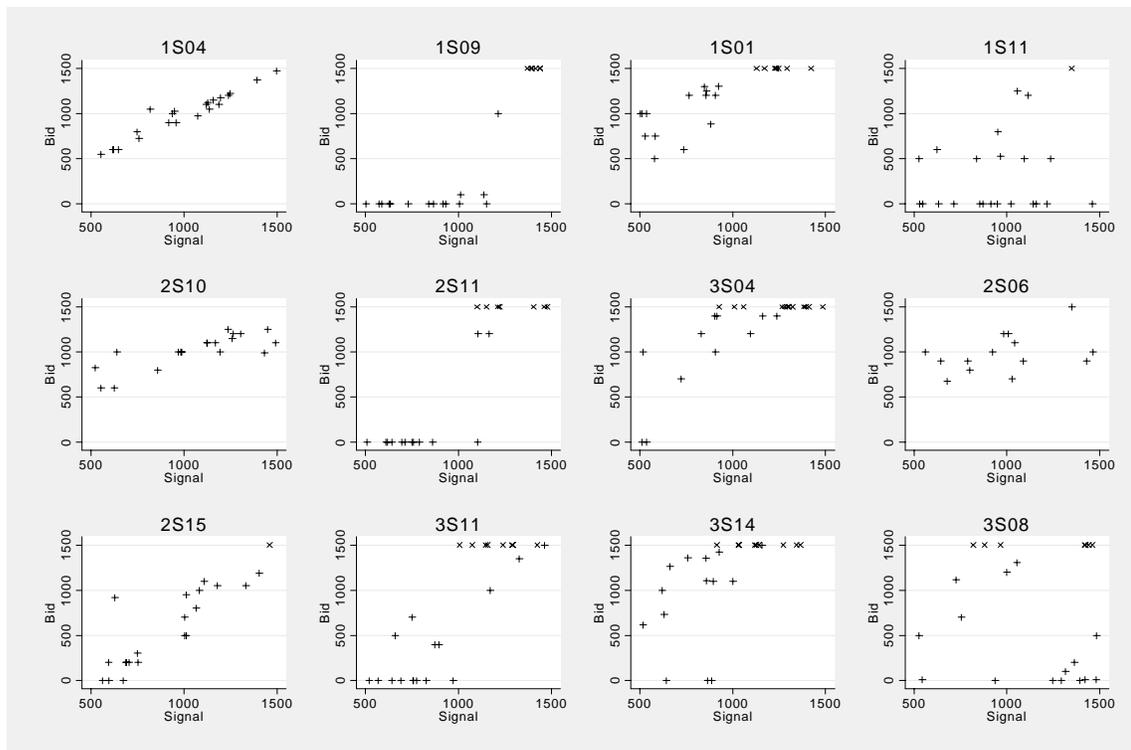
The uniform-price bidding procedure produced entry fees that were significantly higher than predicted: The average realized entry fee (with the standard error of the mean in parentheses) is 426.7 (26.8) over all rounds, and 280.8 (24.2) over the last 13 rounds, versus predicted entry fees of 196.5. These much larger than predicted entry fees contributed substantially to the high bankruptcy rates under the uniform-price procedure.

## 4.2 Individual Subject Stage-One Bid Patterns

The efficiency data reported above relates to market measures of entry efficiency. The data leaves unanswered the question as to individual subject bid patterns that give rise to these efficiency measures. That is, what are the underlying rules of thumb bidders use to produce better than random entry into

stage-two under indicative bidding? What are the individual subject bid patterns that result in less than the predicted full efficiency under the uniform-price procedure?

Figure 1: Stage-One Bid Patterns under Indicative Bidding for Individual Subjects

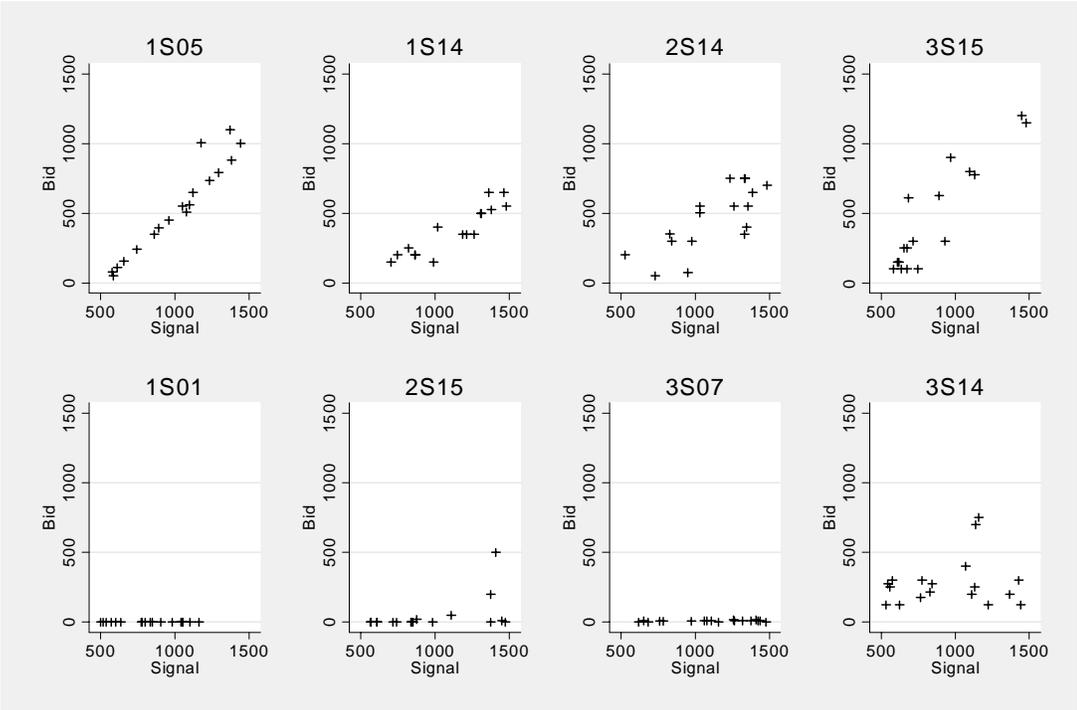


+ represent bids less than or equal to 1,500; × indicate bids greater than 1,500. Subject designation is as follows: xSyy where x is a session number (1-3) and yy is the subject number within the session.

Figure 1 reports individual subject bid patterns under indicative bidding. We have identified three primary rules of thumb that subjects employed. First, a number of subjects (31.8%; 14 out of 44) have essentially increasing bid functions – the higher their first-stage signals, the higher their first-stage bids, with all bids being less than or equal to 1,500. The first column of Figure 1 provides three examples of this type of bidding. The second category (22.7%; 10 out of 44) essentially employs a step function, with low bids for low stage-one valuations and substantially higher bids for higher stage-one valuations. The second column of Figure 1 provides three examples of this type of bidding. Note that in these cases bids above 1,500 are usually well above 1,500; e.g., subject 1S09’s bids above 1,500 are all 99,999 or higher. The third category (15.9%; 7 out of 44) essentially employs an increasing bid function over lower stage-one valuations, followed by relatively high bids for higher stage-one values (bids great than 1,500). The third column of Figure 1 provides examples of this type of bidding. These three types of bidding account for 70.5% of all the first-stage bids. The remaining bidders cannot be easily classified. Some

have essentially flat stage-one bids while others are scattered with no particular pattern. Examples of these types are provided in the last column of Figure 1. While the categories specified here require some subjective interpretation, they do categorize sensible clusters of rules of thumb for stage-one bidding that together serve to produce better than random entry into stage two.

Figure 2: Stage-One Bid Patterns under Uniform-Price Procedure for Individual Subjects



Subject designation is as follows: xSyy where x is a session number (1-3) and yy is the subject number within the session.

Figure 2 reports individual subject stage-one bid patterns under the uniform-price procedure (we have restricted our analysis to subjects who did not go bankrupt). Slightly over half (53.8%; 14 out of 26) employ increasing bid functions, samples of which are shown in the first row of the figure. In addition, there are a number of essentially flat bids (30.8%; 8 out of 26), samples of which are shown in the second row of Figure 2. Given that there are a number of subjects with essentially flat or scattered stage-one bid functions, in conjunction with the fact that those with increasing stage-one bid functions show considerable heterogeneity, the alternative two-stage procedure fails to achieve the 100% efficiency that the theory (unrealistically) predicts.

**Observation 4** *We identify three main rules of thumb individual subjects employ which serve to produce the better than random entry into stage two reported for market outcomes under indicative bidding. Further, there are a sufficiently large number of subjects employing essentially flat or scattered stage-*

*one bids, along with sufficient heterogeneity between those subjects with increasing bid functions, such that the alternative two-stage procedure fails to achieve the 100% efficiency that the theory predicts.*

### 4.3 Second-Stage Bidding, Bidder Profits, and Seller Revenue

Table 4 reports random effect estimates of second-stage bid functions for the two treatments. We have normalized the bid functions to set the origin equal to zero (to account for the minimum bid requirement of 1,000).<sup>22</sup> We employ as explanatory variables bidders' total value in stage two (the sum of their stage-one and stage-two values,  $S_i$ ) and the highest rejected stage-one bid. Total value in stage two is relevant because stage two bid functions should be monotonically increasing in  $S_i$ . The highest rejected stage-one bid is included because it is relevant for the uniform-price procedure.<sup>23</sup> We include it for the indicative bidding case as well for consistency. We also tried regressions with total value squared included, but this variable failed to have any systematic effect in both treatments and are not reported. We report second-stage bid functions for the data as a whole, as well as for auction periods 1-12 and 13-25 separately.

The first thing to note is the large improvement in  $R^2$  for both treatments between auctions 1-12 and 13-25, reflective of a tightening of the variance around the coefficient values particularly with respect to the intercept of the bid function for the indicative bidding case.<sup>24</sup>

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<sup>22</sup>That is, we have subtracted 1,000 from all bids and all values in the regressions.

<sup>23</sup>In the theory, in equilibrium bidders are able to deduce the stage-one value underlying the highest rejected stage-one bid which, in turn, is relevant to the stage-two bid function (see the appendix). Although our subjects are clearly not in equilibrium under the uniform-price mechanism, we still test to see if this variable has any implications for behavior.

<sup>24</sup>We have not estimated equilibrium second-stage bid functions for the uniform-price case as these are highly non-linear which makes them difficult to estimate and they would be misspecified as subjects are clearly not in equilibrium. The regressions reported provide a reasonable fit to the data especially for the later auction periods. We conduct simulations to estimate the  $R^2$  under the specifications reported in the text but with simulated bidders employing the equilibrium bid functions (4) and (5). The average  $R^2$  is .831 (with 95% confidence interval of .794 – .865) when the highest stage-one bid is not included in the regression, and .925 (with 95% confidence interval of .898 – .948) when the highest stage-one bid is included in the regression. Simulations employed 10,000 repetitions of each regression with 108 observations (the number of auctions in the regressions reported in the text).

Table 4: Second-Stage Bid Functions

	Constant	Total value(s)	Highest Rejected Stage-One Bid	R <sup>2</sup>
Indicative (IB) (All Rounds)	-85.6 <sup>***</sup> (32.5)	0.778 <sup>***</sup> (0.021)		0.78
First 12 Rounds	-84.4 <sup>***</sup> (32.8)	0.778 <sup>***</sup> (0.021)	-0.001 (0.003)	0.78
	-86.7 (57.2)	0.759 <sup>***</sup> (0.041)		0.66
	-79.7 (76.4)	0.759 <sup>***</sup> (0.041)	-0.005 (0.038)	0.66
Rounds 13-25	-57.4 <sup>***</sup> (21.5)	0.776 <sup>***</sup> (0.017)		0.93
	-56.8 <sup>***</sup> (21.7)	0.776 <sup>***</sup> (0.017)	-0.000 (0.002)	0.93
Uniform (U) (All Rounds)	-52.4 (49.9)	0.540 <sup>***</sup> (0.032)		0.54
First 12 Rounds	33.4 (82.90)	0.550 <sup>***</sup> (0.032)	-0.197 <sup>**</sup> (0.093)	0.55
	-67.8 (66.5)	0.513 <sup>***</sup> (0.047)		0.45
	-70.7 (95.5)	0.513 <sup>***</sup> (0.048)	0.006 (0.136)	0.45
Rounds 13-25	-41.9 (56.8)	0.587 <sup>***</sup> (0.040)		0.72
	33.2 (72.3)	0.591 <sup>***</sup> (0.040)	-0.265 <sup>*</sup> (0.161)	0.73

\* = Significantly different from zero at the 10% level

\*\* = Significantly different from zero at the 5% level

\*\*\* = Significantly different from zero at the 1% level

For indicative bidding, the highest rejected stage-one bids are nowhere close to achieving statistical significance. This is not unexpected. Focusing on the bid functions absent the highest rejected stage-one bid, estimated intercept values are negative in all cases and significantly less than zero for all auctions and for auctions 13-25. The implication is that bidders are discounting their bids relative to their values somewhat *less* at higher values. This may reflect an attempt by bidders with low total values to recover some portion of their stage-one entry fee.<sup>25</sup>

For the uniform-price case the highest rejected stage-one bid constitutes the common entry fee stage-two bidders must pay. Without the highest rejected stage-one bid as a right hand side variable, the intercepts of the estimated bid functions for the uniform-price case are not significantly different from zero. Further, for both specifications the slopes of the bid functions are flatter, much flatter than with indicative bidding. However, with the highest rejected stage-one bid variable the coefficient is negative

<sup>25</sup>The minimum bid requirement precludes bidding below 1,000 in an effort to avoid losses. For indicative bidding there were 11 cases with  $S_i < 1,200$  in stage two (out of a total of 296 stage-two bids) so that  $S_i - c < 1,000$ . In 10 of these cases subjects bid the minimum of 1,000, thereby suffering losses.

in sign and statistically significant for the pooled data across all treatments, and for auctions 13-25 where the coefficient value is negative and is statistically significant at the 10% level in a two-tailed test.<sup>26</sup> The negative sign for the entry fee variable has minimal impact on average second-stage bids since with its inclusion the intercept of the bid function becomes positive and the coefficient for total value increases a bit.<sup>27</sup>

The theory implies that the highest rejected stage-one bid will be positive in sign at lower stage-one values ( $S_i \leq 2,000$ ) and can possibly (but not necessarily) be negative in sign for  $S_i > 2,000$ . But this is true only in equilibrium, and we already know from the efficiency data that bidders are relatively far from equilibrium. As such we look elsewhere for a possible explanation for the statistical significance of the entry fee variable in the uniform-price case. One possibility is suggested by results from coordination games (games with multiple Pareto ranked equilibria) in which the imposition of an entry fee affects which equilibria subjects will coordinate on, with higher entry fees reliably inducing Pareto improving (i.e., more profitable) equilibria in the coordination game itself (Van Huyck, Battalio, and Beil, 1993; Cachon and Camerer, 1996). Cachon and Camerer’s experiment indicates quite clearly that subjects in these coordination games take account of the entry fees, employing a “loss-avoidance” selection principle in making their choices in the coordination game, and expecting others to do the same. Applying this principle here, higher entry fees in the uniform-price auctions induce bidders to lower their bids in the second stage in an effort to avoid losses, with some confidence that their second-stage rival will do the same, as the entry fees are common and publicly known.

**Observation 5** *Second stage bid functions are monotonically increasing in total value as they should be, with slopes well below 1.0 in both treatments. The highest rejected stage-one bid is statistically significant in the uniform-price procedures suggesting that bidders are attempting to avoid possible losses associated with higher entry fees in this case.*

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<sup>26</sup>In the uniform-price case, the slope of the equilibrium bid function should be close to 1/2 (see (5) in the appendix). It is straightforward to verify that from Table 4, the slope is indeed not different from 1/2 for the pooled data and for auctions 1-12 at the 5% significance level (and for auctions 13-25 at the 10% significance level).

<sup>27</sup>For the uniform-price auctions there were a total of 30 cases with  $S_i \leq 1,500$ , in 22 cases of which subjects would have had to bid below 1,000 to avoid losses. In 3 out of these 22 cases subjects bid the minimum of 1,000. Thus, in this case at least, a majority of subjects appear to have ignored sunk costs in stage-two bidding, as indeed they should.

TABLE 5: BIDDER PROFITS AND SELLER REVENUE  
(STANDARD ERRORS IN PARENTHESIS)

	AVERAGE PROFIT OF STAGE-ONE BIDDERS		AVERAGE WINNER'S PROFIT		ENTRY FEES		SELLER REVENUE	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Indicative (All Rounds)	-6.65** (2.61)	NA	160.10** (15.69)	NA	200	200	2107.18 (30.59)	NA
First 12	-7.59** (3.62)		154.47** (21.72)				2121.11 (44.75)	
Rounds 13-25	-5.76 (3.78)		165.43** (22.70)				2093.99 (42.09)	
Uniform (All Rounds)	-47.44** (10.37)	29.8	164.16** (46.93)	375.0	426.7 (26.8)	196.5	2212.52 (63.55)	1992.0
First 12	-77.26** (14.73)		80.85 (69.88)		528.5 (36.4)		2392.73 (86.56)	
Rounds 13-25	-2.32 (9.72)		290.24** (46.08)		280.8 (24.2)		1939.76 (71.98)	

NA – not applicable.

\*\* Significantly different from zero at the 5% level.

Table 5 reports average profits of stage-one bidders, average profit for the winning stage-two bidder, entry fees, and seller revenue under the two treatments. The first thing to notice is that average bidder profits are negative in both treatments and negative for all time periods. Furthermore, although these losses are minimal for the indicative bidding treatment, they are quite large early on for the uniform-price treatment averaging -77.3 for auctions 1-12. The large losses in this last case are directly accounted for by the much higher than predicted entry fees which averaged 528.5 for auctions 1-12 as opposed to the 196.5 predicted. In both treatments, and in all time periods, the winner of the second-stage auction earned positive average profits after accounting for entry fees. However, these profits were not enough to compensate for the fact that under the auction rules *both* stage-two bidders pay entry fees. These negative average profits account for the bankruptcies reported under both treatments. Furthermore, the much larger early losses under the uniform-price treatment account for the heavy early attrition under that treatment.

A closer look at the profit data shows that for the indicative bidding treatments, when bidders with the highest or second-highest stage-one values got into stage two they earned positive average profits, regardless of whether they won or not. In contrast, bidders getting into stage two with lower stage-one values earned relatively large negative profits, in large measure because they were much less likely to win the stage-two auction, while still paying the 200 entry fee. (Bidders with the highest or second-highest stage-one value won in stage two about twice as often as those with lower stage-one values: 61.5% of the time versus 32.5%.)

For the uniform-price auctions the high entry fees in auctions 1-12 make for negative average profits even for those bidders with the highest or second-highest stage-one values who made it into stage two. Not surprisingly, things were even worse, on average, for those who got into stage-two with lower stage-one values. For auctions 13-25, with their lower stage-one entry fees, bidders with the highest or second-highest stage-one value who made it into stage two earned positive average profits regardless of whether they won the stage-two auction or not. Bidders with lower stage-one values continued to earn negative average profits, largely as a result of winning much less often after making it into stage two (30.3% of the time versus 65.9%).

That these inefficient entries largely account for the negative average profits under both auction institutions is confirmed through simulations in which we use the estimated stage-two bid function to determine average profits conditional on only the highest and second-highest stage-one value holders entering stage two. With the exception of auctions 1-12 in the uniform-price auctions, the simulations show modest positive profits averaged over all bidders due to the extremely high entry fees. While these average per capita profits are still well below the level predicted in the uniform-price case (where we have a prediction against which to evaluate outcomes; 11.8 per capita from the simulation in periods 13-25 versus 29.8 predicted), the simulations indicate that the primary responsibility for the losses is a result of the inefficiencies reported in the stage-one bidding.<sup>28</sup>

**Observation 6** *Losses under indicative bidding and in the uniform-price auctions (after the initial adjustment process) result primarily from the inefficient entry into stage two. Bidders in stage two with the highest or second-highest stage-one values win more often than other bidders in stage two and earn positive average profits overall, after accounting for entry fees. In contrast, bidders with lower stage-one values do not win nearly as often in stage two but must still pay the positive entry fees, resulting in negative average profits.*

Seller revenue is higher under the uniform-price auctions compared to indicative bidding in periods 1-12, primarily as a result of the much higher than predicted entry fees. But they are lower than indicative bidding in periods 13-25, even though entry fees are higher than predicted, and higher, on average than under indicative bidding.<sup>29</sup> This results from the less aggressive bidding in stage two in

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<sup>28</sup>We need to simulate the stage-two values for those stage-one bidders who would have gotten into stage two but did not. Results for the uniform-price case are robust to including or excluding the entry fee variable in the estimated bid function.

<sup>29</sup>Recall that entry costs are fully subsidized under the uniform-price procedure in our experimental design, so the total entry cost incurred is subtracted from the seller's revenue.

the uniform-price auctions (see Table 5, especially the differences in the estimated coefficient for total value). Finally, average revenue is quite close to predicted revenue for the uniform-price auctions in periods 13-25, even though entry fees are higher than predicted.

#### 4.4 Indicative Bidding versus a Discriminatory-Price Procedure

In evaluating the efficiency of the indicative bidding we have focused on a uniform-price first-stage auction procedure, in large measure because we believe that its common entry fee for bidding in stage two is likely to make it the most acceptable of possible alternatives to indicative bidding. Be that as it may, one might argue that given the difficulties a number of subjects have with respect to second-price and uniform-price auction procedures that we have stacked the deck in favor of showing good, or even superior, efficiency properties under indicative bidding.<sup>30</sup> As such we conducted two additional sessions employing the same experimental procedures and subject population as reported on above with the exception that the first-stage auction employed a pay-what-you-bid (discriminatory-price) auction procedure.<sup>31</sup> As with the uniform-price procedure, the discriminatory-price auction generates, in theory at least, fully efficient entry into stage two. Furthermore, after taking into account possible risk aversion on the part of bidders, in single-stage private value auctions, pay-what-you-bid auctions have been shown to conform reasonably closely to Nash equilibrium bidding theory.

Table 6 reports entry efficiency from the two discriminatory sessions along with a summary of outcomes from the indicative bidding sessions. It's clear that all three measures of entry efficiency are higher, on average, under indicative bidding than with discriminatory first-stage bidding.

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<sup>30</sup>See Kagel (1995) for a review of such difficulties in private value auction experiments.

<sup>31</sup>Both sessions recruited 16 subjects and had 25 auctions with two markets operating simultaneously. There were three bankruptcies in each session, comparable to the results reported for the IB sessions. See Ye (2007) for an analysis of the properties of the discriminatory-price two-stage procedure.

Table 6: Entry Efficiency: IB versus Discriminatory-Price Procedure

Session	Percentage of $X_{1,6}$ & $X_{2,6}$			Percentage of $X_{1,6}$ or $X_{2,6}$			Ratio Efficiency		
	Enter Stage Two			Enter Stage Two					
	1-12	13-25	All	1-12	13-25	All	1-12	13-25	All
Discriminatory									
D1	8.3 (5.8)	7.7 (5.3)	8.0 (3.9)	66.7 (9.8)	80.8 (7.9)	74.0 (6.3)	79.4 (3.4)	83.1 (2.5)	81.3 (2.1)
D2	33.3 (9.8)	15.4 (7.2)	24.0 (6.1)	91.7 (5.8)	84.6 (7.2)	88.0 (4.6)	89.1 (2.0)	89.5 (2.0)	89.3 (1.4)
Discr Average	20.8 (5.9)	11.5 (4.5)	16.0 (3.7)	79.2 (5.9)	82.7 (5.3)	81.0 (3.9)	84.3 (2.1)	86.3 (1.6)	85.3 (1.3)
IB Average	29.2 (5.4)	31.6 (5.4)	30.4 (3.8)	86.1 (4.1)	94.7 (2.6)	90.5 (2.4)	88.6 (1.4)	91.6 (1.0)	90.2 (0.8)

**Observation 7** *A robustness check shows that entry is as efficient, if not more efficient, under indicative bidding compared to a discriminatory first-stage auction procedure.*

## 4.5 Discussion and Conclusions

We experimentally investigate indicative bidding, a widely used mechanism for selling high-priced items in field settings. Theoretical analysis of indicative bidding shows that efficient entry cannot be guaranteed, as there is no symmetric increasing equilibrium in which more qualified bidders necessarily submit higher bids in the indicative bidding stage. However, the question that the theory begs is how well does indicative bidding actually perform in practice given the availability of alternative mechanisms that, in theory at least, can achieve 100% efficiency? That is, have field practices settled on an inefficient mechanism with reduced seller revenue in the face of readily available alternative selling mechanisms that promise superior efficiency as well as increased revenue for the seller? Our experiment yields an unambiguous answer to this question: In practice indicative bidding yields efficiency levels that are the same, if not better, than two alternative bidding practices that promise both higher efficiency and higher seller revenue. This occurs both early on when bidders are adjusting to the more complicated alternative bidding procedures as well as after bidders have gained experience, with a number of bidders having gone bankrupt and are no longer bidding.

One natural question that our results beg is that of external validity. That is, are we right to try and extrapolate results from laboratory experiments with undergraduates to field settings with highly motivated business people in high stakes environments who have at their disposal expert advisers? The

answer is no, we cannot extrapolate directly. Rather, what our results do is to show that indicative bidding has the same level of efficiency as opposed to at least two plausible alternatives to it. This puts the burden of proof on those who believe that indicative bidding promotes inefficient entry in field settings to provide evidence to support their case and/or to suggest problems with the internal validity of our results.

In extrapolating results from the laboratory to field settings one also needs to look at the mechanism underlying the laboratory results. Is the same, or a similar, mechanism likely to be present in field settings as well? We believe the answer is yes in this case. The mechanism promoting the relatively high efficiency levels observed in our experiment is the one in which bidders with higher stage-one values can anticipate positive average profits, while those with lower stage-one values earned negative average profits. A similar mechanism for promoting respectable entry efficiency can be expected to operate in field settings since, in addition to possibly earning negative profits as a result of unwise entry, bidders in field settings have alternative income earning opportunities to pursue with their limited resources which should discourage bidders with low stage-one values from bidding very aggressively.

Our results highlight a potential trade-off between different types of mechanisms: Those with relatively simple rules but less clear, or weaker, equilibrium outcomes, and those with relatively more complicated rules but clear underlying equilibrium outcomes. While game theorists are properly concerned with the existence and uniqueness of an equilibrium, selecting a mechanism based only on its underlying equilibrium properties can be misleading. For a mechanism to work in practice, simplicity and transparency are as important, if not more so, than equilibrium considerations.

What precisely underlies the greater simplicity of indicative bidding over the alternative bidding mechanisms employed here? To answer this we first introduce a definition for complexity against which to compare the different mechanisms. A quick look at the literature shows a large number of definitions for measuring complexity.<sup>32</sup> One definition that seems appropriate to our situation is: “The difficulty of making correct predictions about its environment (measured by its error rate) for an agent using the best model it can infer from the information available to it given its computational resources” (Edmonds, 1997). Applying this definition to the present study, the endogenously determined entry fee under the uniform-price procedure would qualify as more complex, due to its inherent variability, compared to the exogenously determined fixed fee under indicative bidding.<sup>33</sup> Under the discriminatory-price procedure

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<sup>32</sup>Just search under “measuring complexity” on the Internet.

<sup>33</sup>This greater complexity still remains even if a second-price auction is employed in stage two, as entry fees are still endogenously determined, hence inherently variable.

bidders determine the entry fee with their first-stage bids. However, these first-stage bids are binding, which requires a rather complicated backward induction calculation to determine the equilibrium first-stage bid. In contrast, under indicative bidding the entry fee/cost for each entrant is fixed and known with certainty. Although stage-one bids remain non-trivial under indicative bidding, as the existence of a sensible equilibrium is an open question, we believe that the fact that the stage-one bidding does not directly impact the entry fee simplifies the stage-one bidding strategy.

Our analysis has focused on efficiency comparison between indicative bidding and the uniform-price procedure, as this would appear to be the relevant and fully efficient auction benchmark for situations where indicative bidding is applied in practice. As noted, a uniform-price procedure would seem to be “fairer” than a discriminatory-price procedure, with the fairness issue becoming more important the higher and more disparate the first-stage bids. But this begs the question as to why we do not compare indicative bidding to a benchmark based on a single-stage procedure where bidders simultaneously and independently make their entry decisions.<sup>34</sup> The answer is that for the type of assets in question, this is not a practical procedure as there may be constraints on how many bidders can be admitted to the due diligence process. For example, in the electrical generating asset sales, the seller may need to consider how many data rooms can be provided (a capacity constraint), or there may be security concerns about information leakage if too many or the wrong bidders are admitted to the due diligence process. The two-stage auction procedure (indicative bidding or the alternative mechanisms considered in our paper) solves this problem by selecting a targeted number of bidders for the final auction.

Our results provide a possible explanation for why indicative bidding is so commonly used, despite the lack of a clear equilibrium solution. The mechanism design problem in auction settings with costly entry is not easy, as the optimal auction in general should not include all the bidders in the final sale. But how to pre-select the most qualified bidders is not trivial. Theoretical analysis reveals that non-binding bidding in the first stage results in the non-existence of a symmetric increasing equilibrium. Therefore, to restore incentive compatibility, it is necessary to make the first-stage bids binding. Bidding for entry rights with entry fee payments is a natural way to solve the problem. No doubt the analysis here has not exhausted the possible alternative bid procedures promising full efficiency, but we conjecture that the rules of the two alternative institutions employed here are among the simplest within the class of selling mechanisms that require binding first-round bidding. Thus, if these alternative bid procedures

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<sup>34</sup>More specifically, in the single-stage benchmark the bidders make entry decisions simultaneously and independently based on their stage-one signals. Once they decide to enter, each bidder incurs an entry cost  $c$  and learns her stage-two signal. Finally, the entrant bidders bid for the asset in a standard sealed bid auction.

fail to exhibit advantages over indicative bidding, there is good reason to appreciate indicative bidding. It is certainly not perfect, but the present results indicate that it works well enough relative to viable and somewhat more complicated alternatives.

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## Appendix: Equilibrium Analysis under The Uniform-Price Procedure

In this appendix we will derive the equilibrium bidding function in the uniform-price procedure. In what follows we assume that  $X$ 's are drawn from  $[L, H]$  uniformly, and  $Y$  takes value either  $H$  (with probability  $p$ ) or  $L$  (with probability  $1 - p$ ). We define  $D = H - L$  to be the spread between the first-stage value upper bound and lower bound. In our experimental design,  $L = 500, H = 1,500$  (the value spread  $D = 1,000$ ), and  $p = 1/2$ .

### First-stage Bidding Functions

Given our specific information structure, for a marginal entrant (say bidder  $i$ ) to win the object, it must be the case that she will obtain a “high” signal ( $Y_i = H$ ) and all the other entrant bidders obtain “low” signals; otherwise bidder  $i$  (the lowest bidder to enter stage two) can never become the highest final bidder to win the object. Thus bidder  $i$ 's expected gain from winning the second-stage auction conditional on being the marginal entrant is

$$\begin{aligned}
 E\Omega(x_i) &= E\left(\mathbf{1}_{S_i > \hat{S}_{1,-i}}(S_i - \hat{S}_{1,-i}) | X_{n,-i} = x_i\right) \\
 &= E\left(\mathbf{1}_{x_i + Y_i > \hat{S}_{1,-i}}(x_i + Y_i - \hat{S}_{1,-i}) | X_{n,-i} = x_i\right) \\
 &= E\left(\mathbf{1}_{x_i + H > X_{1,-i} + L}(x_i + H - (X_{1,-i} + L)) | X_{n,-i} = x_i\right) \mathbf{1}_{Y_i = H, Y_j = L, j \in E \setminus \{i\}} \\
 &= p(1 - p)^{n-1}(x_i + D - E(X_{1,-i} | X_{n,-i} = x_i))
 \end{aligned}$$

where  $E$  denotes the index set of final bidders.

Since

$$\begin{aligned}
 E(X_{1,-i} | X_{n,-i} = x_i) &= \int_{x_i}^H x d\left(\frac{x - x_i}{H - x_i}\right)^{n-1} \\
 &= \frac{n-1}{n}H + \frac{x_i}{n},
 \end{aligned}$$

we have

$$E\Omega(x_i) = p(1 - p)^{n-1} \left[ \frac{D}{n} + \frac{n-1}{n}(x_i - L) \right]$$

Since the entry subsidy  $K = c$  and  $\hat{c} = 0$ , the first-stage equilibrium bid function under a uniform-price scheme is given by

$$\varphi^U(x) = E\Omega(x) - \hat{c} = p(1 - p)^{n-1} \left[ \frac{D}{n} + \frac{n-1}{n}(x - L) \right]$$

In our experimental design,  $n = 2$ , therefore,

$$\varphi^U(x) = E\Omega(x) = p(1-p) \left[ \frac{D}{2} + \frac{1}{2}(x_i - L) \right]$$

With  $p = 1/2, D = 1,000, L = 500$ , we have

$$\varphi^U(x) = 125 + (x - 500)/8 = 62.5 + x/8 \quad (4)$$

The equilibrium range of first-stage bids under a uniform-price auction is thus  $[125, 250]$ .

### Expected Entry Fee Payment

Now we consider the expected total entry fee payment in equilibrium. According to the uniform-price scheme, each entrant bidder pays entry fee  $\varphi^U(X_{n+1;N})$  in equilibrium. The total expected entry fee payment is thus given by

$$\begin{aligned} EM &= n \int_L^H E\Omega(x) dF_{X_{n+1;N}}(x) \\ &= np(1-p)^{n-1} \int_L^H \left( \frac{D}{n} + \frac{n-1}{n}(x-L) \right) \frac{N!}{(N-n-1)!n!} \left( \frac{x-L}{D} \right)^{N-n-1} \left( 1 - \frac{x-L}{D} \right)^n dx \\ &= Dnp(1-p)^{n-1} \int_0^1 \left( \frac{1}{n} + \frac{n-1}{n}z \right) \frac{N!}{(N-n-1)!n!} z^{N-n-1} (1-z)^n dz \quad (\text{where } z = \frac{x-L}{D}) \\ &= D \frac{N!}{(N-n-1)!(n-1)!} p(1-p)^{n-1} \int_0^1 \left[ \frac{1}{n}(1-z)^n z^{N-n-1} + \frac{n-1}{n}(1-z)^n z^{N-n} \right] dz \\ &= D \frac{N!}{(N-n-1)!(n-1)!} p(1-p)^{n-1} \left[ \frac{1}{n} B(N-n, n+1) + \frac{n-1}{n} B(N-n+1, n+1) \right] \\ &= D \frac{N!}{(N-n-1)!(n-1)!} p(1-p)^{n-1} \left[ \frac{1}{n} \frac{(N-n-1)!n!}{N!} + \frac{n-1}{n} \frac{(N-n)!n!}{(N+1)!} \right] \\ &= Dp(1-p)^{n-1} \left[ 1 + \frac{(n-1)(N-n)}{(N+1)} \right] \end{aligned}$$

Plugging the parameters in our experimental design,  $N = 6, n = 2, D = 1,000, p = .5$ , the expected total entry fee payment is approximately 393. The expected entry fee for each entrant is thus  $393/2 = 196.5$ .

By the entry fee equivalence (Ye, 2007), the total expected entry fee payment is the same under a discriminatory-price scheme.

### Second-stage Bidding Function

Now we consider the second-stage bidding (under a first-price auction).

Suppose the first-stage signal possessed by the highest rejected bidder is  $x^*$  (this can be inferred from the highest rejected bid, assuming that a strictly increasing bid function is employed in the first stage). Write  $\hat{S}^* = X + Y$  where  $X \geq x^*$ . Then  $\hat{S}_i^*$  is the total value for the entrant bidder  $i$  assuming efficient entry.

If  $Y = L$ ,  $\hat{S}^* \in [x^* + L, H + L]$ ; if  $Y = H$ ,  $\hat{S}^* \in [x^* + H, 2H]$ .

For  $s \in [x^* + L, H + L]$ ,

$$\begin{aligned} \Pr\{\hat{S}^* \leq s\} &= \Pr(Y = L)\Pr(X \leq s - L | X \geq x^*) \\ &= (1 - p)[F(s - L) - F(x^*)]/(1 - F(x^*)) \\ &= (1 - p)(s - L - x^*)/(H - x^*) \end{aligned}$$

For  $s \in [x^* + H, 2H]$ ,

$$\begin{aligned} \Pr\{\hat{S}^* \leq s\} &= \Pr(Y = L) + \Pr(Y = H)\Pr(X \leq s - H | X \geq x^*) \\ &= (1 - p) + p[F(s - H) - F(x^*)]/(1 - F(x^*)) \\ &= (1 - p) + p(s - H - x^*)/(H - x^*) \end{aligned}$$

Therefore,

$$F_{\hat{S}^*}(s) = \begin{cases} (1 - p)\frac{s - L - x^*}{H - x^*} & : \text{ if } s \in [x^* + L, H + L] \\ 1 - p & : \text{ if } s \in (L + H, x^* + H) \\ (1 - p) + p\frac{s - H - x^*}{H - x^*} & : \text{ if } s \in [x^* + H, 2H] \end{cases}$$

It can be easily verified that under a first-price auction in symmetric IPV setting, the equilibrium bid function  $B(s) = E(\hat{S}_{1;-i}^* | \hat{S}_{1;-i}^* \leq s)$ , that is, the equilibrium bid for bidder  $i$  equals the expected value of the object for her conditional on her being the highest bidder.

1. If  $s \in [x^* + L, H + L]$ ,

$$\begin{aligned} B(s) &= \frac{\int_{x^*+L}^s y dF_{\hat{S}_{1;-i}^*}(y)}{F_{\hat{S}_{1;-i}^*}(s)} \\ &= \frac{\int_{x^*+L}^s y d\left[\frac{(1-p)(y-L-x^*)}{H-x^*}\right]^{n-1}}{\left[(1-p)\frac{s-L-x^*}{H-x^*}\right]^{n-1}} \\ &= s - \frac{\int_{x^*+L}^s (y-L-x^*)^{n-1} dy}{(s-L-x^*)^{n-1}} \\ &= s - \frac{s-L-x^*}{n} \\ &= \frac{n-1}{n}s + \frac{x^*+L}{n} \end{aligned}$$

2. If  $s \in [x^* + H, 2H]$ ,

$$\begin{aligned}
B(s) &= \frac{\int_{x^*+L}^{H+L} y d \left[ (1-p) \frac{y-L-x^*}{H-x^*} \right]^{n-1} + \int_{x^*+H}^s y d \left[ (1-p) + p \frac{y-H-x^*}{H-x^*} \right]^{n-1}}{\left[ (1-p) + p \frac{s-H-x^*}{H-x^*} \right]^{n-1}} \\
&= \frac{\int_{x^*+L}^{H+L} y d [(1-p)(y-L-x^*)]^{n-1} + \int_{x^*+H}^s y d [(1-p)(H-x^*) + p(y-H-x^*)]^{n-1}}{[(1-p)(H-x^*) + p(s-H-x^*)]^{n-1}} \\
&= s - \frac{1}{np} [(1-p)(H-x^*) + p(s-H-x^*)] - \frac{[(1-p)(H-x^*)]^{n-1} \left[ \frac{2p-1}{np} (H-x^*) + x^* - L \right]}{[(1-p)(H-x^*) + p(s-H-x^*)]^{n-1}}
\end{aligned}$$

When  $p = 1/2$ , the equilibrium bid function is as follows:

$$B(s) = \begin{cases} \frac{n-1}{n}s + \frac{x^*+L}{n} & : \text{ if } s \in [x^* + L, H + L] \\ \frac{n-1}{n}s + \frac{2}{n}x^* - \left( \frac{H-x^*}{s-2x^*} \right)^{n-1} (x^* - L) & : \text{ if } s \in [x^* + H, 2H] \end{cases}$$

It can be verified that  $B(s)$  is increasing in  $s$  and  $n$ . In our experimental design,  $n = 2$ , and the equilibrium bid function is given below:

$$B(s) = \begin{cases} \frac{1}{2}s + \frac{x^*+L}{2} & : \text{ if } s \in [x^* + L, H + L] \\ \frac{1}{2}s + x^* - \frac{(H-x^*)(x^*-L)}{s-2x^*} & : \text{ if } s \in [x^* + H, 2H] \end{cases} \quad (5)$$

## Equilibrium Payoffs

It can be shown that revenue equivalence holds in this two-stage auction environment (see Ye for a proof). Let  $\hat{S}_{1;n}$  and  $\hat{S}_{2;n}$  be the highest and second highest total value among all entrant bidders. Then by the revenue equivalence, the expected revenue to the seller is given by

$$ER = E\hat{S}_{2;n} + EM - nc$$

That is, the seller's expected revenue is the sum of the expected winning bid amount and the expected total entry fee payment subtracted by the total entry cost. Note that the seller bears the total entry cost indirectly in equilibrium.

The total expected profit to the bidders is given by

$$E\Pi = (E\hat{S}_{1;n} - E\hat{S}_{2;n}) - EM$$

The term in bracket is the usual private information rent to the winner. So the bidders' total expected profit is their private information rent subtracted by the expected total entry fee payment. Note that the collection of endogenous entry fee serves as the rent extraction scheme in our design.

The bidders' total profit can be decomposed into the loser's loss and the winner's gain. The expected loss for each second-stage bidder who loses the auction is  $EM/n$  (the expected entry fee payment), and the expected winner's profit is  $(E\hat{S}_{1;n} - E\hat{S}_{2;n}) - EM/n$ .

Since  $E\hat{S}_{1;n}$  and  $E\hat{S}_{2;n}$  cannot be obtained analytically in our case, we use Monte Carlo simulations to compute the seller's expected revenue and the bidders' expected payoffs.

The procedure for obtaining  $E\hat{S}_{1;n}$  and  $E\hat{S}_{2;n}$  is as follows:

1. Draw  $N$  first-stage values from Uniform  $[L, H]$  and sort them according to descending order:  $X_1, X_2, \dots, X_N$ .
2. Draw  $n$  second-stage values (either  $L$  or  $H$ , with equal probability). Let the realization be  $Y_1, Y_2, \dots, Y_n$ .
3. Form the total values  $X_1 + Y_1, X_2 + Y_2, \dots, X_n + Y_n$ . Denote the highest total value as  $S_1$  and the second highest value as  $S_2$ .
4. Repeat 1-3 for, say, 100,000 times. Take the average of  $S_1$ 's and  $S_2$ 's. These give the estimates for  $E\hat{S}_{1;n}$  and  $E\hat{S}_{2;n}$ , respectively.

Following the above procedure, our experimental parameters lead to the following simulation results:

$$E\hat{S}_{1;n} = 2,570, E\hat{S}_{2;n} = 1,999$$

Based on this, and  $EM = 393$ , we obtain the following equilibrium payoffs:

The second-stage winner's expected profit:  $(E\hat{S}_{1;n} - E\hat{S}_{2;n}) - EM/n = 375$ .

The second-stage loser's expected loss (the averaged entry fee):  $EM/n = 196.5$ .

Expected profit to all bidders:  $E\Pi = (E\hat{S}_{1;n} - E\hat{S}_{2;n}) - EM = 178.5$ .

Expected profit to each bidder:  $E\Pi/6 = 178.5/6 = 29.8$ .

The seller's expected revenue:  $ER = E\hat{S}_{2;n} + EM - nc = 1,992$  (based on  $c = 200$  and  $n = 2$ ).