Physics 2300: Problem Set #10

These problems are due before class on Wednesday, November 7th. Remember to write a concise problem statement, and to give just a word or two as to where your equations are coming from.

1. Morin 7.12 (Falling into the sun)
2. Morin 7.13 (Intersecting orbits)
3. Morin 7.20 (Ellipse axes) Finish out by using equation (7.26) to express the semimajor axis in terms of E (and L in principle).
4. A satellite is initially in circular orbit at radius R about a planet of mass M. We wish to transfer it to a new circular orbit at radius $2R$, using an intermediate transfer orbit which is an ellipse with perigee R and apogee $2R$. We make a brief rocket burn in the longitudinal direction to switch from circle to ellipse at $r = R$, and another burn at $r = 2R$ to exit the ellipse and start the larger circle. What are the two velocity changes required? What is the eccentricity of the transfer ellipse? How long does the transfer take?
5. In the context of good old $F = -\alpha/r^2$ gravity, a satellite of mass m does circular orbits at radius R about a planet.
   (a) In terms of the inputs α, m and R, what is the orbital speed v_0?
   (b) The satellite then executes a brief rocket burn in the radial direction, resulting in a radial component of velocity $v_r = fv_0$ where f is a dimensionless fraction. Sketch the before and after situations on a graph of the effective potential, indicating which quantities change and which stay the same. (Assume f is small enough that the new orbit is an ellipse.)
   (c) Find the turning points of the elliptical orbit, i.e. the maximum and mininum radii.
   (d) What is the eccentricity?
   (e) What is the ratio of the new period to the old period?
6. Morin 8.26 (Swinging stick)
7. Morin 8.38 (Coin on inclined plane)
8. (BONUS) 8.34 (A triangle, the slick way) and the closely related 8.35 (Fractal triangle)