Physics 2301: Problem Set #1

These problems are due by classtime Wednesday January 16, by uploading a PDF to the Carmen dropbox.

1. There is a worksheet to do.

2. Morin 8.20 (The superball) and 8.21 (Many bounces). Assume the ball is hollow, with $I_0 = (2/3)mR^2$ instead of the solid sphere in the book.

3. Morin 8.75 (Repetitive bouncing) Again take $I_0 = (2/3)mR^2$.

4. Morin 9.37 (Sphere and points)

5. Morin 9.38 (Striking a triangle)

6. Morin 9.42 (Pivot and string)

7. Given the matrices $M_1 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, $M_2 = \begin{pmatrix} 4 & -3 \\ 1 & 2 \end{pmatrix}$ and the vector $\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

   (a) Compute $\vec{v}_1 = M_1 \cdot \vec{v}$ and $\vec{v}_2 = M_2 \cdot \vec{v}$.

   (b) Compute $M_1 \cdot \vec{v}_2$ and $M_2 \cdot \vec{v}_1$. Are these the same vectors?

   (c) Compute the matrix products $M_{12} \equiv M_1 \cdot M_2$ and $M_{21} \equiv M_2 \cdot M_1$. Are these the same matrices?

   (d) Compute $M_{12} \cdot \vec{v}$ and $M_{21} \cdot \vec{v}$, and compare with the vectors of part (b).

8. A rotation $R(\theta)$ by θ transforms the basis vector $\hat{x}$ into $\hat{x}\cos\theta + \hat{y}\sin\theta$ and the basis vector $\hat{y}$ into $\hat{x}(-\sin\theta) + \hat{y}\cos\theta$.

   (a) Express $R(\theta)$ in matrix form.

   (b) If we first rotate by θ and then again by ϕ, the result should be the same as just doing one rotation by the sum $\theta + \phi$. Show that the proposition $R(\phi) \cdot R(\theta) = R(\phi + \theta)$ allows one to derive the trig IDs which express $\sin(\phi + \theta)$ and $\cos(\phi + \theta)$ in terms of sin and cos of θ and ϕ.

9. (BONUS) Morin 9.31 (Rolling wheel)