Physics 2301: Problem Set #1

These problems are due by the end of Wednesday January 15, by uploading a PDF to the Carmen dropbox.

0. There will be weekly Essential Skills drills.

1. Morin 8.20 (The superball) and 8.21 (Many bounces). Assume the ball is hollow, with $I_0 = (2/3)mR^2$ instead of the solid sphere in the book.

2. Morin 8.75 (Repetitive bouncing) Again take $I_0 = (2/3)mR^2$.

3. A certain rigid body rotates about a fixed origin. Relative to that origin, a point A at $\vec{r}_A = a\hat{z}$ has instantaneous velocity $\vec{v}_A = 2v_0\hat{x} - v_0\hat{y}$. At the same instant, point B at $\vec{r}_B = a\hat{x}$ has instantaneous velocity $\vec{v}_B = v_0\hat{y} - 2v_0\hat{z}$.

   (a) What is the spin $\vec{\omega}$ at that moment?

   (b) What is the velocity of point C at location $\vec{r}_C = a\hat{x} + a\hat{z}$?

4. In the context of a rigid body with a stationary pivot point, mass m is located at position $\ell\hat{e}_1$ where the unit vector $\hat{e}_1 \equiv \cos\theta\hat{x} + \sin\theta\hat{y}$.

   (a) If the object spins with $\vec{\omega} = \omega_x\hat{x}$, what is the the resulting angular momentum $\vec{L}$? (Just compute $\vec{v} = \vec{\omega} \times \vec{r}$ and then $\vec{r} \times \vec{p}$.)

   (b) If instead the object spins with $\vec{\omega} = \omega_y\hat{y}$, what is the the resulting angular momentum $\vec{L}$?

   (c) What is $\vec{L}$ when $\vec{\omega} = \omega_1\hat{e}_1$?

   (d) What is $\vec{L}$ when $\vec{\omega} = \omega_2\hat{e}_2$ where $\hat{e}_2 \equiv -\sin\theta\hat{x} + \cos\theta\hat{y}$?

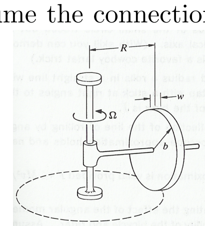
   (e) Redo the first part by finding ω_1 and ω_2 in the expression $\vec{\omega} = \omega_x\hat{x} = \omega_1\hat{e}_1 + \omega_2\hat{e}_2$ and using the linearity property that $\vec{L}(\omega_1\hat{e}_1 + \omega_2\hat{e}_2) = \vec{L}(\omega_1\hat{e}_1) + \vec{L}(\omega_2\hat{e}_2)$.

5. Morin 9.37 (Sphere and points)

6. Morin 9.38 (Striking a triangle)

7. Morin 9.42 (Pivot and string)

8. A millstone of radius b and mass m rolls without slipping in a circle of radius R at angular rate Ω. There is gravity g as usual, and we neglect both the wheel width $w \ll b$ and any friction at the ground. What is the spin vector $\vec{\omega}$? What is the normal force between the wheel and the ground? (Despite the strong extended coupling drawn in the figure, assume the connection between the two axles is a ball joint, i.e. a pivot which applies force at one point.)



9. A rotation $R(\theta)$ by θ transforms the basis vector $\hat{x}$ into $\hat{x}\cos\theta + \hat{y}\sin\theta$ and the basis vector $\hat{y}$ into $\hat{x}(-\sin\theta) + \hat{y}\cos\theta$.

   (a) Express $R(\theta)$ in matrix form.

   (b) If we first rotate by θ and then again by ϕ, the result should be the same as just doing one rotation by the sum $\theta + \phi$. Show that the proposition $R(\phi) \cdot R(\theta) = R(\phi + \theta)$ allows one to derive the trig IDs which express $\sin(\phi + \theta)$ and $\cos(\phi + \theta)$ in terms of sin and cos of θ and ϕ.

   (c) A "Pythagorean triple" is a set of three integers such that $p^2 + q^2 = r^2$, e.g. $3^2 + 4^2 = 5^2$. Corresponding to such a triple, there is an angle θ such that $\sin\theta$ and $\cos\theta$ are both rational numbers, e.g. $\frac{3}{5}$ and $\frac{4}{5}$. Starting from $\theta = \arccos\frac{4}{5}$ compute the matrices $R(\theta)^2$ and $R(\theta)^3$ and thereby discover two new Pythagorean triples.

10. (BONUS) Morin 9.31 (Rolling wheel)