Physics 2301: Problem Set #2

These problems are due by by upload to Carmen by the end of the day on Wednesday January 22.

1. Shankar BTM 8.2.1, 8.2.2 and 8.2.3 p. 213-4.

2. Morin 8.76 (Bouncing under a table) Do this one with a general value of $\beta \equiv \frac{I_0}{mR^2}$, and then evaluate at $\beta = (2/3)$ in particular.

3. Consider two masses moving in one dimension. An m_1 moves at speed v_1 and collides elastically with an m_2 moving at v_2. For convenience, define $\beta = \frac{m_2}{m_1}$.

   (a) Use energy and momentum conservation to find v_{1f} and v_{2f} after the collision, and express the result in matrix form, $\begin{pmatrix} v_{1f} \\ v_{2f} \end{pmatrix} = M \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

   (b) Suppose that the m_1 can bounce off a wall, a process described by the wall matrix W which operates as $W \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -v_1 \\ v_2 \end{pmatrix}$, and suppose we begin with the m_1 at rest with m_2 moving to the left, i.e. $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_0 \end{pmatrix} \equiv \vec{S}_0$. Collisions will occur, producing a sequence of states $\vec{S}_1 = M \cdot \vec{S}_0$, $\vec{S}_2 = W \cdot M \cdot \vec{S}_0$, $\vec{S}_3 = M \cdot W \cdot M \cdot \vec{S}_0$ until we reach a final state with $v_2 > v_1 > 0$. For the specific case $\beta = 2$, how many collisions occur and what are the final state velocities? Do this part "by hand".

   (c) How many collisions occur for the case $\beta = 100$? Do this part in Mathematica, using the function MatrixPower for convenience.

4. Consider a rigid trianglular frame with three point masses, an m located at $\vec{r}_A = a\hat{x} + 2a\hat{y} - a\hat{z}$, a $2m$ located at $\vec{r}_B = a\hat{x} - a\hat{y} - a\hat{z}$ and a $3m$ at $\vec{r}_C = -a\hat{x} + a\hat{z}$ relative to the CM. Express the moment of inertia matrix for each of the three masses, and add them to get the matrix for the system. Then ask Mathematica to find the Eigenvectors and Eigenvalues, i.e. the principal moments and the associated moments of inertia.

5. Morin 9.23 (Rolling coin) The book solution uses the CM as the origin; let's agree to instead use the center of the circle of radius R.

6. Morin 9.39 (Striking another triangle)

7. Morin 9.41 (Circling stick again)
8. Morin 9.43 (Rotating sheet) By "look like" we mean "comment on the aspect ratio a/b." Is it for example large, small, or unity?
9. Morin 9.45 (Stick on a ring)
10. (BONUS) Morin 8.77 (Bouncing under a table again) Feel free to let Mathematica do the matrix multiplies.