Physics 2301: Problem Set #4

These problems are due by the end of the day on Wednesday Feb 5, by upload of a PDF to Carmen.

1. Here are three matrices.

$$A \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad B \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad C \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

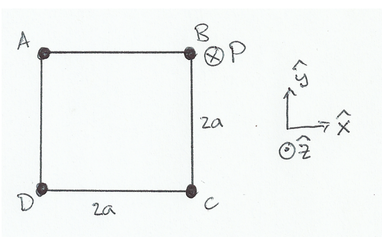
   (a) Which one(s) are symmetric?

   (b) Which one(s) are orthogonal?

   (c) Which one(s) have $\vec{v}_1 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ as an eigenvector? If so, what is the eigenvalue?

   (d) Which one(s) have $\vec{v}_2 \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ as an eigenvector? If so, what is the eigenvalue?

   (e) Which one(s) are *idempotent* (mathspeak for $M^n = M \cdot M \cdots M = M$ for some integer $n > 1$)? Give the value of n if true.

2. 

Four identical masses (each size m) are attached to the corners of a square frame of dimensions $2a \times 2a$. The frame is of negligible mass. Initially the system rotates in its plane with $\vec{\omega} = \omega\hat{z}$, with the CM at rest. We briefly fire a rocket thrusters in to the page at corner B, endowing the system with momentum $-P\hat{z}$. **As it happens, $P = 2m\omega a$.**

   (a) Find the new angular momentum $\vec{L}$ (relative to an origin at the CM) and spin $\vec{\omega}$, expressed in the $\hat{x}$, $\hat{y}$, $\hat{z}$ basis.

   (b) Find the velocities of each of the four masses just after the thrust, and verify that the total momentum is $-P\hat{z}$.

   (c) Find the kinetic energies of the masses, and verify that the sum matches the expression $\frac{1}{2}M_{\text{tot}}{v_{\text{CM}}}^2 + \frac{1}{2}\vec{L}_0 \cdot \vec{\omega}$.

   (d) How long will it take before the square again lies in an $\hat{x}$- $\hat{y}$ plane (granted, at a new z).

3. Morin 9.52 (Horizontal ω) p. 426

4. Morin 10.19 (Circular pendulum) p. 483. Also add part (c): find ω when we replace the point mass m by a uniform stick of length ℓ. Work in the rotating frame, and do an integral over all the dm's to find the torque from the centrifugal force.

5. Morin 10.20 (Spinning bucket) p. 483
6. Morin 10.22 (Bug on a hoop) p. 483
7. Morin 10.23 (Maximum normal force) p. 484
8. Morin 10.25 (Free particle motion) p. 484. Your job is to verify that the solution works by taking derivatives "by hand" and plugging in. Then go to Mathematica, DSolve for the motion with initial conditions of your choice and ParametricPlot. See how exotic a trajectory you can make.
9. (BONUS) Morin 10.26 (Coin on a turntable) p. 484