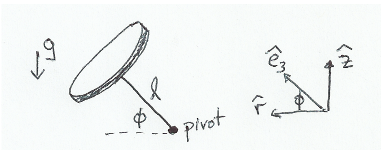
Physics 2301: Problem Set #5

These problems are due Wednesday Feb 12 by upload of a PDF to Carmen.

0. Essential Skills

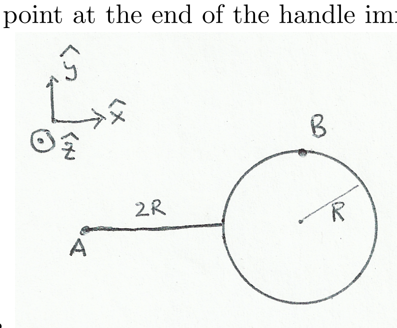
1. 

Consider a gyroscope comprising a uniform disk (of mass m and radius R) and a (massless) handle of length ℓ. There is gravity g in what we'll call the $-\hat{z}$ direction, and the handle makes an angle ϕ relative to the horizontal as it attaches to a frictionless pivot. We are given the spin rate ω_s, and as you understand, the full spin vector is $\vec{\omega} = \omega_s \hat{e}_3 + \Omega\hat{z}$ where Ω is a quantity we seek to determine.

   (a) For the special case $\phi = 0$, express $\vec{L}$ and find the precession rate Ω.

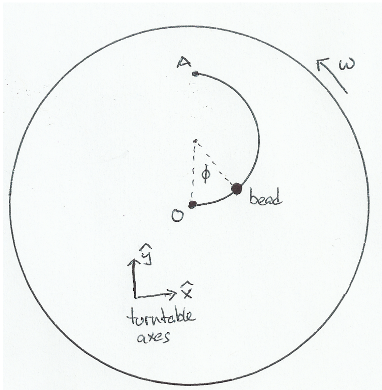
   (b) For generic ϕ, work in the "gyroscopic approximation" where we assume $\omega_s \gg \Omega$ (and that we can therefore ignore the contribution of $\Omega\hat{z}$ to $\vec{L}$, again find (approximately) the precession rate Ω for general angle ϕ.

   (c) Finally, make no approximation as you include the $\Omega\hat{z}$ contribution to $\vec{L}$, and find the quadratic equation which Ω must satisfy. (No need to solve the quadratic.)

2. Consider a crude model of a tennis racquet, consisting of a hoop of mass m and radius R attached to a handle of length $2R$ which also has mass m. The racquet is initially floating motionless in outer space when it is struck by a downward blow at point B, giving the system momentum P in to the page. Find the resulting velocity ($\vec{v}_A$) of the point at the end of the handle immediately after the blow. What is


the kinetic energy of the racquet?

3. Consider a "half-pipe" (a frictionless rail in the shape of a semi-circle of radius R) on a turntable rotating counterclockwise at ω. We work in the rotating frame, and represent Shaun White by a bead of mass m which starts essentially at rest at the origin ($\phi = 0$) and slides along the rail to point A



($\phi = \pi$).

(a) Parameterizing $\vec{r}(\phi) = R\sin\phi\hat{x} + R(1 - \cos\phi)\hat{y}$, compute the work done by the centrifugal force from the origin to generic point ϕ, and verify that it equals the change in centrifugal potential, $\frac{1}{2}m\omega^2|\vec{r}|^2$.

(b) Find the speed of the bead (relative to the turntable) and the normal force on the bead as a function of ϕ.

4. Morin 10.18 (Oscillations across the equator) p. 482

5. Morin 10.29 (Bead on a hoop) p. 485

6. Morin 10.31 (Roche limit) and 10.32 (Roche limit with rotation) p. 486

7. (BONUS) Morin 10.12 (Shape of the Earth) p. 479. Make a Mathematica notebook following the book's discussion of this 4-star problem, doing the needed numerical integrals with NIntegrate.