# Physics 2301: Problem Set #7

These problems are due by the end of Wednesday March 4 by upload to Carmen.

0. Essential Skills

1. Given a real number $x$ near $x = 0$, define $y$ to be the solution to the transcendental equation $y = e^{-xy}$. Find the Taylor series for $y(x)$ about $x = 0$, up to order $x^3$. That is, take the "ansatz" $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \cdots$, plug it in to the equation and demand that terms match order by order in powers of $x$.

2. A mass at the end of a spring oscillates with motion $x(t) = x_0 \cos(\omega t)$ where $\omega x_0 \ll c$. A clock attached to the moving mass runs slow relative to the ground frame. What is the reading on the moving clock after one cycle? Write an integral, then expand in the small parameter $\epsilon \equiv (\omega x_0/c)^2$ and integrate term by term as you give the first two terms in $T_{\text{clock}} = \frac{2\pi}{\omega}(1 + a_1\epsilon + a_2\epsilon^2 + \cdots)$.

3. For the function $f(x) \equiv (1 + \exp x)^{-1}$, use your knowledge of the series for $e^x$ and for $1/(1 + x)$ to find the series for $f(x)$ up to $\mathcal{O}(x^3)$ (and ignoring $\mathcal{O}(x^4)$). Then create a small Mathematica notebook which plots $f(x)$ together with that third order approximant. Finally use Mathematica to plot the approximation for increasingly large orders and determine the radius of convergence to three digits.

4. Morin 11.39 (Twice Simultaneous) p. 558—use the Mathematica notebook to draw spacetime diagrams for both frames.

5. Morin 12.22 (CM frame) p. 616

6. Morin 12.25 (A decay) p. 616

7. Morin 12.27 (Perpendicular photon) p 616

8. Morin 12.36 (Pushing on a mass) p. 618

9. A photon of energy $E$ collides with a proton (of mass $M$) at rest in our lab frame, producing a final state with a proton and a pion (of mass $m_\pi$), i.e. $\gamma + p \to \pi^0 + p$. What is the smallest $E$ which makes this possible?

10. (BONUS) Morin 11.58 (Time dilation and $Lv/c^2$) p. 562.