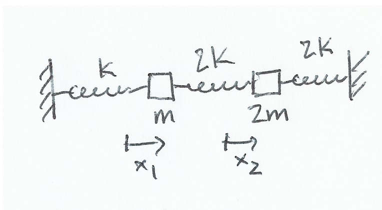
Physics 2301: Problem Set #10

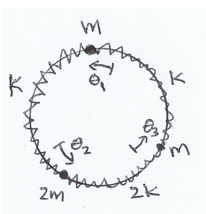
These problems are due at the end of the day on Wednesday, April 15

1. Express the following complex numbers in $x + \imath y$ format. If there is an ambiguity, in each case give the most conventional response, e.g. $\sqrt{4} = +2$ as opposed to -2. What are: $\sqrt{1+\imath}$, $\log(-2)$, $\cos(1 + 2\imath)$, $\cosh(1+\imath\pi)$ and $\arcsin(2)$? In each case show the steps as you compute it, i.e. don't just ask Mathematica.

2. Morin 4.33 (Beads on angled rails) p. 126.



3. Consider the spring system drawn, and suppose we start at at rest at $t = 0$ with $x_1(0) = a$ and $x_2(0) = -a$. What are $x_1(t)$ and $x_2(t)$ in that case? Repeat for the case when we instead start at the equilibrium position $x_1(0) = 0 = x_2(0)$ and with velocities $\dot{x_1}(0) = v_0$ and $\dot{x}_2(0) = -v_0$.



4. Three masses (two m's and one $2m$) are constrained to slide around a frictionless hoop of radius R. There are springs (two of constant k and one of constant $2k$) coiled around the hoop which connect the pairs of masses. Using the θ_j variables drawn, write the three couple equations of motion, put them in matrix format, and find the three normal modes. That is, find the most general solution.

5. Consider an RLC circuit where in some units $R = 2$, $L = 1$ and $C = \frac{1}{4}$ and the applied voltage is $V(t) = \cos 2t$. Find the charge on the capacitor $Q(t)$ if we start with $Q = 0$ and $\dot{Q} = 0$ at $t = 0$.

6. (Rephrased for clarity:) An oscillator with natural frequency ω_0 also experiences a friction force $-2\gamma m\dot{x}$ and a driving force $F_0 \cos(\omega t)$. At late times $x(t)$ approaches the steady state solution $x(t) = A\cos(\omega t) + B\sin(\omega t)$ where the coefficients A and B are given in Morin's equation 4.31 (page 112) among other places. Note his ω_d is our ω and his ω is our ω_0. Over the course of one cycle ($T = 2\pi/\omega$) how much work is done by friction? For simplicity, I suggest writing your intermediate expressions terms of A and B and waiting until the end to write A and B in terms of the inputs. Likewise, find the work done by the driving force and verify that it balances the work from friction.

7. Morin 4.34 (Coupled and damped) p. 126. That is, consider our "base case" system (discussed in Morin as the example in figure 4.9) and add friction. Specifically find the normal modes and amend Morin's formula (4.55) which give general solution for $x_1(t)$ and $x_2(t)$ in the absence of friction.

8. Morin 4.35 (Coupled and driven) p. 126. Again the system in question the example in figure 4.9.

9. (BONUS) Animate the beads on a rail (4.33) problem in Mathematica. For numerical parameters, please choose an angle θ such that the two eigenfrequencies are *commensurate* (meaning that ω_1/ω_2 is a nice rational number like 2, 3 or 3/2. That way whatever the motion is will be exactly periodic for any initial conditions.