Physics 2301: Problem Set #11

These problems are due at the end of the day on Thursday April 23

1. In some units, a coordinate $x(t)$ obeys the driven damped equation

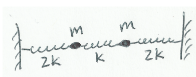
$$\ddot{x}(t) + 4\dot{x}(t) + 3x(t) = \cos(2t)$$

   (a) If we choose to express the long time solution in real format as $x(t) = A\cos(2t) + B\sin(2t)$, what are the constants A and B?

   (b) If we choose to express the same solution as the real part of $z(t) = z_0 \exp(2it)$, how should we choose the complex number z_0?

   (c) What are the two "transient" solutions, i.e. solutions to the homogeneous equation?

   (d) If we start the system at rest with $x = 0$ at $t = 0$, what is $x(t)$?

2. 

Consider a system of 2 equal masses m and 3 springs attached to two walls, as drawn. Note that the springs are not of equal strength.

   (a) Find the frequencies of the normal modes.

   (b) Suppose now that in addition to the springs, the right mass is driven by an external force $F(t) = F_0 \cos(\sqrt{\frac{k}{m}}t)$ (with no driving force on the left mass). If the system starts at $t = 0$ with both masses at rest at their equilibrium positions, find the motion. That is, find $x_1(t)$ and $x_2(t)$.

3. Consider the space of functions on the interval $x \in (0, 1)$.

   (a) Choose as a basis a set of polynomials of degree n, starting with $P_0(x) = 1$. If $P_1(x) = \alpha + \beta x$, how should we choose α and β so that $P_1(x)$ is both properly normalized and orthogonal (perpendicular) to $P_0(x)$?

   (b) Similarly, find $P_2(x)$, a quadratic function which is normalized and orthogonal to both $P_0(x)$ and $P_1(x)$.

   (c) What are b_0, b_1 and b_2 in the expansion $\exp x = \sum_n b_n P_n(x)$?

4. Shankar, problem 9.7.3 pg. 283

5. Consider a function $f(x)$ defined on the domain $-\pi < x < \pi$ by the rules $f(x) = -x$ for $x < 0$ and $f(x) = 0$ for $x \geq 0$. Find the coeffients a_n in the series $f(x) = \sum_n a_n \phi_n(x)$, where the basis functions are $\phi_n(x) = \exp(inx)/\sqrt{2\pi}$. At the end, please write $f(x)$ in terms of $\sin(nx)$ and $\cos(nx)$ as well.

6. Similarly, consider a function $f(x)$ defined on the domain $-\pi < x < \pi$ by the rules $f(x) = 0$ for $x < 0$ and $f(x) = \sin x$ for $x \geq 0$. As before use the exponential basis to compute Fourier coefficients, and also express in terms of sines and cosines. (BTW between these two problems you will have done Destin's homework.)

7. Morin 6.31 (Explicit minimization) p. 252.

8. Morin 6.42 (Spring on a spoke) p. 254.

9. (BONUS) Use Mathematica to animate a Lagrangian problem. Anything from the chapter 6 exercises is good (other than 6.45, which will be solved in an example notebook), or make up your own combination of springs, pendulums, rolling hoops and sliding blocks. No need to solve anything "by hand." Your job is to draw pictures, choose coordinates and write the Lagrangian. A sample notebook will show you how Mathematica can do the rest.