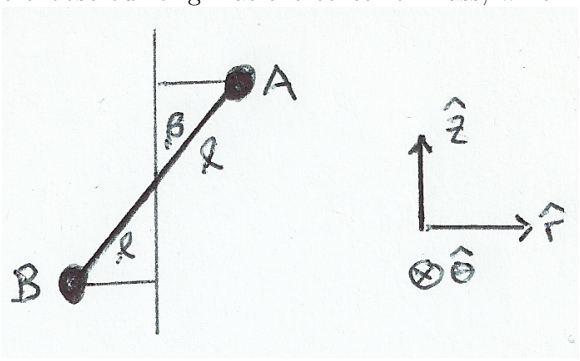
Physics 2300: Worksheet #1: The Baton

Consider the simplest rigid body, consisting of two point objects each of mass m, connected by a *massless* stick of length 2ℓ. Suppose that the motion of this baton is to spin about an axis we'll choose to call $\hat{z}$, i.e. $\vec{\omega} = \omega\hat{z}$. We choose our origin at the center of mass, which is stationary. The angle between the AB axis and $\hat{z}$ is β.



1. Express the following vectors in terms of the parameters and basis vectors drawn:
   - $\vec{r}_A$, the position of mass A.
   - $\vec{v}_A = \vec{\omega} \times \vec{r}_A$, its velocity.
   - $\vec{L}_A$, the angular momentum of A.
   - Likewise, find the angular momentum for B, $\vec{L}_B$.
2. Taking $\frac{d}{dt}$, what is the external torque on the baton system?
3. Suppose this external torque comes from two strings, applying forces $-T\hat{r}$ at A and $T\hat{r}$ at B. Express T in terms of inputs.
4. The *principal directions* of this baton are its symmetry axis (AB) and any vector perpendicular to it. Specifically we define $\hat{e}_3 \equiv \vec{r}_A/\ell$, $\hat{e}_2 \equiv -\hat{\theta}$ and $\hat{e}_1 \equiv \hat{e}_2 \times \hat{e}_3$. Express $\hat{e}_3$ and $\hat{e}_1$ in terms of inputs.
5. What are the moments of inertia in the directions $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$?
6. Express $\vec{\omega}$ in the $\hat{e}_k$ basis. Likewise, express the system's angular momentum $\vec{L}$, and verify it agrees with the previous expression.
7. Suppose the very thin stick of length 2ℓ has mass $M > 0$. Express $\vec{L}$ and find the tension T.