Analytic and Numeric Methods of Physics

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Electromagnetic Field Theory I

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Grading: 30% HW, 30% Midterm, 40% Final

Midterm: ~ Oct. 24 or 27

Final: Dec. 11

Class Notes: posted online

Syllabus: online

Textbooks: J. D. Jackson "Classical Electrodynamics"
A. Zangwill "Modern Electrodynamics"
Math methods:

G. B. Arfen et al. "Mathematical Methods for Physicists"
Special Theory of Relativity

Lorentz Transformations

Need to find relation between \((t, x, y, z)\) and \((t', x', y', z')\) in different frames.

Einstein's Postulates:

1. The laws of nature are independent of inertial frame we are in.
2. The speed of light is a finite constant and is independent of inertial frame.

Assuming that space-time is homogeneous and isotropic one can use Einstein's postulates to derive Lorentz transformations:

\[
\begin{align*}
    x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} , \\
    y' &= y , \\
    z' &= z , \\
    t' &= \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{align*}
\]
The velocity transformation:

\[
\frac{dx'}{dt'} = \frac{dx - v dt}{dt - \frac{v}{c^2} dx} = \frac{u'}{u} = \frac{dx}{dt} = \frac{u - v}{1 - uv/c^2}
\]

\[
\Rightarrow \quad u' = \frac{u - v}{1 - uv/c^2} \quad \text{or} \quad u = \frac{u' + v}{1 + uv/c^2}
\]

if \( u = c \) \( \Rightarrow \) \( u' = c \) \( \Rightarrow \) speed of light is \( c \) in all frames.

**Def.** 4-dimensional coordinates: \( x^\mu, \mu = 0, 1, 2, 3 \)

\[x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z\] (Greek indices \( 0, 1, 3 \))

**Def.** \( \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}} \)

\( \Rightarrow \) Lorentz transformation in matrix form is

\[
\begin{pmatrix}
  x'^0 \\
  x'^1 \\
  x'^2 \\
  x'^3
\end{pmatrix} = \begin{pmatrix}
  \gamma & -\beta \gamma & 0 & 0 \\
  -\beta \gamma & \gamma & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x^0 \\
  x^1 \\
  x^2 \\
  x^3
\end{pmatrix}
\]

\( \Rightarrow \) in more compact form: \( x'^\mu = \Lambda^{\mu}_{\nu} x^\nu \).
The inverse Lorentz transform:
\[
\begin{pmatrix}
  x_0' \\
  x_1' \\
  x_2' \\
  x_3'
\end{pmatrix} = \begin{pmatrix}
  \delta & \beta \delta & 0 & 0 \\
  \beta \delta & \delta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix}
\]

Invariant interval: flash a light at the origin(s) at \( t = t' = 0 \)

\( \Rightarrow \) light reaches point \((x,y,z)\)

after time \( t \) such that

\[ c^2 t^2 - x^2 - y^2 - z^2 = 0 \]

In moving frame \((K')\) have

\[ c^2 t'^2 - x'^2 - y'^2 - z'^2 = 0. \]

\( \Rightarrow \) as \( y' = y, z' = z \) \( \Rightarrow c^2 t^2 - x^2 = c^2 t'^2 - x'^2 \)

\( \Rightarrow \) can explicitly check that it works:

\[
c^2 t'^2 - x'^2 = \frac{1}{1 - \frac{v^2}{c^2}} \left[ (c t - \frac{v}{c} x)^2 - (x - v t)^2 \right] =
\]

\[
= \frac{1}{1 - \frac{v^2}{c^2}} \left[ c^2 t^2 \left( 1 - \frac{v^2}{c^2} \right) - x^2 \left( 1 - \frac{v^2}{c^2} \right) \right] = c^2 t^2 - x^2
\]

as expected.
Quantity \( S_{12}^2 = c^2 (t_1 - t_2)^2 - |\vec{x}_1 - \vec{x}_2|^2 \) is called the square of the interval between two events at times \( t_1 \) and \( t_2 \) at locations \( \vec{x}_1 \) and \( \vec{x}_2 \). \( S_{12}^2 \) is Lorentz invariant.

(i) \( S_{12}^2 > 0 \Rightarrow \) timelike separation \( \Rightarrow \) there exists a frame where \( \vec{x}_1' = \vec{x}_2' \Rightarrow S_{12}^2 = c^2 (t_1'^2 - t_2'^2) \Rightarrow \) the events take place at the same space point, but at different times.

(ii) \( S_{12}^2 < 0 \Rightarrow \) spacelike separation \( \Rightarrow \) there exists a frame where \( t_1'' = t_2'' \Rightarrow S_{12}^2 = - (\vec{x}_1'' - \vec{x}_2'')^2 \Rightarrow \) events take place at the same time but at different locations.

(iii) \( S_{12}^2 = 0 \Rightarrow \) lightlike separation.