Analytic and Numeric Methods of Physics

aka (Physics 7701)

Electromagnetic Field Theory I

Instructor: Yuri Kovchegov

Office: PRB M2042 , phone: 292-9628

E-mail : kovchegov.1@osu.edu

Office hours: stop by

Course website: http://www.physics.ohio-state.edu\~yuri\7701.php

grading: 30% HW, 30% Midterm, 40% Final

Grader: Cheng Li, 2041

Midterm: ~ Oct. 24 or 27

Final: Dec. 11

Class Notes: posted online

Syllabus: online

Textbooks: J. D. Jackson "Classical Electrodynamics"

A. Zangwill "Modern Electrodynamics"

L. D. Landau, E. M. Lifshitz "The Classical Theory of Fields" vol. 2

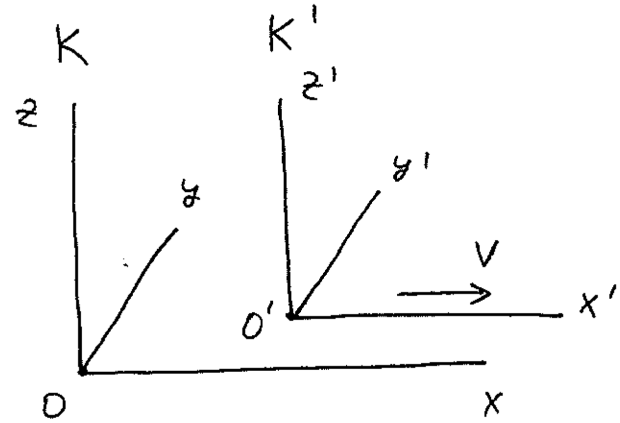
Math methods :

G. B. Arfken et al, "Mathematical Methods for Physicists"

Special Theory of Relativity

Lorentz Transformations

Need to find relation between (t, x, y, z) and (t', x', y', z') in different frames.



Einstein's Postulates:

1. The laws of Nature are independent of inertial frame we are in.

2. The speed of light is a finite constant and is independent of inertial frame.

Assuming that space-time is homogeneous and isotropic one can use Einstein's postulates to derive Lorentz transformations:

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

The velocity transformation:

$$u' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx} = \Big| u = \frac{dx}{dt} = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Rightarrow \boxed{u' = \frac{u-v}{1-\frac{uv}{c^2}}} \quad \text{OR} \quad \boxed{u = \frac{u'+v}{1+\frac{u'v}{c^2}}}$$

if $u = c \Rightarrow u' = c \Rightarrow$ speed of light is c in all frames.

Def. 4-dimensional coordinates: x^μ, $\mu = 0,1,2,3$

$$x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z$$

(Greek indices $= 0,1,2,3$)

Def. $\beta \equiv \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$.

$\Rightarrow$ Lorentz transformation in matrix form is

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{=\Lambda^\mu{}_\nu} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

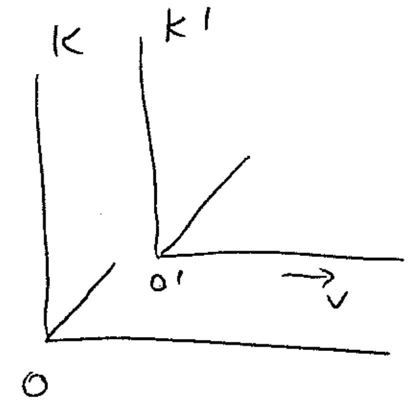
$\Rightarrow$ in more compact form: $x'^\mu = \Lambda^\mu{}_\nu x^\nu$.

The <u>inverse</u> Lorentz transform:

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

Invariant <u>interval</u>: flash a light at the origin(s) at $t = t' = 0$

$\Rightarrow$ light reaches point (x, y, z) after time t such that



$$c^2 t^2 - x^2 - y^2 - z^2 = 0$$

In moving frame (K') have

$$c^2 t'^2 - x'^2 - y'^2 - z'^2 = 0.$$

$\Rightarrow$ as $y' = y$, $z' = z$ $\Rightarrow$ $c^2 t^2 - x^2 = c^2 t'^2 - x'^2$

$\Rightarrow$ can explicitly check that it works:

$$c^2 t'^2 - x'^2 = \frac{1}{1 - \frac{v^2}{c^2}} \left[\left(ct - \frac{v}{c} x \right)^2 - (x - vt)^2 \right] =$$

$$= \frac{1}{1 - \frac{v^2}{c^2}} \left[c^2 t^2 \left(1 - \frac{v^2}{c^2} \right) - x^2 \left(1 - \frac{v^2}{c^2} \right) \right] = c^2 t^2 - x^2$$

as expected.

Quantity $s_{12}^2 = c^2 (t_1 - t_2)^2 - |\vec{x}_1 - \vec{x}_2|^2$

is called the square of the interval between 2 events

at times t_1 & t_2 & locations $\vec{x}_1$ & $\vec{x}_2$. s_{12}^2 is Lorentz-invariant.

(i) $s_{12}^2 > 0$ => timelike separation => there exists a frame where $\vec{x}_1' = \vec{x}_2'$ => $s_{12}^2 = c^2(t_1'^2 - t_2'^2)$

=> the events take place at the same space point, but at diff. times

(ii) $s_{12}^2 < 0$ => spacelike separation =>

there exists a frame where $t_1'' = t_2''$ =>

=> $s_{12}^2 = -(\vec{x}_1'' - \vec{x}_2'')^2$ => events take place at the same time but at different locations

(iii) $s_{12}^2 = 0$ => lightlike separation