$$\Rightarrow \vec{E} = \left(\hat{\varepsilon}_1 E_1 + \hat{\varepsilon}_2 E_2\right) e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

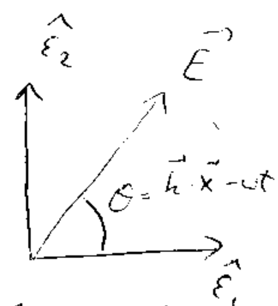
E_1, E_2 ~ are coefficients, generally complex.

Def. if E_1 & E_2 have the same phases the wave is linearly polarized in the direction defined by angle $\theta = \tan^{-1}\left(\frac{E_2}{E_1}\right)$, with amplitude $E = \sqrt{|E_1|^2 + |E_2|^2}$.

Suppose phases of E_1 & E_2 differ by $90°$: $E_2 = \pm i E_1$

$$\Rightarrow \vec{E} = E_1\left(\hat{\varepsilon}_1 \pm i \hat{\varepsilon}_2\right) e^{i(\vec{k}\cdot\vec{x} - \omega t)} = \text{taking Re} =$$

$$= E_1\left(\hat{\varepsilon}_1 \cos(\vec{k}\cdot\vec{x} - \omega t) \mp \hat{\varepsilon}_2 \sin(\vec{k}\cdot\vec{x} - \omega t)\right)$$



the wave direction rotates with time t at any fixed location $\vec{x}$.

Def. $\Rightarrow$ Such wave is called circularly polarized.

$\hat{\varepsilon}_1 + i\hat{\varepsilon}_2$ ~ counterclockwise ~ left circ. polar. ~ positive helicity

$-$ clockwise ~ right — " — ~ negative helicity

Change the basis: $\hat{\varepsilon}_\pm = \frac{1}{\sqrt{2}}\left(\hat{\varepsilon}_1 \pm i \hat{\varepsilon}_2\right)$

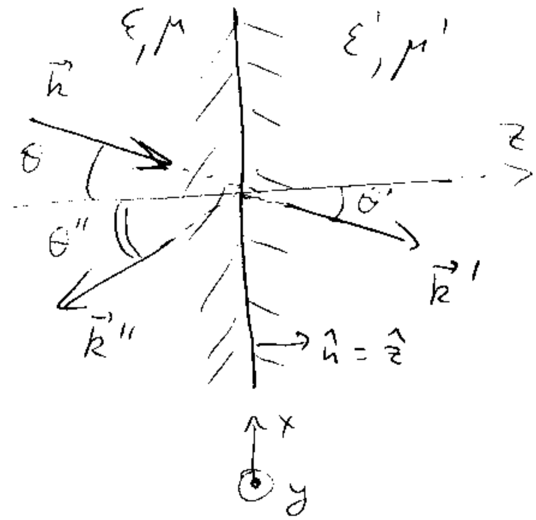
$$\Rightarrow \vec{E} = \left(E_+ \hat{\varepsilon}_+ + E_- \hat{\varepsilon}_-\right) e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

left- & right- polarized waves

If $E_+ = \pm E_-$ $\Rightarrow$ get linearly polarized wave

Any linearly polarized wave is a superposition of 2 circularly polarized waves with opposite helicities.

Reflection and Refraction



Incident wave:

$$\begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} \\ \vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E} = \sqrt{\mu\varepsilon} \frac{\vec{k}\times\vec{E}}{k} \end{cases}$$

Refracted wave:

$$\begin{cases} \vec{E}' = \vec{E}_0' e^{i(\vec{k}'\cdot\vec{x} - \omega' t)} \\ \vec{B}' = \sqrt{\mu'\varepsilon'} \frac{\vec{k}'\times\vec{E}'}{k'} \end{cases}$$

Reflected wave: $\begin{cases} \vec{E}'' = \vec{E}_0'' e^{i(\vec{k}''\cdot\vec{x} - \omega'' t)} \\ \vec{B}'' = \sqrt{\mu\varepsilon} \frac{\vec{k}''\times\vec{E}''}{k''} \end{cases}$

Match boundary conditions: $\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow D_n$ is cont.

$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow B_n$ is continuous

$\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow \vec{\nabla}\times\vec{E} - i\omega\vec{B} = 0 \Rightarrow E_t$ is continuous

as $\vec{B}$ has no δ-function singularity at $z=0$

$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t} = -i\omega\vec{D} \Rightarrow H_t$ is continuous (same reason)

to have any boundary conditions at $z=0$ need

$$\omega = \omega' = \omega'' \Rightarrow k = k'' = \sqrt{\mu\varepsilon}\,\omega, \quad k' = \sqrt{\mu'\varepsilon'}\,\omega$$

spatial phase factors should also be equal

at $z=0$: $\vec{k}\cdot\vec{x}\big|_{z=0} = \vec{k}'\cdot\vec{x}\big|_{z=0} = \vec{k}''\cdot\vec{x}\big|_{z=0}, \ \forall x, y$

$\Rightarrow$ choose $\vec{k} = (k_x, 0, k_z) \Rightarrow \vec{k}\cdot\vec{x}\big|_{z=0} = k_x \cdot X \Rightarrow$ no y-dep.

$\Rightarrow$ there should be no y-dependence in $\vec{k}'\cdot\vec{x}$ and in $\vec{k}''\cdot\vec{x}$ as well $\Rightarrow$ $k'_y = k''_y = 0$ $\Rightarrow$ all lie in the <u>same plane</u>

$$k \cdot \sin\Theta = k' \cdot \sin\Theta' = k'' \cdot \sin\Theta''$$

$\Rightarrow$ as $k = k''$ $\Rightarrow$ $\Theta = \Theta''$ $\approx$ angle of reflection is equal to angle of incidence!

as $k = \sqrt{\mu\varepsilon}\,\omega$ and $k' = \sqrt{\mu'\varepsilon'}\,\omega$ $\Rightarrow$

$\sqrt{\mu\varepsilon}\,\sin\theta = \sqrt{\mu'\varepsilon'}\,\sin\Theta'$. Remember $n = c\sqrt{\mu\varepsilon}$ (index of refraction) $\Rightarrow$ $n\sin\Theta = n'\sin\Theta'$

Snell's law!

The only thing left is to find $\vec{E}_0'$ & $\vec{E}_0''$ using b.c.'s:

D_n ($= \varepsilon E_n$) continuous $\Rightarrow$ $\hat{n}\cdot\left[\varepsilon(\vec{E}_0 + \vec{E}_0'') - \varepsilon'\vec{E}_0'\right] = 0$

B_n continuous $\Rightarrow$ $\hat{n}\cdot\left[\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right] = 0$
(and $\omega = \omega' = \omega''$)

E_t continuous $\Rightarrow$ $\hat{n}\times\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right] = 0$

H_t continuous $\Rightarrow$ $\left[\frac{1}{\mu}(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'') - \frac{1}{\mu'}(\vec{k}'\times\vec{E}_0')\right]\times\hat{n} = 0$

Consider 2 cases:

(I) $\vec{E}_0$ $\perp$ to the plane of incidence

$\vec{E}_0, \vec{E}_0', \vec{E}_0'' \parallel \hat{y}$