$\Rightarrow$ choose $\vec{k} = (k_x, 0, k_z) \Rightarrow \vec{k}\cdot\vec{x}\big|_{z=0} = k_x \cdot x \Rightarrow$ no y-dep.

$\Rightarrow$ there should be no y-dependence in $\vec{k}'\cdot\vec{x}$ and in $\vec{k}''\cdot\vec{x}$ as well $\Rightarrow k_y' = k_y'' = 0 \Rightarrow$ all lie in the <u>same plane</u>

$$k \cdot \sin\theta = k' \cdot \sin\theta' = k'' \cdot \sin\theta''$$

$\Rightarrow$ as $k = k''$ $\Rightarrow$ $\theta = \theta''$ ~ angle of reflection is equal to angle of incidence!

as $k = \sqrt{\mu\varepsilon}\,\omega$ and $k' = \sqrt{\mu'\varepsilon'}\,\omega$ $\Rightarrow$

$\sqrt{\mu\varepsilon}\,\sin\theta = \sqrt{\mu'\varepsilon'}\,\sin\theta'$. Remember $n = c\sqrt{\mu\varepsilon}$

(index of refraction) $\Rightarrow$ $n\sin\theta = n'\sin\theta'$

Snell's law!

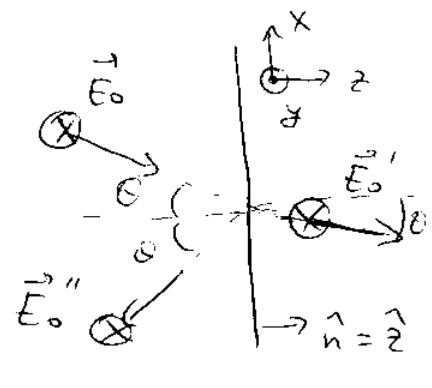
The only thing left is to find $\vec{E}_0'$ & $\vec{E}_0''$ using b.c.'s:

$$\left.\begin{aligned}
&D_n \,(= \varepsilon E_n) \text{ continuous} \Rightarrow \hat{n}\cdot\left[\varepsilon(\vec{E}_0 + \vec{E}_0'') - \varepsilon'\vec{E}_0'\right] = 0\\
&B_n \text{ continuous} \Rightarrow \hat{n}\cdot\left[\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right] = 0\\
&(\text{and } \omega = \omega' = \omega'')\\
&E_t \text{ continuous} \Rightarrow \hat{n}\times\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right] = 0\\
&H_t \text{ continuous} \Rightarrow \left[\frac{1}{\mu}(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'') - \frac{1}{\mu'}(\vec{k}'\times\vec{E}_0')\right]\times\hat{n} = 0
\end{aligned}\right\}$$

Consider 2 cases:

(I) <u>$\vec{E}_0 \perp$ to the plane of incidence</u>

$\vec{E}_0, \vec{E}_0', \vec{E}_0'' \parallel \hat{y}$



3rd & 4th equations: $(\hat{n} = \hat{z})$

$$\begin{cases} E_0 + E_0'' - E_0' = 0 \\ \frac{1}{\mu}\left(k E_0 \cos\theta - k'' E_0'' \cos\theta''\right) - \frac{1}{\mu'} k' E_0' \cos\theta' = 0 \end{cases}$$

as $k = k'' = \sqrt{\mu\varepsilon}\,\omega$, $k' = \sqrt{\mu'\varepsilon'}\,\omega$ $\Rightarrow$ and $\theta = \theta''$

$$\begin{cases} E_0 + E_0'' - E_0' = 0 \\ \sqrt{\frac{\varepsilon}{\mu}}\left(E_0 - E_0''\right)\cos\theta - \sqrt{\frac{\varepsilon'}{\mu'}}\cdot E_0' \cdot \cos\theta' = 0 \end{cases}$$

1st eqn. $0 = 0$; 2nd eqn.: $k E_0 \sin\theta + k'' E_0'' \sin\theta'' -$

$- k' E_0' \sin\theta' = 0 \Rightarrow \left(E_0 + E_0''\right)\sin\theta - \sqrt{\frac{\mu'\varepsilon'}{\mu\varepsilon}}\, E_0' \sin\theta' = 0$

as $\sqrt{\mu\varepsilon}\sin\theta = \sqrt{\mu'\varepsilon'}\sin\theta'$ (Snell's law) $\Rightarrow$ $E_0 + E_0'' - E_0' = 0$

$\Rightarrow$ duplicates the 3rd one.

Using Snell's law $(n\sin\theta = n'\sin\theta')$ to get rid of n' we write (work it out yourself):

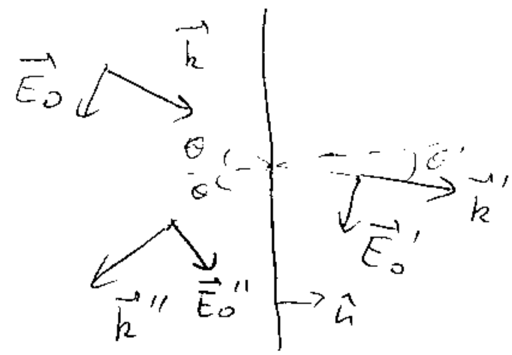
$$\boxed{\begin{aligned} \frac{E_0'}{E_0} &= \frac{2n\cos\theta}{n\cos\theta + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2\theta}} \\ \frac{E_0''}{E_0} &= \frac{n\cos\theta - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2\theta}}{n\cos\theta + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2\theta}} \end{aligned}}$$

$n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

$n' = \sqrt{\frac{\mu'\varepsilon'}{\mu_0\varepsilon_0}}$

(II) $\vec{E}_0 \parallel$ plane of incidence (xz plane)

2 independent equations (3rd & 4th):



$$\begin{cases} \left(E_0 + E_0''\right)\cos\theta - E_0'\cos\theta' = 0 \\ \sqrt{\frac{\varepsilon}{\mu}}\left(E_0 - E_0''\right) - \sqrt{\frac{\varepsilon'}{\mu'}}\, E_0' = 0 \end{cases}$$

(other two can be reduced to those)

Solve: (using Snell's Law)

$$\boxed{\frac{E_0'}{E_0} = \frac{2nn'\cos\theta}{\frac{\mu}{\mu'}n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}} \qquad \frac{E_0''}{E_0} = \frac{-\frac{\mu}{\mu'}n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}{\frac{\mu}{\mu'}n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}}$$

Normal incidence: $\theta = 0 \Rightarrow$ both (I) and (II) give

$$\frac{E_0'}{E_0} = \frac{2n}{n + \frac{\mu}{\mu'}n'}, \qquad \frac{E_0''}{E_0} = \frac{n - \frac{\mu}{\mu'}n'}{n + \frac{\mu}{\mu'}n'}$$

Polarization by reflection: put $\mu = \mu'$ for simplicity

(I): $$\frac{E_0''}{E_0} = \frac{n\cos\theta - \sqrt{n'^2 - n^2\sin^2\theta}}{n\cos\theta + \sqrt{n'^2 - n^2\sin^2\theta}}$$ ← different ⇒

(II): $$\frac{E_0''}{E_0} = \frac{-n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}{n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}$$ ↙ ⇒ reflected light is polarized!

in case I $\quad \frac{E_0''}{E_0}$ never vanishes (always <0 if $n'>n$)

in case II $\quad \frac{E_0''}{E_0} = 0$ for $\boxed{\theta_B = \tan^{-1}\left(\frac{n'}{n}\right)}$ Brewster's angle

~~Explan~~

$\Rightarrow$ reflected light is polarized.

if $\theta = \theta_B \Rightarrow$ polarization is linear, $\perp$ to the plane of incidence.

(fish in the ocean reflect light ~ squids with polarized vision can see them)

Total internal reflection: Snell's law:

$$n \sin\theta = n' \sin\theta' \leq n' \Rightarrow \theta \leq \sin^{-1}\left(\frac{n'}{n}\right) \Rightarrow$$

$\Rightarrow$ for $\boxed{\theta > \sin^{-1}\left(\frac{n'}{n}\right)}$ get total reflection

"evanescent wave"

$$k' = \sqrt{\mu'\varepsilon'}\,\omega \Rightarrow k'_x = -\sqrt{\mu'\varepsilon'}\,\omega \sin\theta' = -\sqrt{\mu_0\varepsilon_0}\,\omega\, n' \sin\theta' = -\frac{\omega}{c} n \sin\theta$$

$$'' \; n'\frac{\omega}{c} \qquad k'_y = 0$$

$$k'_z = \sqrt{k'^2 - k'^2_x} = \sqrt{n'^2 - n^2 \sin^2\theta}\;\frac{\omega}{c}$$

$\Rightarrow$ for $\theta > \sin^{-1}\left(\frac{n'}{n}\right)$: k'_z becomes imaginary

$$\Rightarrow e^{i k_z z} \sim e^{-|k_z| z} \sim \text{exponential falloff}$$

effectively the wave gets reflected from a different surface ~ violation of geom. optics, Goos-Hänchen effect