in case (I) $\frac{E_0''}{E_0}$ never vanishes (always < 0 if $n' > n$)

in case (II) $\frac{E_0''}{E_0} = 0$ for $\boxed{\theta_B = \tan^{-1}\left(\frac{n'}{n}\right)}$ Brewster's angle

~~explan~~

$\Rightarrow$ reflected light is <u>polarized</u>.

if $\theta = \theta_B \Rightarrow$ polarization is linear, $\perp$ to the plane of incidence.

(fish in the ocean reflect light ~ squids with polarized vision can see them)

<u>Total internal reflection</u>: Snell's law:

$n \sin\theta = n' \sin\theta' \leq n' \Rightarrow \theta \leq \sin^{-1}\left(\frac{n'}{n}\right) \Rightarrow$

$\Rightarrow$ for $\boxed{\theta > \sin^{-1}\left(\frac{n'}{n}\right)}$ get total reflection "evanescent wave"

$k' = \sqrt{\mu'\varepsilon'}\,\omega \Rightarrow k'_x = -\sqrt{\mu'\varepsilon'}\,\omega \sin\theta' = -\sqrt{\mu_0\varepsilon_0}\,\omega\, n' \sin\theta' =$
$\quad = -\frac{\omega}{c} n \sin\theta$

$\quad \overset{\prime\prime}{n' \frac{\omega}{c}}$

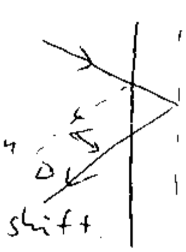
$k'_y = 0$

$k'_z = \sqrt{k'^2 - k_x'^2} = \sqrt{n'^2 - n^2 \sin^2\theta}\,\frac{\omega}{c}$

$\Rightarrow$ for $\theta > \sin^{-1}\left(\frac{n'}{n}\right)$: k'_z becomes imaginary

$\Rightarrow e^{i k_z z} \sim e^{-|k_z| z}$ ~ exponential falloff

effectively the wave gets reflected from a different surface ~ violation of geom. optics, Goos-Hänchen effect



transmission coefficient $\vec{S} = \vec{E}\times\vec{H} = Re[\vec{E}\times\vec{H}^*]$

$$T = \frac{|\vec{S}'|}{|\vec{S}|} = \frac{E_0'\cdot H_0'\cdot\frac{1}{2}}{E_0\cdot H_0\cdot\frac{1}{2}} = \frac{\mu}{\mu'}\frac{E_0'\cdot B_0'}{E_0 B_0} = \frac{\mu}{\mu'}\frac{\sqrt{\mu'\varepsilon'}}{\sqrt{\mu\varepsilon}}\left(\frac{E_0'}{E_0}\right)^2 =$$

↑ $\langle\cos^2\rangle$ ~ phase

if $\mu = \mu'$: $T = \frac{4nn'}{(n+n')^2}$; $R = \left(\frac{n-n'}{n+n'}\right)^2$.

$$= \Big|_{\theta=0}^{\text{for}} = \sqrt{\frac{\mu\varepsilon'}{\mu'\varepsilon}}\cdot\frac{4n^2}{\left(n+\frac{\mu}{\mu'}n'\right)^2}.$$

~ fraction of incident power that got through

reflection coefficient ~ fraction of inc. power reflected. $(T+R=1)$

$$R = \frac{|\vec{S}''|}{|\vec{S}|} = \frac{E_0''H_0''}{E_0 H_0} = \frac{E_0''B_0''}{E_0 B_0} = \frac{|E_0''|^2}{|E_0|^2} = \left(\frac{n-\frac{\mu}{\mu'}n'}{n+\frac{\mu}{\mu'}n'}\right)^2.$$

Electromagnetic Waves in Conductors

maxwell equations: $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ $\quad\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

$\vec{\nabla}\cdot\vec{D} = \rho \underset{\text{assume}}{=} 0$ $\quad\vec{\nabla}\cdot\vec{B} = 0$

Ohm's law: $\vec{J} = \sigma\vec{E}$; $\vec{B} = \mu\vec{H}$, $\vec{D} = \varepsilon\vec{E}$

Look for plane-wave solutions:

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}\ ; \quad \vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\Rightarrow \vec{\nabla}\times\vec{E} = i\omega\vec{B} = i\omega\mu\vec{H} \qquad \vec{\nabla}\times\vec{H} = \sigma\vec{E} - i\omega\varepsilon\vec{E}$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\nabla^2\vec{E} = i\omega\mu\,\vec{\nabla}\times\vec{H} = i\omega\mu(\sigma - i\omega\varepsilon)\vec{E}$$

$\Rightarrow$ we get $(\nabla^2 + k^2)\vec{E} = 0$ with $k^2 = \mu\varepsilon\omega^2 + i\omega\mu\sigma$

$$\Rightarrow k = \pm\sqrt{\mu\varepsilon}\,\omega\sqrt{1+\frac{i\sigma}{\varepsilon\omega}} \quad \boxed{\equiv k_1 + i k_2}$$

As $\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{k}\cdot\vec{E}_0 = 0$. ~ still transverse

Good conductor: $\frac{\sigma}{\varepsilon\omega} \gg 1$

Bad conductor: $\frac{\sigma}{\varepsilon\omega} \ll 1$

Assume that $\vec{k} \parallel \hat{z}$ and $\vec{E} \parallel \hat{x}$ (linear polarization)

$$\vec{E} = \hat{x} E_0 e^{i(kz-\omega t)} = \hat{x} E_0 e^{ik_1 z - k_2 z - i\omega t}$$

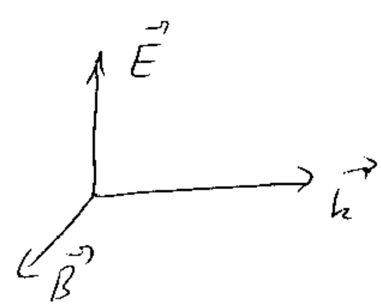
$\Rightarrow$ wavelength $\lambda = \frac{2\pi}{k_1}$

$E \sim e^{-k_2 z}$ ~ exponentially decaying

$\vec{B}$-field: $\vec{B} = -\frac{i}{\omega}\vec{\nabla}\times\vec{E} = \frac{1}{\omega}\vec{k}\times\vec{E}_0 e^{i(kz-\omega t)} =$

$= \frac{k}{\omega}\hat{y} E_0 e^{i(kz-\omega t)}$

$\Rightarrow \vec{B} \perp \vec{k}, \quad \vec{B} \perp \vec{E}$



but: as k is complex, $\vec{B}$ & $\vec{E}$ are <u>out of phase</u>.

Time-averaged Poynting vector:

$$\langle S_z \rangle = \frac{1}{2}\,\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right) = \frac{1}{2\mu}\mathrm{Re}\left(\frac{k^*}{\omega}|E_0|^2 e^{-2k_2 z}\right) =$$

$$= \frac{1}{2\mu} \frac{k_1 |E_0|^2}{\omega} e^{-2k_2 z} \propto e^{-z/\delta} \Rightarrow \delta = \frac{1}{2k_2}$$ "skin depth"

$$k_2 = \mathrm{Im}\left[\sqrt{\mu\varepsilon}\, \omega \sqrt{1 + \frac{i\sigma}{\varepsilon\omega}} \right]$$

Bad conductor. $k_2 \simeq \sqrt{\mu\varepsilon}\, \cancel{\omega} \frac{\sigma}{2\varepsilon\cancel{\omega}} = \sqrt{\frac{\mu}{\varepsilon}} \frac{\sigma}{2}$

$\Rightarrow \delta = \frac{1}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$ (if σ is small $\Rightarrow \delta$ is large)

Good conductor. $k_2 \simeq \sqrt{\mu\varepsilon}\, \omega \frac{1}{\sqrt{2}} \sqrt{\frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\sigma\mu\omega}{2}}$

$\Rightarrow \delta = \sqrt{\frac{1}{2\sigma\mu\omega}}$. (if σ is large $\Rightarrow \delta$ is small)

Frequency - dependent ε, μ, σ.

we just showed that $k = \sqrt{\mu\varepsilon}\, \omega \sqrt{1 + \frac{i\sigma}{\varepsilon\omega}} =$

$= \omega \sqrt{\mu\left(\varepsilon + \frac{i\sigma}{\omega}\right)} \Rightarrow$ if we want $k = \sqrt{\mu\varepsilon}\, \omega$

as in non-conductors, we have to, in general, assume that $\varepsilon = \varepsilon(\omega)$, $\mu = \mu(\omega)$, $\sigma = \sigma(\omega)$

and, here redefine $\varepsilon \to \varepsilon(\omega) = \varepsilon_b + \frac{i\sigma}{\omega}$ due to bound charges

"complex dielectric function" (not a constant!)

$n(\omega) = \sqrt{\frac{\mu(\omega)\varepsilon(\omega)}{\mu_0 \varepsilon_0}}$ ~ "complex index of refraction"

$$V_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n(\omega)} \text{ still.} \Rightarrow k = \frac{n(\omega)\cdot\omega}{c}$$

<u>a simple model for $\varepsilon(\omega)$</u>: consider an electron in external electric field:

$$m\ddot{\vec{x}} = \vec{F} = -K\vec{x} - m\gamma\dot{\vec{x}} - e\vec{E}$$

$\uparrow$ mass $\quad$ K: spring const $\quad$ γ: damping

$$\omega_0^2 = \frac{K}{m} \Rightarrow m\left(\ddot{\vec{x}} + \gamma\dot{\vec{x}} + \omega_0^2\vec{x}\right) = -e\vec{E}$$

if $\vec{E} = \vec{E}_0 e^{-i\omega t} \Rightarrow \vec{x} = \vec{x}_0 e^{-i\omega t} \Rightarrow$

$$\Rightarrow m\left(-\omega^2 - i\omega\gamma + \omega_0^2\right)\vec{x}_0 = -e\vec{E}_0$$

$$\Rightarrow \vec{x}_0 = \frac{e\vec{E}_0}{m(\omega^2 + i\omega\gamma - \omega_0^2)}$$

$\Rightarrow$ the amplitude of the ~~atomic~~ molecular dipole moment

$$\vec{p} = -e\vec{x}_0 = \frac{e^2\vec{E}_0}{m(\omega_0^2 - i\omega\gamma - \omega^2)}$$

$\Rightarrow$ if there are n electrons per unit volume

$$\Rightarrow \vec{P} = n\vec{p} = \frac{ne^2(\underbrace{\vec{E}_0 e^{-i\omega t}}_{=\vec{E}})}{m(\omega_0^2 - i\omega\gamma - \omega^2)} \Rightarrow$$

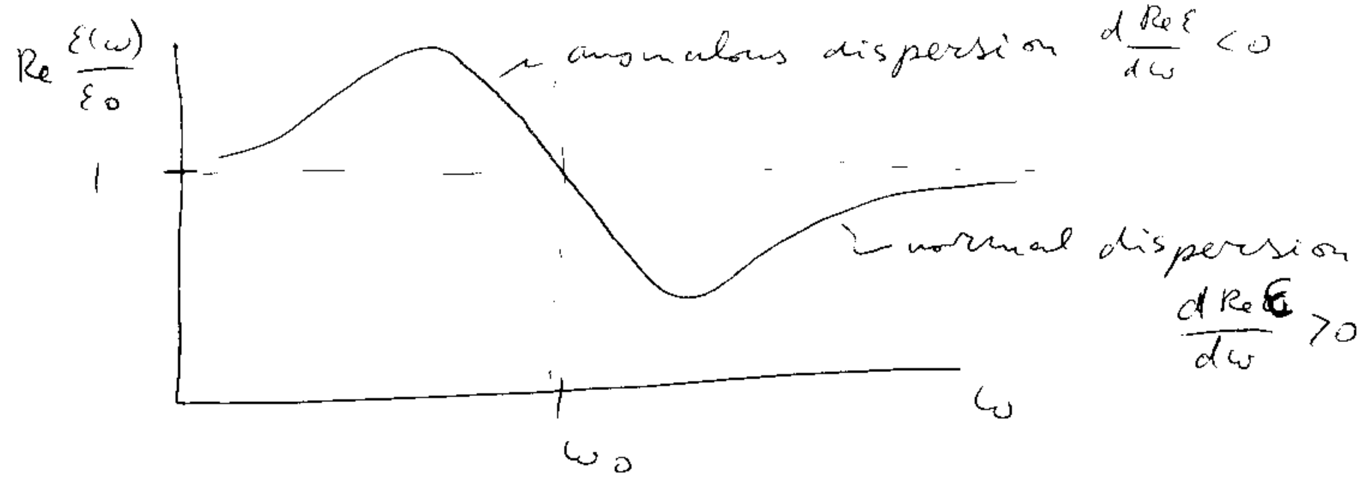
$$\Rightarrow \vec{D} = \varepsilon_0\vec{E} + \vec{P} = \left[\varepsilon_0 + \frac{ne^2}{m(\omega_0^2 - i\omega\gamma - \omega^2)}\right]\vec{E} = \varepsilon(\omega)\vec{E}$$

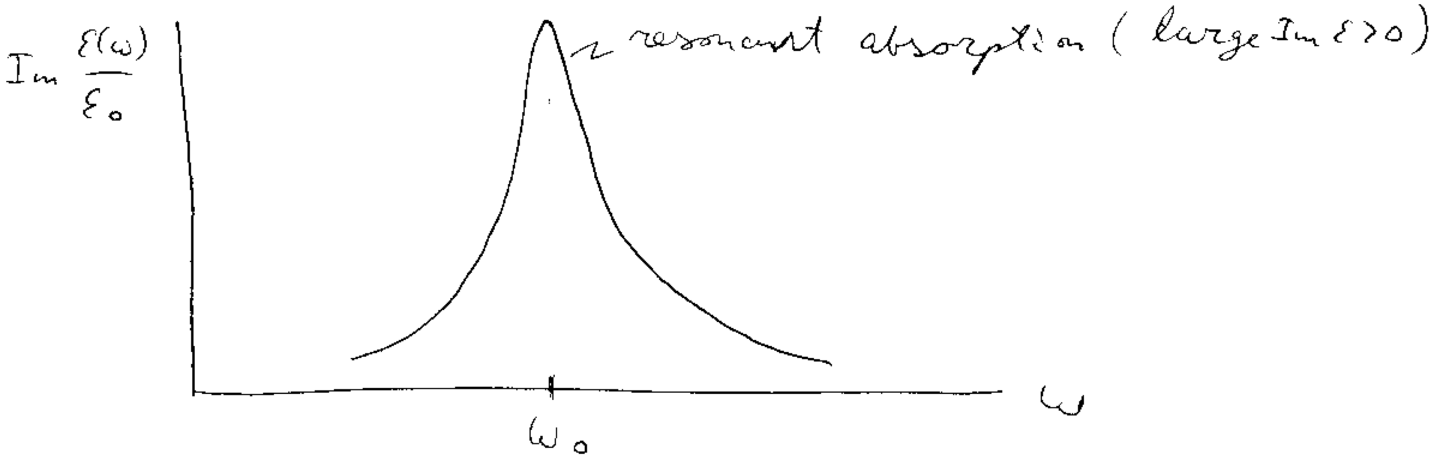
$$\Rightarrow \boxed{\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{ne^2}{m\varepsilon_0(\omega_0^2 - i\omega\gamma - \omega^2)}}$$

In general various electron states have different frequencies ω_i, γ_i & damping const's $\Rightarrow$ $\dfrac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \dfrac{ne^2}{\varepsilon_0 m} \sum_i \dfrac{f_i}{\omega_i^2 - i\omega\gamma_i - \omega^2}$.

$$\mathrm{Re}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{ne^2}{m\varepsilon_0}\,\frac{\omega_0^2 - \omega^2}{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}$$

$$\mathrm{Im}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{ne^2}{m\varepsilon_0}\,\frac{\omega\gamma}{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}$$





$k = k_1 + ik_2 \Rightarrow |\vec{E}|^2 \sim e^{-2k_2 z} \equiv e^{-\alpha z}$

$\alpha = 2k_2 = \dfrac{1}{\delta}$ = absorption coefficient

$k = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$ if $\mu(\omega) = \mu_0 \Rightarrow$

$$\Rightarrow k = \omega\sqrt{\mu_0\varepsilon_0}\,\sqrt{1 + \frac{ne^2}{m\varepsilon_0(\omega_0^2 - i\omega\gamma - \omega^2)}}$$

$\Rightarrow k_2 \neq 0$ is due to $\gamma \neq 0$ $\Rightarrow$ absorption is due to damping.
due to $\operatorname{Im}\varepsilon \neq 0$, which is

Low frequency: if ~~some~~ electrons are free

$$\Rightarrow \omega_0 = 0 \Rightarrow \frac{\varepsilon(\omega)}{\varepsilon_0} = 1 - \frac{ne^2}{m\varepsilon_0\omega(\omega + i\gamma)} =$$

$$= 1 + \frac{ne^2\, i}{m\varepsilon_0\omega(\gamma - i\omega)} = 1 + \frac{i\sigma}{\varepsilon_0\omega} \Rightarrow$$

$$\Rightarrow \sigma_{(\omega)} = \frac{ne^2}{m}\,\frac{1}{\gamma - i\omega}$$

Drude model (1900) of conductivity

High frequency: $\dfrac{\varepsilon(\omega)}{\varepsilon_0} \simeq 1 - \dfrac{ne^2}{m\varepsilon_0\omega^2} = 1 - \dfrac{\omega_p^2}{\omega^2}$

where $\omega_p^2 = \dfrac{ne^2}{m\varepsilon_0}$ is the plasma frequency

$$k = \omega\sqrt{\mu_0\varepsilon_0}\,\sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}$$

$\Rightarrow$ if $\omega < \omega_p \Rightarrow k = \dfrac{i}{c}\sqrt{\omega_p^2 - \omega^2}$ ~ imaginary $\Rightarrow$

$\Rightarrow$ waves do not propagate! ~ screening

Reflectivity $R = \left|\dfrac{1 - n(\omega)}{1 + n(\omega)}\right|^2 = \left|\dfrac{1 - \sqrt{1 - \frac{\omega_p^2}{\omega^2}}}{1 + \sqrt{1 - \frac{\omega_p^2}{\omega^2}}}\right|^2 = \begin{cases} 1, & \omega < \omega_p \\ <1, & \omega > \omega_p \end{cases}$