$$\begin{cases} \vec{\nabla} \times \vec{E} = i\omega \vec{B} & \vec{\nabla}\cdot\vec{E} = 0 \\ \vec{\nabla} \times \vec{B} = -i\mu\varepsilon\omega \vec{E} & \vec{\nabla}\cdot\vec{B} = 0 \end{cases}$$

$$\vec{E} = \vec{E}_z + \vec{E}_t \,, \qquad \vec{B} = \vec{B}_z + \vec{B}_t$$

first $\vec{E} = e^{i(kz-\omega t)}\, \vec{E}(x,y)$; $\vec{B} = e^{i(kz-\omega t)}\, \vec{B}(\vec{x},\vec{y})$

$$\Rightarrow \vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & ik \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}\left(\frac{\partial E_z}{\partial y} - ik E_y\right) +$$

$$+ \hat{y}\left(ik E_x - \frac{\partial}{\partial x} E_z\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = i\vec{k} \times \vec{E}_t +$$

$$+ \hat{z}\cdot\left(\vec{\nabla}_t \times \vec{E}_t\right) + \underbrace{\hat{x}\frac{\partial E_z}{\partial y} - \hat{y}\frac{\partial E_z}{\partial x}}_{\vec{\nabla}\times\vec{E}_z} = \underbrace{i\vec{k}\times\vec{E}_t + \vec{\nabla}_t \times \vec{E}_z}_{\text{transverse}}$$

$$+ \underbrace{\hat{z}\cdot\left(\vec{\nabla}_t \times \vec{E}_t\right)}_{\text{longitudinal}}$$

$$\Rightarrow \begin{cases} i\vec{k}\times\vec{E}_t + \vec{\nabla}_t \times \vec{E}_z = i\omega \vec{B}_t & \vec{\nabla}_t \cdot \vec{E}_t + ik E_z = 0 \\ \hat{z}\cdot\left(\vec{\nabla}_t \times \vec{E}_t\right) = i\omega \cancel{B_t} B_z & \\ i\vec{k}\times\vec{B}_t + \vec{\nabla}_t \times \vec{B}_t = -i\mu\varepsilon\omega \vec{E}_t & \vec{\nabla}_t \cdot \vec{B}_t + ik B_z = 0 \\ \hat{z}\cdot\left(\vec{\nabla}_t \times \vec{B}_t\right) = -i\mu\varepsilon\omega E_z & \end{cases}$$

$$\hat{z} \times i\omega \vec{B}_t = i\,\hat{z}\times(\vec{k}\times\vec{E}_t) + \hat{z}\times(\vec{\nabla}_t \times \vec{E}_z) = -ik\vec{E}_t + \vec{\nabla}_t E_z$$

$$\begin{cases} ik\vec{E}_t + i\omega\,\hat{z}\times\vec{B}_t = \vec{\nabla}_t E_z & \vec{\nabla}_t\cdot\vec{E}_t + ikE_z = 0 \\ \hat{z}\cdot(\vec{\nabla}_t\times\vec{E}_t) = i\omega B_z & \\ ik\vec{B}_t - i\omega\mu\varepsilon\,\hat{z}\times\vec{E}_t = \vec{\nabla}_t B_z & \vec{\nabla}_t\cdot\vec{B}_t + ikB_z = 0 \\ \hat{z}\cdot(\vec{\nabla}_t\times\vec{B}_t) = -i\mu\varepsilon\omega E_z & \end{cases}$$

multiply the 1st equation by $\frac{\omega\mu\varepsilon}{k}\,\hat{z}\times$ & add to the 3rd one:

$$i\underbrace{\frac{\omega^2\mu\varepsilon}{k}}\,\underbrace{\hat{z}\times(\hat{z}\times\vec{B}_t)}_{-\vec{B}_t} + ik\vec{B}_t = \vec{\nabla}_t B_z + \frac{\omega\mu\varepsilon}{k}\,\hat{z}\times\vec{\nabla}_t E_z$$

$$i\left(k - \frac{\omega^2\mu\varepsilon}{k}\right)\vec{B}_t = \vec{\nabla}_t B_z + \frac{\omega\mu\varepsilon}{k}\,\hat{z}\times\vec{\nabla}_t E_z$$

$$\Rightarrow \boxed{\vec{B}_t = \frac{i}{\omega^2\mu\varepsilon - k^2}\left[k\vec{\nabla}_t B_z + \mu\varepsilon\omega\,\hat{z}\times\vec{\nabla}_t E_z\right]}$$

Similarly:

$$\boxed{\vec{E}_t = \frac{i}{\mu\varepsilon\omega^2 - k^2}\left[k\vec{\nabla}_t E_z - \omega\,\hat{z}\times\vec{\nabla}_t B_z\right]}$$

$\Rightarrow$ all is expressed in terms of E_z, B_z

<u>Except</u>) TEM waves: $E_z = B_z = 0$

$\Rightarrow \vec{\nabla}_t \cdot \vec{E}_t = 0$, $\vec{\nabla}_t \times \vec{E}_t = 0 \Rightarrow \vec{E} = -\vec{\nabla}_t \Phi$, $\nabla_t^2 \Phi = 0$

$\Rightarrow$ get Laplace equation; from 3rd eqn get

$\vec{B}_t = \frac{\omega\mu\varepsilon}{k} \hat{z} \times \vec{E}_t$ and from 1st eqn get

$\vec{E}_t = -\frac{\omega}{k} \hat{z} \times \vec{B}_t \Rightarrow \frac{\omega\mu\varepsilon}{k} = \frac{k}{\omega} \Rightarrow k = \omega\sqrt{\mu\varepsilon}$

TEM modes have the dispersion relations like waves in medium!

<u>TE modes</u>: $E_z = 0 \Rightarrow \vec{E}_t = \frac{-i\omega}{\mu\varepsilon\omega^2 - k^2} \hat{z} \times \vec{\nabla}_t B_z$

$$\vec{B}_t = \frac{ik}{\mu\varepsilon\omega^2 - k^2} \vec{\nabla}_t B_z$$

$\Rightarrow$ all is expressed in terms of B_z (or H_z)!

<u>TM modes</u>: $B_z = 0 \Rightarrow \vec{E}_t = \frac{ik}{\mu\varepsilon\omega^2 - k^2} \vec{\nabla}_t E_z$

$$\vec{B}_t = \frac{i\mu\varepsilon\omega}{\omega^2\mu\varepsilon - k^2} \hat{z} \times \vec{\nabla}_t E_z$$

$\Rightarrow$ all is expressed in terms of E_z.