$\Rightarrow$ To find the relativistic generalization of this formula, need to use the complete $\vec{E}_{rad}$ in the expression for $dP/d\Omega$ ~ tedious...

$\Rightarrow$ Insted use the following line of arguments:

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\frac{d\vec{p}}{dt}\cdot\frac{d\vec{p}}{dt}$$

P is a Lorentz-invariant quantity (why?)

$P = \frac{d\mathcal{E}}{dt}$, $\mathcal{E}$ is 0th component of p^μ

t is 0th component of x^μ.

Generalize Larmor formula by writing

$$\boxed{P = -\frac{2}{3}\frac{e^2}{m^2c^3}\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}}$$

(in NR case $\tau \simeq t$ $\Rightarrow$ get back $|\dot{\vec{p}}|^2$)

$$\Rightarrow P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\vec{p}}{d\tau}\frac{d\vec{p}}{d\tau} - \frac{1}{c^2}\left(\frac{d\mathcal{E}}{d\tau}\right)^2\right) = \left| \begin{array}{l} \vec{p} = \gamma m\vec{v} \\ \mathcal{E} = mc^2\gamma \\ d\tau = dt/\gamma \end{array}\right.$$

$$= \frac{2}{3}\frac{e^2}{m^2c^3}\left(\gamma^2 m^2\left|\frac{d(\vec{v}\gamma)}{dt}\right|^2 - m^2c^2\gamma^2\left(\frac{d\gamma}{dt}\right)^2\right) =$$

$$= \frac{2}{3}\frac{e^2}{m^2c^3}\Big[\gamma^4 m^2|\dot{\vec{v}}|^2 + \gamma^2 m^2\dot{\gamma}^2\underbrace{(v^2-c^2)}_{-c^2\frac{1}{\gamma^2}} +$$

$$+ 2\gamma^2 m^2\,\vec{v}\cdot\dot{\vec{v}}\;\gamma\cdot\dot{\gamma}\Big] \Rightarrow$$

as $\frac{d\gamma}{dt} = \dot{\gamma} = -\frac{1}{2}\gamma^3 \cdot (-2)\frac{\vec{V}\cdot\dot{\vec{V}}}{c^2} = \gamma^3\,\vec{\beta}\cdot\dot{\vec{\beta}}$

$$\Rightarrow P = \frac{2}{3}\frac{e^2}{\cancel{m^2}c^3}\left[\gamma^4 \cancel{m^2}c^2|\dot{\vec{\beta}}|^2 - \cancel{m^2}c^2\gamma^6(\vec{\beta}\cdot\dot{\vec{\beta}})^2 + \right.$$

$$\left. + 2\cancel{m^2}c^2\gamma^6(\vec{\beta}\cdot\dot{\vec{\beta}})^2\right] = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\frac{|\dot{\vec{\beta}}|^2}{\gamma^2} + \right.$$

$$\left. + (\vec{\beta}\cdot\dot{\vec{\beta}})^2\right]$$

Finally, as $(\vec{\beta}\times\dot{\vec{\beta}})\cdot(\vec{\beta}\times\dot{\vec{\beta}}) = \vec{\beta}^2\dot{\vec{\beta}}^2 - (\vec{\beta}\cdot\dot{\vec{\beta}})^2$

$$\Rightarrow \dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2 = \dot{\vec{\beta}}^2\underbrace{(1-\beta^2)}_{1/\gamma^2} + (\vec{\beta}\cdot\dot{\vec{\beta}})^2$$

$$\Rightarrow \boxed{P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right]}$$

Lienard (1898)

$\Rightarrow$ acceleration leads to radiation

$\Rightarrow$ in particle accelerators energy loss due to radiation limits maximum attainable energy...

$$P \sim \frac{1}{m^2}\left(\frac{d\vec{p}}{dt}\right)^2 \quad \text{and} \quad \vec{F} = \frac{d\vec{p}}{dt} \text{ a force} \Rightarrow$$

$\Rightarrow$ for the same applied force $P \sim \frac{1}{m^2} \Rightarrow$ The lighter the particle the more it radiates $\Rightarrow$ electrons radiate <u>most</u>.

Examples: linear accelerator

$$\vec{\beta} \parallel \dot{\vec{\beta}}\ , \quad \dot{\vec{\beta}} = \text{const} \Rightarrow P = \frac{2}{3}\frac{e^2}{c}\gamma^6\, \dot{\vec{\beta}}^{\,2}$$

$$\frac{dp}{dt} = m\frac{d}{dt}(\gamma v) = m\gamma\dot{v} + m v \gamma^3 \frac{v\cdot\dot{v}}{c^2} =$$

$$= m\gamma\dot{v}\left(1 + \frac{v^2}{c^2}\gamma^2\right) = m\gamma^3\dot{v} = mc\gamma^3\dot{\beta}$$

$$\Rightarrow \dot{\beta}^2 = \frac{1}{m^2c^2\gamma^6}\left(\frac{dp}{dt}\right)^2 \Rightarrow P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp}{dt}\right)^2$$

$$\frac{dp}{dt} = \text{force} = \frac{d\mathcal{E}}{dx} \Rightarrow P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2$$

force $\cdot\, dx$ = work

$$\frac{\text{power radiated}}{\text{power supplied}} = \frac{P}{v\cdot\frac{dE}{dx}} = \frac{2}{3}\frac{e^2}{m^2c^3v}\frac{dE}{dx} \xrightarrow[v\to c]{}$$

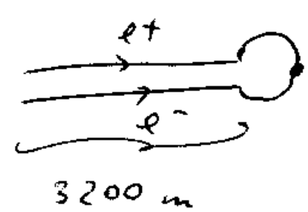
$$\to \frac{2}{3}\frac{e^2}{m^2c^4}\frac{dE}{dx} = \frac{2}{3}\frac{e^2}{mc^2}\frac{1}{mc^2}\frac{dE}{dx} =$$

$$= \frac{2}{3}\, 2.8\cdot 10^{-15}\,\text{m}\,(0.511\ \text{MeV})^{-1}\cdot\frac{dE}{dx} \Rightarrow \text{to have this} \sim 1$$

need $\frac{dE}{dx} \sim \frac{3}{2}\frac{.5}{2.8\cdot 10^{-15}}\frac{\text{MeV}}{\text{m}} \approx 2.5\cdot 10^{14}\,\frac{\text{MeV}}{\text{m}}$

Usually $\frac{dE}{dx} < 50\ \text{MeV/m} \Rightarrow$ losses are tiny $\Rightarrow$

good to build linear colliders. (SLAC



$\sqrt{s} \simeq 90\ \text{GeV} \Rightarrow \frac{dE}{dx} \simeq \frac{45\ \text{GeV}}{3200\,\text{m}} \simeq 14\ \frac{\text{MeV}}{\text{m}}$.)