$$\boxed{\begin{aligned} E_1' &= E_1 & B_1' &= B_1 \\ E_2' &= \gamma(E_2 - \beta B_3) & B_2' &= \gamma(B_2 + \beta E_3) \\ E_3' &= \gamma(E_3 + \beta B_2) & B_3' &= \gamma(B_3 - \beta E_2) \end{aligned}}$$

if $v \ll c \Rightarrow$ get $\vec{E}' = \vec{E} + \frac{v}{c} \times \vec{B}$ ~ cf. 1st quarter

$$\vec{B}' = \vec{B} - \frac{1}{c}\vec{v} \times \vec{E}.$$

Lorentz - invariants:

$F^{\mu\nu} F_{\mu\nu} = 2\,(\vec{B}^2 - \vec{E}^2)$ ~ by construction this is Lorentz-inv.

$F^{\mu\nu} \tilde{F}_{\mu\nu} = 4\,\vec{B}\cdot\vec{E}$ ~ also Lorentz-inv.

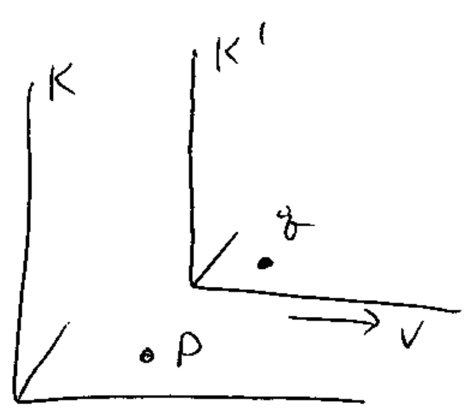
<u>Example</u>: plane waves, $\vec{E} = \frac{c}{\omega}\vec{k} \times \vec{B} \Rightarrow$

$\Rightarrow \vec{E}\cdot\vec{B} = 0$, $|\vec{E}| = |\vec{B}| \Rightarrow E^2 - B^2 = 0$

~ true in all frames!

<u>Example</u>: moving point charge: in it's rest frame the field is given by Coulomb's law: $\vec{E} = \frac{q}{r^3}\vec{r}$

(note Gaussian units)



$\Rightarrow$ observer at P has coordinates (x_1', x_2', x_3')

$\Rightarrow$ $\vec{E}'(P) = q \dfrac{\vec{x}'}{\vec{x}'^3}$ (assume that charge q is at the origin in frame K')

$\vec{B}' = 0$

$\Rightarrow$ boost $\rightarrow$ $E_x = E_x' = q \dfrac{x'}{(x'^2 + y'^2 + z'^2)^{3/2}} =$

$$= q \frac{\gamma(x - vt)}{[\gamma^2 (x - vt)^2 + y^2 + z^2]^{3/2}}$$

$$E_y = \gamma(E_y' + \beta \underset{0}{\cancel{B_z'}}) = q \frac{\gamma y}{[\gamma^2 (x-vt)^2 + y^2 + z^2]^{3/2}}$$

$$E_z = \gamma(E_z' - \beta \underset{0}{B_y'}) = q \frac{\gamma z}{[\gamma^2 (x-vt)^2 + y^2 + z^2]^{3/2}}$$

$$B_x = B_x' = 0 ; \quad B_y = \gamma(\overset{=0}{B_z'} - \beta E_z') =$$

$$= -\gamma\beta \frac{q z}{[\gamma^2 (x-vt)^2 + y^2 + z^2]^{3/2}}$$

$$B_z = \gamma(\underset{0}{B_z'} + \beta E_y') = \gamma\beta \frac{q y}{[\gamma^2 (x-vt)^2 + y^2 + z^2]^{3/2}}$$

In the non-relativistic case: $\beta \ll 1$, $\gamma \simeq 1$

$$\Rightarrow B_y \simeq -\frac{v}{c} \frac{q z}{[(x-vt)^2 + y^2 + z^2]^{3/2}} ; \quad B_z \simeq \frac{v}{c} \frac{q y}{[(x-vt)^2 + y^2 + z^2]^{3/2}}$$

$$\Rightarrow \vec{B} = \frac{q}{c}\,\frac{\vec{v}\times\vec{r}}{r^3} \quad \sim \text{Biot-Savart law!}$$

In the UR limit, $\beta \to 1$, $\gamma \to \infty$ $\Rightarrow$

$$\Rightarrow B_y \simeq -\frac{q\,z}{\gamma^2 (x-vt)^3} = -E_z \,; \quad B_z \simeq \frac{q\,y}{(x-vt)^3} = E_y \,;$$

$$E_x \simeq \frac{q}{\gamma^2 (x-vt)^2}\,\mathrm{sign}(x-vt) \,.$$

Relativistic Particles in Electromagnetic Fields.

Start with Lorentz force:

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{1}{c}\,\vec{v}\times\vec{B}\right) = \vec{F}$$

$\Rightarrow$ want to transform this into an equation for 3-components of 4-vector p^μ $\Rightarrow$

$$\Rightarrow p^\mu = m u^\mu = m\gamma(c, \vec{v}) \quad \Rightarrow$$

$$\frac{d\vec{p}}{d\tau} = \gamma\,\frac{dp}{dt} = \frac{q}{c}\left[u^0 \vec{E} + \vec{v}\times\vec{B}\right]$$

On the other hand $\dfrac{dE}{dt} = \vec{F}\cdot\vec{v}$ $\Rightarrow$