Last time | **Review of First Semester** (cont'd)

Among other things, spent some time reviewing non-Abelian gauge theories:

$$\mathcal{L}_{quarks} = \bar{q}\,[\,i\partial\!\!\!/ - m\,]\,q$$

is SU(3) symmetric under <u>global</u> SU(3) transformations:

$$\left[\begin{array}{l} q(x) \to \underset{\substack{\uparrow \\ 3\times 3 \text{ matrix}}}{S}\, q(x) \\ \bar{q}(x) \to \bar{q}(x)\, S^\dagger \end{array}\right. , \quad SS^\dagger = 1$$

If we want <u>local</u> SU(3) symmetry, $S \to S(x)$,

$\Rightarrow$ need to introduce gauge field A_μ^a, $a = 1, \ldots, 8$ (gluons), such that

$$\boxed{\mathcal{L}_{QCD} = \bar{q}\,[\,iD\!\!\!\!/ - m\,]\,q - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$

where $D_\mu = \partial_\mu - ig A_\mu$ (covariant derivative)

$$A_\mu = \sum_{a=1}^{8} A_\mu^a T^a$$

$$F_{\mu\nu} = \frac{i}{g}\,[\,D_\mu, D_\nu\,] = \sum_{a=1}^{8} F^a_{\mu\nu}\, T^a$$

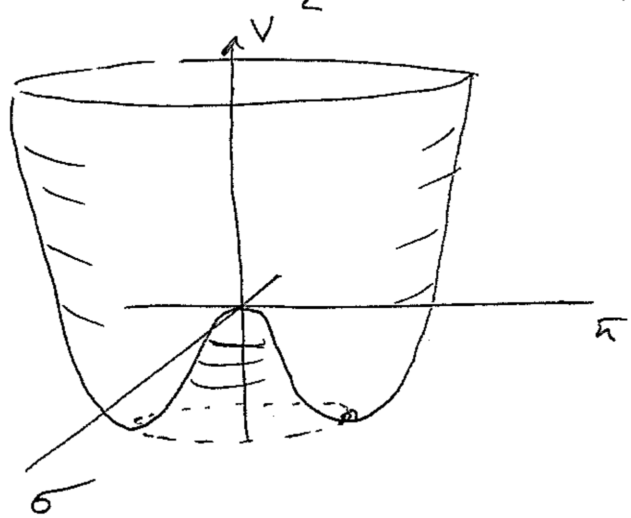
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu .$$

Spontaneous Symmetry Breaking (SSB)

- ~ symmetry manifest in $\mathcal{L}$, but not respected by ground state.
- ~ <u>Nambu - Goldstone theorem</u>: spontaneous breakdown of a continuous symm. $\Rightarrow$ massless spinless particles (Nambu-Goldstone bosons).

<u>Example</u>: Abelian σ - Model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\partial_\mu\bar{\pi}\,\partial^\mu\bar{\pi} + \frac{\mu^2}{2}(\sigma^2+\bar{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\bar{\pi}^2)^2$$



at the minimum

$$\sigma^2+\bar{\pi}^2 = v^2 = \frac{\mu^2}{\lambda}$$

$\Downarrow$

pick vacuum at

$$\langle 0|\sigma|0\rangle = v = \frac{\mu}{\sqrt{\lambda}}$$

$$\langle 0|\bar{\pi}|0\rangle = 0$$

$\Rightarrow$ exand near the vacuum: $\sigma = v + \sigma'$, $\bar{\pi}$:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma'\,\partial^\mu\sigma' + \frac{1}{2}\partial_\mu\bar{\pi}\,\partial^\mu\bar{\pi} - \mu^2\sigma'^2 - \lambda v\sigma'(\sigma'^2+\bar{\pi}^2) - \frac{\lambda}{4}(\sigma'^2+\bar{\pi}^2)^2$$

$\Rightarrow$ $\bar{\pi}$ is <u>massless</u>: Goldstone boson

σ' has mass $\sqrt{2}\mu$.

$U(1)$ symmetry broken spontaneously