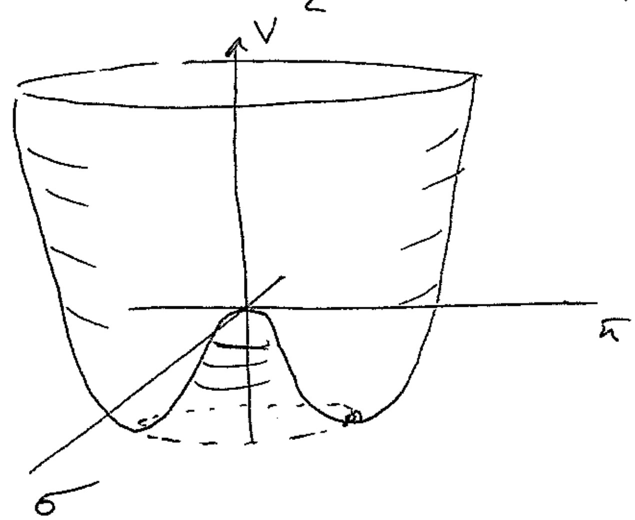
Spontaneous Symmetry Breaking (SSB)

~ symmetry manifest in $\mathcal{L}$, but not respected by ground state.

~ <u>Nambu - Goldstone theorem</u>: spontaneous breakdown of a continuous symm. $\Rightarrow$ massless spinless particles (Nambu-Goldstone bosons).

<u>Example</u>: Abelian σ - Model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\,\partial^\mu\pi + \frac{\mu^2}{2}(\sigma^2+\pi^2) - \frac{\lambda}{4}(\sigma^2+\pi^2)^2$$



at the minimum

$$\sigma^2 + \pi^2 = v^2 = \frac{\mu^2}{\lambda}$$

$$\Downarrow$$

pick vacuum at

$$\langle 0|\sigma|0\rangle = v = \frac{\mu}{\sqrt{\lambda}}$$

$$\langle 0|\pi|0\rangle = 0$$

$\Rightarrow$ exand near the vacuum: $\sigma = v + \sigma'$, π :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma'\,\partial^\mu\sigma' + \frac{1}{2}\partial_\mu\pi\,\partial^\mu\pi - \mu^2\sigma'^2 - \lambda v\sigma'(\sigma'^2+\pi^2) - \frac{\lambda}{4}(\sigma'^2+\pi^2)^2$$

$\Rightarrow$ π is <u>massless</u>: Goldstone boson

σ' has mass $\sqrt{2}\mu$.

$U(1)$ symmetry broken spontaneously

Example: non-abelian σ-model:

$\pi \to \vec{\pi} = (\pi^1, \pi^2, \pi^3)$ ~ pion field.

$q^N = \begin{pmatrix} p \\ n \end{pmatrix}$ ~ fermions.

$SU(2)_L \otimes SU(2)_R$ symmetric

after SSB get: σ' has mass $\mu\sqrt{2}$

q^N have mass $g\,v$

$\vec{\pi}$ have mass $\emptyset$ (Goldstone bosons)

$SU(2)_L \otimes SU(2)_R$ spont. broken to $SU(2)$.

In QCD: pions (π^+, π^-, π^0) are Goldstone bosons of chiral SSB, $m_\pi = 0$ (as $m_u \neq m_d \neq 0$ not exact)

$v = \langle 0 | \bar{q} q | 0 \rangle = -(230\,MeV)^3$.

The Electroweak Theory.

local vs global gauge symmetries:

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu - ig A_\mu \quad \Rightarrow \begin{cases} \psi \to e^{i\alpha(x)}\psi \\ A_\mu \to A_\mu + \frac{1}{g}\partial_\mu \alpha \end{cases}$$

local $U(1)$ symmetry (abelian)

non-abelian:

$$\boxed{\mathcal{L} = \bar{\psi}\left[i\gamma^\mu D_\mu - m \right]\psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$

$$D_\mu = \partial_\mu - igA_\mu, \quad A_\mu = \sum_a T^a A_\mu^a, \quad F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu \right] =$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu \right].$$

$$\begin{cases} \psi \to S(x)\psi \\ A_\mu \to S(x) A_\mu S^{-1}(x) - \frac{i}{g}(\partial_\mu S) S^{-1} \end{cases}$$

$S(x)$ a unitary $N \times N$ matrix $\Rightarrow$ $SU(N)$ local symm.

<u>Higgs mechanism</u>: $\mathcal{L} = (D_\mu \varphi)^* (D^\mu \varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} +$

$U(1)$ model

$$+ \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

$\Rightarrow$ $U(1)$ gauge symm. $\Rightarrow$ pick a VEV: $\langle 0|\varphi|0\rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}$

$\Rightarrow$ write $\varphi = \frac{\rho'(x)}{\sqrt{2}} e^{i\theta(x)}$, $B_\mu(x) = A_\mu - \frac{1}{g}\partial_\mu \theta(x)$

& expand $\rho' = v + \rho$ around the VEV. One gets:

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \mu^2 \rho^2 - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} g^2 v^2 B_\mu B^\mu + \frac{1}{2} g^2 B_\mu B^\mu (2v\rho + \rho^2) - \lambda v \rho^3 - \frac{\lambda}{4}\rho^4}$$

ρ has mass $\sqrt{2}\mu$

B_μ a massive gauge field $m_B = gv$

(no Goldstone bosons) θ - would-be Goldstone boson

$SU(2)\otimes U(1)$ Electroweak theory: $\psi_{L,R} = \frac{1 \pm \gamma_5}{2}\psi$

leptons

$=-1$ $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$, $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$, $\boxed{Q = I_3 + \frac{Y}{2}}$

$=-2$ $R_e = e_R$, $R_\mu = \mu_R$, $R_\tau = \tau_R$. weak isospin, weak hypercharge

gauge field: $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$, $\vec{F}_{\mu\nu}$

B_μ with $f_{\mu\nu}$

Higgs field: $\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}$, $Y=+1$, $\phi^\dagger = (\phi^{(-)}, \phi^{(0)*})$

$$\boxed{\begin{aligned}\mathcal{L}_{\text{leptons + gauge + Higgs}} &= \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu)R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - \right.\\
&\left. - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e + (\mu,\tau) - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\
&+ \left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^\dagger\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right] \\
&+ \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - G_e\left[\underbrace{\bar{L}_e\phi R_e}_{Y=+1,+1,-2\,=\,0} + \text{c.c.}\right] - (\mu,\tau)\end{aligned}}$$

$SU(2)_L\otimes U(1)_Y$ symmetric

VEV $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $\phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)}\begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$

(to break the $SU(2)_L\otimes U(1)_Y \to U(1)_{EM}$)

Def.

$$\begin{cases} W_\mu = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) \\ W_\mu^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \\ Z_\mu = -B_\mu\sin\theta_W + W_\mu^3\cos\theta_W \\ A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W \end{cases}$$

$\tan\theta_W = g'/g$ Weinberg angle

get $$\mathcal{L} = \bar{e}\, i\gamma\cdot\partial e + \bar{\nu}_{eL}\, i\gamma\cdot\partial \nu_{eL} - \frac{G_e}{\sqrt{2}}(v+\eta)\bar{e}e -$$
$$- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$
$$+ \frac{g^2}{4}(v+\eta)^2 W_\mu^+ W^\mu + \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu +$$
$$+ \frac{g}{2\cos\theta_W}\left[2\sin^2\theta_W\, \bar{e}_R\gamma\cdot Z e_R + (2\cdot\sin^2\theta_W - 1)\,\bar{e}_L\gamma\cdot Z e_L\right]$$
$$- e\,\bar{e}\gamma\cdot A e + \frac{g}{2\cos\theta_W}\bar{\nu}_{eL}\gamma\cdot Z \nu_{eL} - \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\gamma\cdot W e_L + e.c.\right] +$$
$$+ (\mu, \tau)$$

$$M_W = \frac{gv}{2} \simeq 80.4 \text{ GeV}$$

$$M_Z = \frac{gv}{2\cos\theta_W} \simeq 91.2 \text{ GeV}$$

$$m_\gamma = 0$$

$$m_e = \frac{G_e v}{\sqrt{2}}, \quad m_\mu = \frac{G_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{G_\tau v}{\sqrt{2}}$$

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \qquad (\geqslant .04\text{ eV in reality})$$

$$\theta_W \simeq 30°, \quad \frac{g^2}{4\pi} \simeq \frac{1}{30} \sim \text{small}$$

$$m_H = \mu\sqrt{2} = v\sqrt{2\lambda}, \quad v \simeq 246 \text{ GeV}$$

$$m_H \approx 125 \text{ GeV}$$

Quarks in EW theory:

$$\boxed{\begin{aligned}\mathcal{L}_{\text{quarks + gauge}} &= \bar{L}_u\, i\gamma^\mu\Big(\partial_\mu - i\underbrace{\frac{g'}{6}}_{\frac{1}{2}\cdot\frac{1}{3}} B_\mu - ig\,\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big)L_u \\ &+ \bar{R}_u\, i\gamma^\mu\Big(\partial_\mu - i\overbrace{\frac{2}{3}}^{=\frac{1}{2}\cdot\frac{4}{3}} g' B_\mu\Big)R_u + \bar{R}_d\, i\gamma^\mu\Big(\partial_\mu + i\underbrace{\frac{g'}{3}}_{\frac{1}{2}\cdot(-\frac{2}{3})} B_\mu\Big)R_d \\ &+ 2 \text{ more generations.}\end{aligned}}$$

$2(Q-I_3) = Y$

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad 1/3$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R \qquad 4/3$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_R \qquad -2/3$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — CKM matrix — mass eigenstates

Cabibbo-Kobayashi-Maskawa

(unitary)

quarks-Higgs coupling: $\tilde{\phi} = i\tau^2\phi^*$, $Y = -1$ — Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\Rightarrow$ write $\mathcal{L} = -G_1\left[\bar{L}_u\tilde{\phi}R_u + \bar{R}_u\tilde{\phi}^\dagger L_u\right] -$

$$- G_2\left[\bar{L}_u\phi R_d + \bar{R}_d\phi^\dagger L_u\right] + \text{other flavors } (c,t)$$

(all terms with $Y=0$ ($U(1)$ inv) & $SU(2)$ inv.)

$\Rightarrow$ get quark masses.