Interactions of W's and Z's with Quarks:

$$U_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L , \quad D_L = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L , \quad \psi = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim \text{six spinors}$$

U_R, D_R ~ by analogy

$$M \equiv \begin{pmatrix} \mathbb{1}_{3\times3} & 0 \\ 0 & V \end{pmatrix} , \quad V = \text{CKM matrix}$$

(i) Charged current (coupling to $W^\pm$):

$$\boxed{\begin{aligned} \mathcal{L}_{c.c.} = \frac{g}{2\sqrt{2}} \Big\{ & \bar{u}\,\gamma\cdot W(1-\gamma_5)\left[V_{ud}\, d + V_{us}\, s + V_{ub}\, b\right] \\ & + \bar{c}\,\gamma\cdot W(1-\gamma_5)\left[V_{cd}\, d + V_{cs}\, s + V_{cb}\, b\right] + \bar{t}\,\gamma\cdot W(1-\gamma_5)\cdot \\ & \cdot\left[V_{td}\, d + V_{ts}\, s + V_{tb}\, b\right] + h.c. \Big\} \end{aligned}}$$

(ii) Neutral Current (coupling to γ and Z):

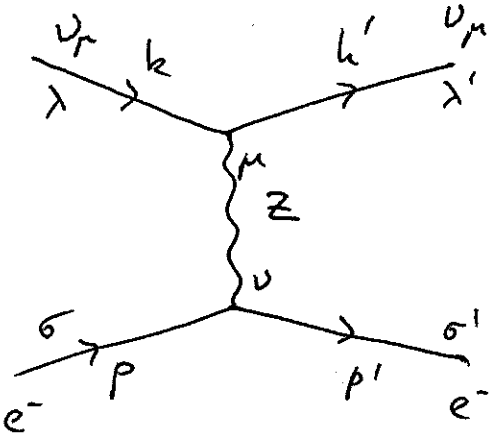
$$\mathcal{L}_{photons} = \sum_f e_f\, \bar{q}_f\, \gamma\cdot A\, q_f , \quad e_f = e \begin{cases} 2/3, & u,c,t \\ -1/3, & d,s,b \end{cases} \qquad e = g \sin\theta_W$$

$$\boxed{\begin{aligned} \mathcal{L}_Z = \frac{g}{4\cos\theta_W} \Big\{ & \bar{U}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\tfrac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5)\tfrac{4}{3}\sin^2\theta_W\right] U \\ & - \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\tfrac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\tfrac{2}{3}\sin^2\theta_W\right] D \Big\} \end{aligned}}$$

$$\mathcal{L}_{nc} = \mathcal{L}_Z + \mathcal{L}_{photons} .$$

Elastic electron-neutrino scattering.

Consider $\nu_\mu + e \to \nu_\mu + e$



We know that

$$\mathcal{L}_Z = \frac{g}{4\cos\theta_W}\Big\{ \bar\nu_e \gamma\cdot Z(1-\gamma_5)\nu_e$$

$$+ 2\sin^2\theta_W\, \bar e\, \gamma\cdot Z(1+\gamma_5) e$$

$$+ (2\sin^2\theta_W - 1)\, \bar e\, \gamma\cdot Z(1-\gamma_5) e \Big\} + (\mu,\tau)$$

$$\equiv \frac{g}{2\cos\theta_W}\Big\{ g_L^\nu\, \bar\nu_e \gamma\cdot Z(1-\gamma_5)\nu_e + g_R^e\, \bar e \gamma\cdot Z(1+\gamma_5) e$$

$$+ g_L^e\, \bar e\gamma\cdot Z(1-\gamma_5)e\Big\} + \dots$$

↑
definition

Scattering amplitude:

$$iM = \left(\frac{ig}{2\cos\theta_W}\right)^2 g_L^\nu \bar u_{\lambda'}(k')\,\gamma_\mu(1-\gamma_5)\,u_\lambda(k)\;\bar u_{\sigma'}(p')\big[g_L^e\gamma_\nu(1-\gamma_5)$$

$$+ g_R^e\gamma_\nu(1+\gamma_5)\big]u_\sigma(p)\quad \underbrace{\frac{-i\left[g^{\mu\nu} - \frac{(k-k')^\mu(k-k')^\nu}{M_Z^2}\right]}{(k-k')^2 - M_Z^2 + i\varepsilon}}_{\simeq\ \frac{i}{M_Z^2}g^{\mu\nu}\ \text{at low energy} \ll M_Z^2}$$

$$\Rightarrow\quad M \simeq \frac{-g^2}{4M_Z^2\cos^2\theta_W} g_L^\nu \bar u_{\lambda'}(k')\gamma_\mu(1-\gamma_5)u_\lambda(k)\;\bar u_{\sigma'}(p')\big[g_L^e\gamma^\mu(1-\gamma_5)$$

$$+ g_R^e\gamma^\mu(1+\gamma_5)\big]u_\sigma(p) \quad\Rightarrow$$

$$\Rightarrow \sum_{\lambda,\lambda',\sigma,\sigma'} |M|^2 \simeq \frac{g^4}{16 M_Z^4 \cos^4\theta_W} L^-_{\mu\nu}(k,k') (g_L^\nu)^2 \cdot \left[(g_L^e)^2 L^{-\mu\nu}(p,p') + (g_R^e)^2 L^{+\mu\nu}(p,p') \right] = \frac{g^4}{16 M_Z^4 \cos^4\theta_W} (g_L^\nu)^2 \left[(g_L^e)^2 \cdot L^-_{\mu\nu}(k,k') L^{-\mu\nu}(p,p') + (g_R^e)^2 L^-_{\mu\nu}(k,k') L^{+\mu\nu}(p,p') \right],$$

where we have used the following result and definition: [Use $tr[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$, $tr[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\varepsilon^{\mu\nu\rho\sigma}$]

$$\sum_{\lambda,\lambda'} \bar{u}_{\lambda'}(k') \gamma_\mu (1 \mp \gamma_5) u_\lambda(k) \cdot u^+_\lambda(k) (1 \mp \overset{\overset{\gamma_5}{\parallel}}{\gamma_5^+}) \gamma_\nu^+ \gamma^0 u_{\lambda'}(k')$$

$$= tr[\gamma_\mu (1\mp\gamma_5) \not{k} \gamma^0 (1\mp\gamma_5) \gamma_\nu^+ \gamma^0 \not{k}'] = 2 \cdot tr[\gamma_\mu (1\mp\gamma_5) \not{k} \underbrace{\gamma^0 \gamma_\nu^+ \gamma^0}_{\gamma_\nu} \not{k}'] = 2\, tr[\gamma_\mu (1\mp\gamma_5) \cdot \not{k} \gamma_\nu \not{k}'] = 8 [k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i \varepsilon^\alpha{}_\nu{}^\beta{}_\mu k_\alpha k'_\beta]$$

$$= 8 [k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i \underbrace{\varepsilon^{\alpha\nu\beta\mu}}_{=\varepsilon_{\mu\nu\alpha\beta}} k_\alpha k'_\beta] \equiv L^\mp_{\mu\nu}(k,k')$$

Now, $L^-_{\mu\nu}(k,k') L^{\mp\mu\nu}(p,p') = 64 [k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} + i\varepsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta][p^\mu p'^\nu + p^\nu p'^\mu - p\cdot p' g^{\mu\nu} \pm i \varepsilon^{\mu\nu\rho\sigma} p_\rho p'_\sigma]$

$$= 64 [2p\cdot k\, p'\cdot k' + 2p\cdot k'\, p'\cdot k \cancel{- 4p\cdot p' k\cdot k'} + \cancel{4 p\cdot p' k\cdot k'}^{\,g_{\mu\nu}g^{\mu\nu}}$$

$$\mp \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} k^\alpha k'^\beta p_\rho p'_\sigma] = \Big|\ \text{as } \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = -2\delta_\alpha{}^\rho \delta_\beta{}^\sigma + 2\delta_\alpha{}^\sigma\delta_\beta{}^\rho$$