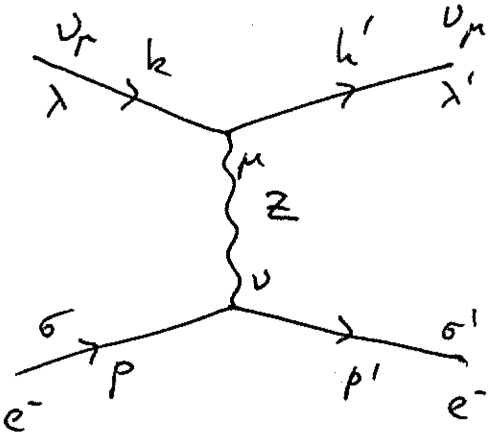
Elastic electron-neutrino scattering

Consider $\nu_\mu + e \to \nu_\mu + e$



We know that

$$\mathcal{L}_Z = \frac{g}{4\cos\theta_W}\Big\{ \bar\nu_e\, \gamma\cdot Z(1-\gamma_5)\nu_e + 2\sin^2\theta_W\, \bar e\, \gamma\cdot Z(1+\gamma_5)\, e + (2\sin^2\theta_W - 1)\, \bar e\, \gamma\cdot Z(1-\gamma_5)\, e \Big\} + (\mu,\tau)$$

$$\equiv \frac{g}{2\cos\theta_W}\Big\{ g_L^{\nu_e}\, \bar\nu_e\, \gamma\cdot Z(1-\gamma_5)\nu_e + g_R^e\, \bar e\, \gamma\cdot Z(1+\gamma_5)\, e + g_L^e\, \bar e\, \gamma\cdot Z(1-\gamma_5)\, e\Big\} + \dots$$

$\uparrow$ definition

Scattering amplitude:

$$iM = \left(\frac{ig}{2\cos\theta_W}\right)^2 g_L^\nu\, \bar u_{\lambda'}(k')\, \gamma_\mu(1-\gamma_5)\, u_\lambda(k)\, \bar u_{\sigma'}(p')\left[g_L^e\, \gamma_\nu(1-\gamma_5) + g_R^e\, \gamma_\nu(1+\gamma_5)\right] u_\sigma(p)\; \underbrace{\frac{-i\left[g^{\mu\nu} - \frac{(k-k')^\mu (k-k')^\nu}{M_Z^2}\right]}{(k-k')^2 - M_Z^2 + i\varepsilon}}_{\simeq \frac{i}{M_Z^2} g^{\mu\nu} \text{ at low energy} \ll M_Z^2}$$

$$\Rightarrow\quad M \simeq \frac{-g^2}{4M_Z^2\cos^2\theta_W}\, g_L^\nu\, \bar u_{\lambda'}(k')\gamma_\mu(1-\gamma_5)u_\lambda(k)\, \bar u_{\sigma'}(p')\left[g_L^e\, \gamma^\mu(1-\gamma_5) + g_R^e\, \gamma^\mu(1+\gamma_5)\right]u_\sigma(p) \quad\Rightarrow$$

$$\Rightarrow \sum_{\lambda,\lambda',\sigma,\sigma'} |M|^2 \simeq \frac{g^4}{16 M_Z^4 \cos^4\theta_W} L^-_{\mu\nu}(k,k')\,(g_L^{\nu_\mu})^2 \cdot \left[(g_L^e)^2 L^{-\mu\nu}(p,p') + (g_R^e)^2 L^{+\mu\nu}(p,p') \right] = \frac{g^4}{16 M_Z^4 \cos^4\theta_W} (g_L^{\nu_\mu})^2 \left[(g_L^e)^2 \cdot L^-_{\mu\nu}(k,k') L^{-\mu\nu}(p,p') + (g_R^e)^2 L^-_{\mu\nu}(k,k') L^{+\mu\nu}(p,p') \right],$$

where we have used the following result and definition: [Use $tr[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$, $tr[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\varepsilon^{\mu\nu\rho\sigma}$]

$$\sum_{\lambda,\lambda'} \bar u_{\lambda'}(k')\,\gamma_\mu(1\mp\gamma_5)\,u_\lambda(k)\cdot u^\dagger_\lambda(k)\,(1\mp\overset{\overset{\gamma_5}{\shortparallel}}{\gamma_5^\dagger})\,\gamma_\nu^\dagger\gamma^0 u_{\lambda'}(k')$$

$$= tr\left[\gamma_\mu(1\mp\gamma_5)\,\not k\,\gamma^0(1\mp\gamma_5)\gamma_\nu^\dagger\gamma^0\not k'\right] = 2\cdot tr\left[\gamma_\mu(1\mp\gamma_5)\not k\,\underbrace{\gamma^0\gamma_\nu^\dagger\gamma^0}_{\gamma_\nu}\not k'\right] = 2\,tr\left[\gamma_\mu(1\mp\gamma_5)\cdot\not k\,\gamma_\nu\not k'\right] = 8\left[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i\,\varepsilon^\alpha{}_\nu{}^\beta{}_\mu k_\alpha k'_\beta\right]$$

$$= 8\left[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i\,\underbrace{\varepsilon^{\alpha\nu\beta\mu}_{\ \mu\nu\alpha\beta}}_{=\varepsilon} k_\alpha k'_\beta\right] \equiv L^{\mp}_{\mu\nu}(k,k')$$

Now, $L^-_{\mu\nu}(k,k')\,L^{\mp\mu\nu}(p,p') = 64\left[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} + i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right]\left[p^\mu p'^\nu + p^\nu p'^\mu - p\cdot p' g^{\mu\nu} \pm i\varepsilon^{\mu\nu\rho\sigma}p_\rho p'_\sigma\right]$

$$= 64\left[2p\cdot k\, p'\cdot k' + 2p\cdot k'\, p'\cdot k - \cancel{4p\cdot p'\,k\cdot k'} + \overset{g_{\mu\nu}g^{\mu\nu}}{\cancel{4\,p\cdot p'\,k\cdot k'}} \mp \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma}k^\alpha k'^\beta p_\rho p'_\sigma\right] = \Big|\ \text{as } \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = -2\delta_\alpha{}^\rho\delta_\beta{}^\sigma + 2\delta_\alpha{}^\sigma\delta_\beta{}^\rho$$

$$= 128\left[p\cdot k\, p'\cdot k' + p\cdot k'\, p'\cdot k \mp \left(-p\cdot k\, p'\cdot k' + p\cdot k'\, p'\cdot k\right)\right]$$

$$= 128 \cdot \begin{cases} 2\, p\cdot k\, p'\cdot k' & \text{"}-\text{"} \\ 2\, p\cdot k'\, p'\cdot k & \text{"}+\text{"} \end{cases}$$

$$\underbrace{\frac{1}{2}\sum_{\lambda,\lambda',\sigma,\sigma'} |M|^2}_{\langle |M|^2\rangle\ \parallel} = \frac{1}{2}\frac{g^4}{16\cdot M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2 128\cdot 2$$

$\uparrow$ averaging over electron polarizations

$$\cdot\left[\left(g_L^e\right)^2 p\cdot k\, p'\cdot k' + \left(g_R^e\right)^2 p\cdot k'\, p'\cdot k\right]$$

$$= \frac{8 g^4}{M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2\left[\left(g_L^e\right)^2 \underbrace{p\cdot k\, p'\cdot k'}_{(p\cdot k)^2} + \left(g_R^e\right)^2 \underbrace{p\cdot k'\, p'\cdot k}_{(p\cdot k')^2}\right]$$

$$p+k = p'+k' \Rightarrow p\cdot k = p'\cdot k' \Rightarrow$$

$$p-k' = p'-k \Rightarrow p\cdot k' = p'\cdot k$$

The cross section is

$$d\sigma = \frac{1}{2E_p\, 2E_K\, |\vec{v}_p - \vec{v}_k|}\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{d^3k'}{(2\pi)^3 2E_{k'}}\langle |M|^2\rangle (k')$$

$$\cdot (2\pi)^4 \delta^4(p'+k'-p-k) =$$

work in the lab frame = rest frame of electron $\Rightarrow \vec{v}_p = 0$, $E_p = m_e$
$|\vec{v}_k| = 1$ as $m_\nu \simeq 0$. $\Rightarrow p\cdot k = m_e E_k$
$p\cdot k' = m_e E_{k'}$

$$= \frac{1}{2m_e\, 2E_K}\frac{d^3k'}{(2\pi)^3 2E_{k'}}\frac{1}{2E_{p'}}\, 2\pi\, \delta(\underbrace{E_{p'}}_{\sqrt{(\vec{k}-\vec{k}')^2+m_e^2}} + E_{k'} - \underbrace{E_p}_{m_e} - E_K)\,\cdot$$

$$\cdot \frac{8 g^4 m_e^2}{M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2\left[\left(g_L^e\right)^2 E_k^2 + \left(g_R^e\right)^2 E_{k'}^2\right]$$

Now, we can integrate over angles:

$$\frac{d^3k'}{(2\pi)^3 2E_{k'}} \frac{1}{2E_{p'}} \cdot 2\pi\, \delta\left(\sqrt{(\vec{k}-\vec{k}')^2 + m_e^2} + \underbrace{k'}_{|\vec{k}'|} - m_e - \underbrace{k}_{|\vec{k}|}\right) =$$

$$= \frac{k'^{\cancel{2}}\, dk' \cdot \cancel{2\pi} \cdot d\cos\theta}{(2\pi)^{\cancel{2}}\; 2\cancel{k'}\; 2E_{p'}} \;\delta\left(\sqrt{k^2+k'^2-2kk'\cos\theta+m_e^2} + k' - m_e - k\right)$$

$$= \frac{\cancel{k'}\,dk'}{8\pi\, E_{p'}} \cdot \frac{1}{\frac{1}{\cancel{2E_{p'}}}\;\cancel{2}k\cancel{k'}} = \frac{dk'}{8\pi k} = \frac{dE_{p'}}{8\pi k} \leftarrow \text{can measure recoil electron}$$

since $E_{p'} + k' = m_e + k \Rightarrow |dk'| = |dE_{p'}|$

$$\Rightarrow \frac{d\sigma}{dE_{p'}} = \frac{1}{\cancel{8}\pi k}\frac{1}{4m_e E_k}\frac{\cancel{8}g^4 m_e^2}{M_Z^4\cos^4\theta_W}(g_L^{\nu_\mu})^2\left[(g_L^e)^2 E_k^2 + (g_R^e)^2 \cdot E_{k'}^2\right]$$

$$\boxed{\frac{d\sigma}{dE_{p'}} = \frac{g^4 m_e}{4\pi M_Z^4 \cos^4\theta_W}(g_L^{\nu_\mu})^2\left[(g_L^e)^2 + (g_R^e)^2\left(\frac{E_{k'}}{E_k}\right)^2\right]}$$

Here $E_{k'} = m_e + E_k - E_{p'}$.

$\Rightarrow$ can measure $|g_L^e|$ and $|g_R^e|$ (given $g_L^{\nu_\mu}$), to test SM predictions

Can we augment the Standard Model to include right-handed neutrinos?

Postulate a right-handed neutrino singlet ν_R ($Y = 2(Q - I_3) = 0$). Add the following to the SM Lagrangian:

$$\mathcal{L}_{R.H.\nu} = G_R [\underbrace{\bar{L}}_{Y=+1} \underbrace{\tilde{\varphi}}_{Y=-1} \underbrace{\nu_R}_{Y=0} + c.c.] + \ldots$$

$\Rightarrow$ since $\langle \psi_0 | \tilde{\varphi} | \psi_0 \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ ~ the VEV

$\Rightarrow$ near the VEV get

$$\mathcal{L}_{R.H.\nu} = G_R \left[(\bar{\nu}_L \ \bar{e}_L) \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \nu_R + c.c. \right] =$$

$$= G_R [\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L] \frac{v}{\sqrt{2}} = \frac{G_R v}{\sqrt{2}} \bar{\nu} \nu$$ $\Rightarrow$ a mass term for ν's!

$\Rightarrow \boxed{m_\nu = \frac{G_R v}{\sqrt{2}}}$ $\Rightarrow$ for $m_\nu \simeq 0.04$ eV, $v \simeq 246$ GeV

$\Rightarrow G_R = \frac{m_\nu \sqrt{2}}{v} \simeq 2 \times 10^{-13}$ ~ seems to require a lot of fine-tuning in this coupling (why not?)