Last time: Neutrino Oscillations

if neutrinos are massive & mix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\text{PMNS matrix}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\Rightarrow$ have oscillations ($\nu_\mu \to \nu_e \to \nu_\tau$ etc)

For 2 generations have

$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta)\, \sin^2\left(\frac{1.27\, \Delta m^2 L}{E}\right)$$

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

EW eigenstates

mass eigenstates

Quantum Chromodynamics (QCD): theory of quarks and gluons. SU(3) gauge group

Quark fields: q_α^{if} ← color, $i=1,2,3$; ← flavor index, $f=u,d,s,c,b,t$

↑ spinor index $\alpha=1,2,3,4$

A_μ^a ~ gluon fields

← color, $a=1,\dots,8$

← Lorentz index $\mu=0,1,2,3$

The Lagrangian is

$$\boxed{\mathcal{L}_{QCD} = \bar{q}^{if}\left[i\gamma\cdot D_{ij} - m_f\right]q^{jf} - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}}$$

$$D_\mu = \partial_\mu - i g A_\mu^a T^a, \qquad T^a = \frac{\lambda^a}{2}$$ ~ Gell-Mann matrices

⇒ Sum over flavors and colors assumed.

⇒ Other local non-Abelian theories in nature:

electroweak interactions (SU(2) group).

Faddeev - Popov Quantization

We want to quantize a gauge theory:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \quad \text{(consider a general non-Abelian case).}$$

The generating functional is

$$Z[0] = \int \mathcal{D} A_\mu \, e^{iS} = \int \mathcal{D} A_\mu \, e^{i\int d^4x \left(-\frac{1}{4}\right) F^a_{\mu\nu} F^{a\mu\nu}} =$$

$$= \int \mathcal{D} \bar{A}_\mu \, e^{iS} \cdot \int \mathcal{D} \Lambda$$

where $\bar{A}_\mu$ is the field in one particular gauge, Λ is the gauge transformation.

Problem: $\int \mathcal{D}\Lambda = \infty \Rightarrow Z = \infty \Rightarrow$ bad!

Even worse is the need to pick a gauge: consider abelian field A_μ: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) = \frac{1}{2} A^\mu \underbrace{[g_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu]}_{(D^{-1})_{\mu\nu}} A^\nu$

$\Rightarrow$ to find photon propagator need to solve

$$[g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu] D^{\nu\rho}(x) = \delta_\mu^\rho \, \delta^4(x)$$

$\Rightarrow$ act with $\partial^\mu \Rightarrow (\partial_\nu \partial^2 - \partial^2 \partial_\nu) D^{\nu\rho} = 0 = \partial^\rho \delta^4(x)$

$\Rightarrow$ this can not be true $\Rightarrow$ the operator has no inverse! $\Rightarrow$ no photon propagator?

However, if we choose a gauge, e.g. $\partial_\mu A_\mu = 0$

$$\Rightarrow \quad \mathcal{L} = \frac{1}{2} A^\mu \Box A^\nu \Rightarrow g_{\mu\nu} \Box D^{\nu\rho}(x) = \delta_\mu{}^\rho \delta^4(x).$$

$\Rightarrow$ easy to invert!

$\Rightarrow$ Need to fix the gauge!

Start with $\quad Z(0) = \int \mathcal{D} A_\mu \, e^{iS}$.

Insert into the integrand $\left(A_\mu^\Lambda = \Lambda A_\mu \Lambda^+ - \frac{i}{g}(\partial_\mu \Lambda)\Lambda^+\right)$

$$1 = \int \mathcal{D}\Lambda \; \delta(\Lambda - 1) = \int \mathcal{D}\Lambda \cdot \delta(G(A^\Lambda)) \det\left(\frac{\delta G(A^\Lambda)}{\delta \Lambda}\right)$$

evals at $\Lambda = 1$ due to δ-fn

where $G(A) = 0$ is the gauge condition we want to impose, e.g. $G(A) = \partial_\mu A^\mu$ for covariant gauge. Now

$$Z(0) = \int \mathcal{D} A_\mu \, e^{iS(A_\mu)} \int \mathcal{D}\Lambda \; \delta(G(A^\Lambda)) \det\left(\frac{\delta G(A^\Lambda)}{\delta \Lambda}\right)\Bigg|_{\Lambda = 1}$$

Change the order of integration & define a new field $A_\mu' = A_\mu^\Lambda$ to write (dropping the prime)

(as in QED $\mathcal{D} A_\mu = \mathcal{D} A_\mu^\Lambda$, $\underline{S(A_\mu) = S(A_\mu^\Lambda)}$) Λ = unitary