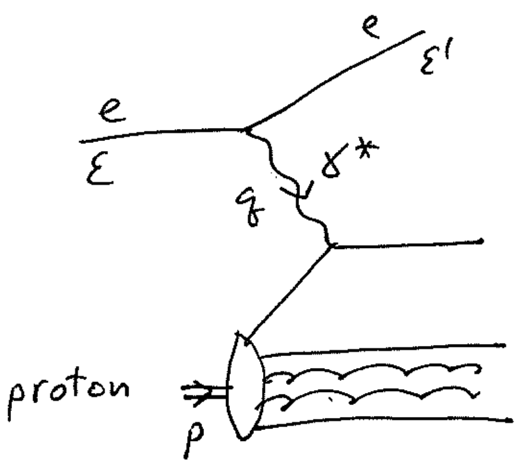
Last time | Parton Model and DIS

DIS (cont'd)



$e + p \to e + X$

$\boxed{Q^2 = -q^2}$ photon virtuality

$\boxed{x = \frac{Q^2}{2p\cdot q}}$ Bjorken x

$$\boxed{\frac{d\sigma}{d^3k'} = \frac{\alpha_{EM}^2}{Q^4 \mathcal{E}\mathcal{E}'} l_{\mu\nu} W^{\mu\nu}}$$ ← rest frame of the proton

$$l_{\mu\nu} = 2\left(k_\mu k_\nu' + k_\nu k_\mu' - g_{\mu\nu} k\cdot k'\right)$$ leptonic tensor (put $m_e = 0$)

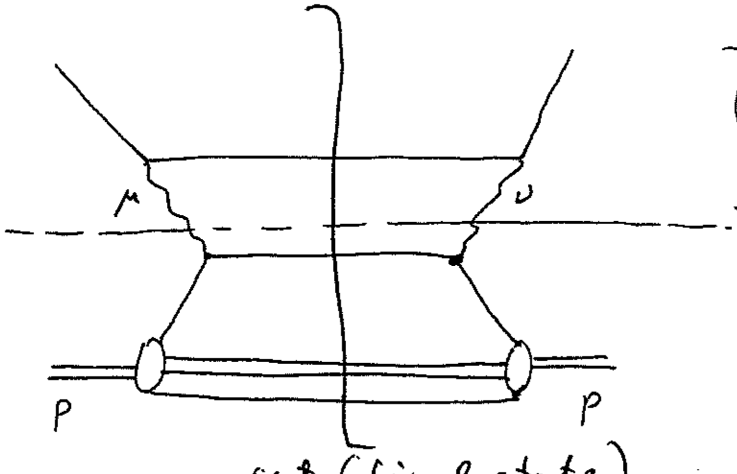
$$W_{\mu\nu} = \frac{1}{4\pi m_p}\int d^4x\, e^{iq\cdot x} \langle p | j_\mu(x)\, j_\nu(0) | p\rangle$$

hadronic tensor

$$j_\mu(x) = \sum_f e_f\, \bar{q}_f(x)\gamma_\mu q_f(x)$$ ~ EM current

$$\boxed{W_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x\, e^{iq\cdot x} \langle p | j_\mu(x)\, j_\nu(0) | p \rangle}$$

(averaged over σ) ← proton polarization



$\} \; l_{\mu\nu}$ (EM part)

$\} \; W_{\mu\nu}$ (hadronic part)

cut (final state)

$$W_{\mu\nu}(p,q) = A(x,Q^2)\, p_\mu p_\nu + B(x,Q^2)\, q_\mu q_\nu + C(x,Q^2)\, g_{\mu\nu} +$$

$$+ D(x,Q^2)\,(p_\mu q_\nu + p_\nu q_\mu) + E(x,Q^2)\,(p_\mu q_\nu - p_\nu q_\mu) +$$

$$+ F(x,Q^2)\, \varepsilon_{\mu\nu\rho\sigma}\, p^\rho q^\sigma$$

$F = 0$ in $\gamma^* p$, $\gamma^* A$ (F comes from γ_5's, appears in ν DIS).

(1) $q_\mu W^{\mu\nu} = 0$ (current conservation)

$q_\mu \to \partial_\mu \Rightarrow \partial_\mu j^\mu(x) = 0$

$$A\, p_\nu\, (p\cdot q) + B\, q_\nu \cdot q^2 + C\, q_\nu + D\,(p\cdot q\, q_\nu + q^2 p_\nu)$$

$$+ E\,(p\cdot q\; q_\nu - q^2 p_\nu) = 0$$

(2) $q_\nu W^{\mu\nu} = 0 = A\, p_\mu (p\cdot q) + B\, q^2 q_\mu + C q_\mu + D(p\cdot q\; q_\mu +$

$$+ q^2 p_\mu) + E\,(p_\mu q^2 - p\cdot q\; q_\mu) = 0$$

$(1) - (2) = 0 \Rightarrow (E = 0.)$

p_μ and q_μ are independent $\Rightarrow$

$$0 = A\, p\cdot q + D q^2 \qquad 0 = B q^2 + C + D\, p\cdot q$$

$$D = -A \frac{p\cdot q}{q^2} \qquad B = -\frac{1}{q^2} C + A\left(\frac{p\cdot q}{q^2}\right)^2$$

$$W_{\mu\nu} = A\left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu\right] + C\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]$$

Usually one writes

$$W_{\mu\nu} = -W_1(x, Q^2)\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2(x, Q^2)}{m_p^2} \cdot \left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu\right].$$

W_1 & W_2 are <u>structure</u> <u>functions</u> (Def.)

Using $q_\mu l^{\mu\nu} = q_\nu l^{\mu\nu} = 0$ yields

$$l_{\mu\nu} W^{\mu\nu} = -W_1 (-4\underbrace{k\cdot k'}_{2\varepsilon\varepsilon'\sin^2\frac{\theta}{2}}) + \frac{2W_2}{m_p^2}\underbrace{\left[2\, p\cdot k\; p\cdot k' - m^2 k\cdot k'\right]}_{2m^2\varepsilon\varepsilon' - 2m^2\varepsilon\varepsilon'\sin^2\frac{\theta}{2} = 2m^2\varepsilon\varepsilon'\cos^2\frac{\theta}{2}}$$

$$q_\mu l^{\mu\nu} = (k-k')_\mu \, 2\left(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k'\right) =$$

$$= 2\left(k^2 k'^\nu + \cancel{k^\nu k\cdot k'} - \cancel{k^\nu k\cdot k'} - \cancel{k'\cdot k\, k'^\nu} - k^\nu k'^2\right.$$

$$\left. + \cancel{k'^\nu k\cdot k'}\right) = 2\left(k^2 k'^\nu - k'^2 k^\nu\right) \simeq 0 \quad \text{as } k^2 \simeq k'^2 \simeq 0$$

(neglect electron's mass), $q_\nu l^{\mu\nu} = 0$ (similar)

$$\Rightarrow l_{\mu\nu} W^{\mu\nu} = l^{\mu\nu}\left[-W_1\left(g_{\mu\nu} - \cancelto{0}{\frac{q_\mu q_\nu}{q^2}}\right) + \frac{W_2}{m_p^2}\cdot\right.$$

$$\left.\left(p_\mu p_\nu - \frac{p\cdot q}{q^2}\left(p_\mu \cancelto{0}{q_\nu} + p_\nu \cancelto{0}{q_\mu}\right) + \left(\frac{p\cdot q}{q^2}\right)^2 \cancelto{0}{q_\mu q_\nu}\right)\right]$$

$$= -l^\mu{}_\mu W_1 + \frac{W_2}{m_p^2} p_\mu p_\nu l^{\mu\nu} = \left| \begin{matrix} \text{as } l^{\mu\nu} = 2(k^\mu k'^\nu + \\ + k^\nu k'^\mu - g^{\mu\nu} k\cdot k') \end{matrix} \right.$$

$$= -2\left(2k\cdot k' - \underbrace{4}_{= g^\mu{}_\mu} k\cdot k'\right) W_1 + \frac{W_2}{m_p^2} 2\left(2p\cdot k\; p\cdot k' - p^2 k\cdot k'\right)$$

$$= 4 k\cdot k' W_1 + 2\frac{W_2}{m_p^2}\left(2p\cdot k\; p\cdot k' - m_p^2 k\cdot k'\right)$$

remember: $k = (\varepsilon, 0, 0, \varepsilon)$, $k' = (\varepsilon', \varepsilon'\sin\theta, 0, \varepsilon'\cos\theta)$

$p = (m_p, \vec{0})$

$\Rightarrow k\cdot k' = 2\varepsilon\varepsilon' \sin^2(\theta/2)$; $p\cdot k = m_p \varepsilon$, $p\cdot k' = \varepsilon' m_p$

$$l_{\mu\nu} W^{\mu\nu} = 4 \varepsilon \varepsilon' \left[2 W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

$$\frac{d\sigma}{d^3k'} = \frac{4 \alpha_{EM}^2}{Q^4} \left[2 W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

By varying the angle θ can separate W_1 & W_2 contributions in experiments. $\Rightarrow$ Rosenbluth separation

Usually one defines $F_1(x, Q^2) = m_p W_1(x, Q^2)$, $F_2(x, Q^2) = \nu\, W_2(x, Q^2)$

The Parton Model.

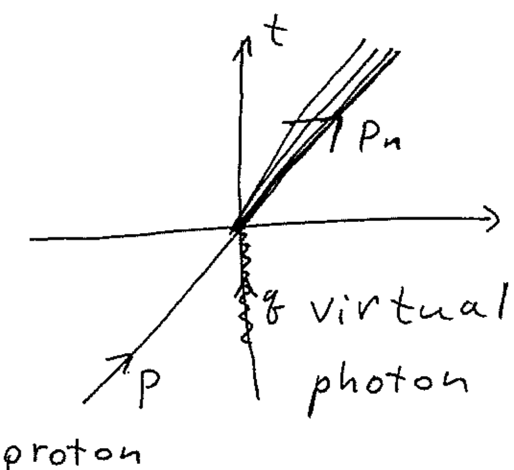
Sterman ch 14, Peskin 17.5
Y.K. & Levin, ch. 2.2

Go to Infinite Momentum Frame:

$$p_\mu \approx \Big(\underset{0}{p + \frac{m^2}{2p}}, \underset{1}{0}, \underset{2}{0}, \underset{3}{p} \Big),$$

$\underline{q} = (q^1, q^2) \sim$ 2d vector in transverse plane

$$q_\mu = \Big(\underset{0}{q_0}, \underset{1,2}{\underline{q}}, \underset{3}{0} \Big),$$



Q^2 and x are 2 invariants

large, $Q \gg \Lambda_{QCD}$

$$p \cdot q = m\nu = q_0 \cdot p$$

$$\Rightarrow q_0 = \frac{m\nu}{p} \sim \text{small as } p \text{ goes large, } p \gg Q$$

$\Rightarrow$ as $\nu = \dfrac{Q^2}{2m_p x}$ $\Rightarrow q_0 = \dfrac{Q^2}{2xp}$

$$\Rightarrow Q^2 = -q^2 = \underline{q}^2$$

$\Rightarrow q_0 \ll Q$ since $xp \gg Q$.

$$F_1(x, Q^2) = m_p W_1(x, Q^2)$$

$$F_2(x, Q^2) = \nu W_2(x, Q^2) = \frac{Q^2}{2 m_p x} W_2(x, Q^2)$$

$$\frac{d\sigma}{d^3k'} = \frac{4\alpha_{EM}^2}{Q^4}\left[2 \cdot \frac{1}{m_p} F_1(x, Q^2) \sin^2\frac{\theta}{2} + \frac{2 m_p x}{Q^2} F_2(x, Q^2) \cdot \cos^2(\theta/2)\right]$$

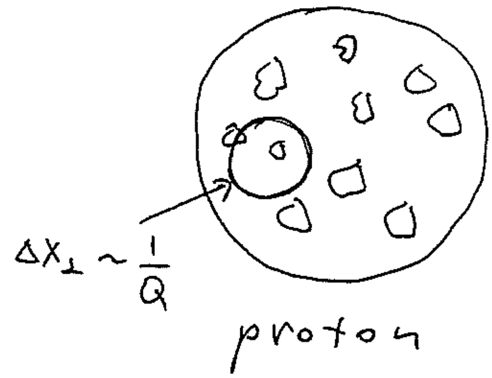
F_1, F_2 are dimensionless

$Q^2 = q^2 \Rightarrow$ photon acts like a <u>microscope</u>

in <u>transverse</u> plane:

$\Delta x_\perp \cdot q_\perp \sim 1 \qquad (\hbar = 1)$

$\Delta x_\perp \sim \frac{1}{q_\perp} \sim \frac{1}{Q}$

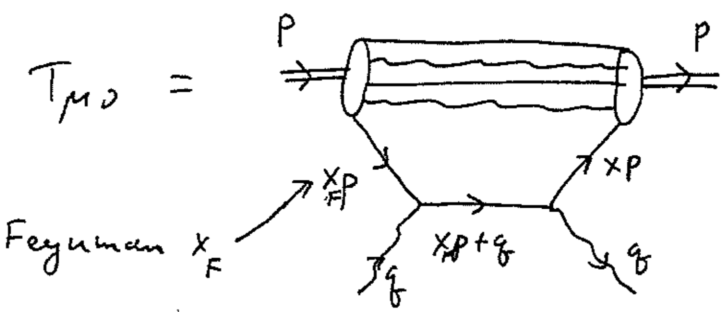


large Q ~ resolve just 1 quark

Define $$T_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x \, e^{iq\cdot x} \frac{1}{2}\sum_\sigma \langle p,\sigma | T j_\mu(x) j_\mu(0) | p,\sigma\rangle$$

$$W_{\mu\nu} = 2\,\mathrm{Im}\,(i\,T_{\mu\nu}) \qquad \text{(optical theorem)}$$

$T_{\mu\nu} =$



"Forward Amplitude"

Def Feynman-x: the fraction of proton's longitudinal momentum carried by struck quark

typical interaction time in proton's rest frame is $\frac{1}{\Lambda_{QCD}} \Rightarrow$ boost to get $\frac{p}{m}\frac{1}{\Lambda} \equiv \tau_\Lambda$

int. time of DIS is $\tau_{DIS} \simeq \frac{1}{q^0}$, where

$q^0 \simeq \frac{Q^2}{2x_B p}$ $\left(= \frac{m\nu}{p} \leftarrow \text{as } m\nu = \frac{Q^2}{2x}\right)$ is struck quark's velocity: $\tau_{DIS} \simeq \frac{2xp}{Q^2}$.

time-ordered product: (denoted T)

$$\boxed{T\, j_\mu(x)\, j_\nu(y) \equiv \Theta(x^0-y^0)\, j_\mu(x)\, j_\nu(y) + \Theta(y^0-x^0)\, j_\nu(y)\, j_\mu(x)}$$

note: currents do not commute with each other in general $\Rightarrow$ not a trivial object.

$$2\,\mathrm{Im}\left(i\,T_{\mu\nu}\right) = 2\,\mathrm{Im}\Big[i \cdot \frac{1}{4\pi m_p}\int d^4x\, e^{iq\cdot x}\langle p|\,\Theta(x^0)\, \underbrace{j_\mu(x)}^{e^{ip\cdot x} j_\mu(0) e^{-ip\cdot x}} \underbrace{\;}_{\sum_n |n\rangle\langle n|} j_\nu(0) +$$

$$+\,\Theta(-x^0)\, j_\nu(0)\, j_\mu(x)\,|p\rangle\Big] = 2\cdot\frac{1}{4\pi m_p}\sum_n \mathrm{Re}\Big\{\int d^4x\, e^{iq\cdot x + ip\cdot x - ip_n\cdot x}$$

$$\cdot\,\Theta(x^0)\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle + \int d^4x\, e^{iq\cdot x + ip_n\cdot x - ip\cdot x}\,\Theta(-x^0)$$

$$\cdot\,\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\Big\} = \sum_n 2\,\frac{1}{4\pi m_p}\Big\{(2\pi)^3\,\delta(\vec q+\vec p-\vec p_n)\cdot \mathrm{Re}$$

$$\left(\frac{-1}{i(q^0+p^0-p_n^0+i\varepsilon)}\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle\right) + (2\pi)^3\,\delta(\vec q+\vec p_n-\vec p)\cdot$$

$$\cdot\,\mathrm{Re}\left(\frac{1}{i(q^0+p_n^0-p^0-i\varepsilon)}\,\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\right)\Big\}$$

$\propto \delta(q^0+p_n^0-p^0)$

$\overset{\shortparallel}{0}$ not physical $\Rightarrow$ drop (after including $\delta(q^0+p_n^0-p^0)$)

$$= 2\,\frac{1}{4\pi m_p}\sum_n (2\pi)^3\,\delta(\vec q+\vec p-\vec p_n)\,(-)\left(\mathrm{Im}\,\frac{1}{q^0+p^0-p_n^0+i\varepsilon}\right)\cdot\langle p|j_\mu(0)|n\rangle$$

$$\langle n|j_\nu(0)|p\rangle = \Big|\text{as } \mathrm{Im}\,\frac{1}{x+i\varepsilon} = -\pi\,\delta(x) = \frac{1}{4\pi m_p}\sum_n (2\pi)^4\,\delta^4(q+p-p_n)$$

$$\cdot\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle = W_{\mu\nu} \quad \text{as desired.}$$