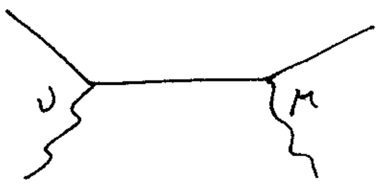
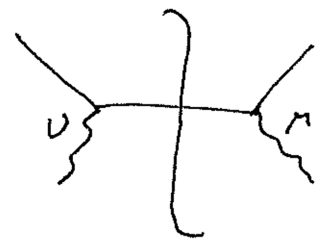
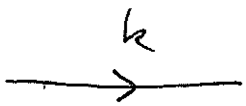
Last time | The Parton Model (cont'd)

We discussed the optical theorem in terms of its manifestation in DIS: $\boxed{W_{\mu\nu} = 2\,\mathrm{Im}\left(i\,T_{\mu\nu}\right)}$

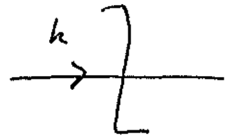
$T_{\mu\nu} =$ 

$W_{\mu\nu} =$ 

e.g. scalar propagator:



$\Rightarrow$

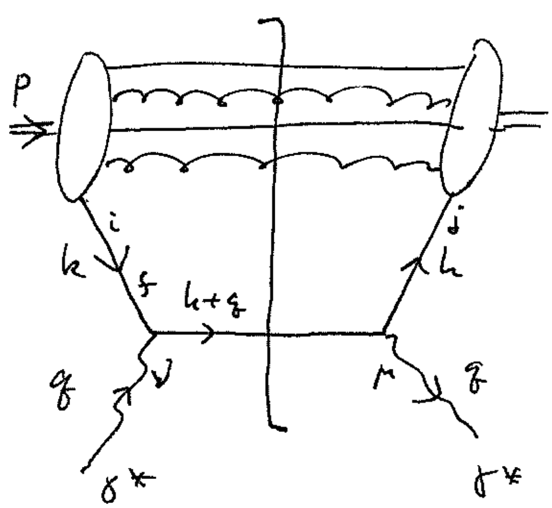


$$\frac{i}{k^2-m^2+i\varepsilon} \qquad\qquad 2\pi\,\delta^{(+)}(k^2-m^2)$$

$$= 2\pi\,\theta(k^0)\,\delta(k^2-m^2)$$

$\uparrow$ positive energy.

Indeed, $$2\,\mathrm{Im}\left(i\,\frac{i}{k^2-m^2+i\varepsilon}\right) = 2\pi\,\delta(k^2-m^2)$$

$W_{\mu\nu} =$ 

$\Big\}\ A^f_{\underbrace{ij}_{\text{Dirac indices}}}(p,k)$

We got

$$W_{\mu\nu} = \frac{1}{2m_p} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ij}(p,k) \left[\gamma_\mu (\not{k}+\not{q}) \gamma_\nu \right]_{ji} \cdot \delta\left((k+q)^2 \right)$$

Using $Q^2 \gg k^2, \underline{k} \cdot q$ and $k^+ \gg k^-$ we got

$$\delta\left((k+q)^2\right) \simeq \frac{x}{Q^2}\, \delta\left(x - \frac{k^+}{p^+}\right)$$

$\Rightarrow$ Bjorken x is the light-cone momentum fraction of the struck quark!

if x is small ($\lesssim 1$) and Q is large

$\Rightarrow$ $\tau_{DIS} \ll \tau_\Lambda$ interaction is "instantaneous"

as $\frac{2xp}{Q^2} \ll \frac{p}{m}\frac{1}{\Lambda} \Rightarrow$ as $2xm\Lambda < 2m\Lambda \ll Q^2$

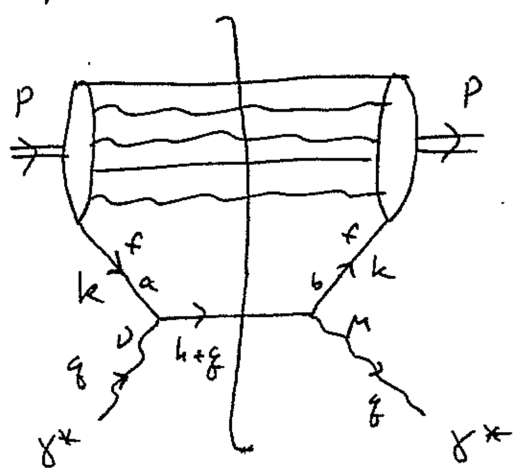
Define light cone variables:

for vector V^μ one has $V^+ = \frac{1}{\sqrt{2}}(V^0 + V^3)$

$\underline{V} = (V^1, V^2)$ $\qquad V^- = \frac{1}{\sqrt{2}}(V^0 - V^3)$

(2d transverse vector)

$$V_1 \cdot V_2 = V_{1\mu} V_2^\mu = V_{1+} V_{2-} + V_{1-} V_{2+} - \underline{V}_1 \cdot \underline{V}_2$$

$W_{\mu\nu} =$ 

$\} A^f_{ab}(p,k)$ Dirac indices

e.g.

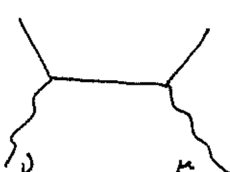
$p^+ = \frac{p}{\sqrt{2}}$

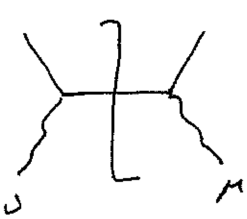
$p^- = \frac{m^2}{2\sqrt{2}p}$

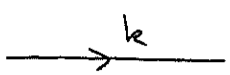
$p^+ \gg p^-$

as $W_{\mu\nu} = 2\,\mathrm{Im}(i\,T_{\mu\nu})$

cut (Im part of a propagator)

$T_{\mu\nu} =$ 

$W_{\mu\nu} =$ 



$\frac{i}{k^2 - m^2 + i\varepsilon}$ $\Rightarrow$ 

$2\pi\delta^{(+)}(k^2 - m^2)$

as $2\,\mathrm{Im}\frac{-1}{k^2 - m^2 + i\varepsilon} = +2\pi\delta^{(+)}(k^2 - m^2)$

We write

$$W_{\mu\nu} = \frac{1}{\cancel{4\pi} 2 m_p} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k) \left[\gamma_\mu \gamma \cdot (k+q) \gamma_\nu\right]_{ba}$$

$$\cdot \cancel{(2\pi)}\, \delta\left((k+q)^2\right)$$ where A^f_{ab} is the rest of the diagram (see p. 90).

Start calculating assuming that

$$Q^2 \gg k^2,\ \underline{k}\cdot\underline{q} \quad , \quad k^+ \gg k^- \ (\text{IMF})$$

$$\therefore \quad (k+q)^2 = k^2 + 2k^+q^- + 2k^-q^+ - \underline{k}\cdot\underline{q} - Q^2$$

$q_3 = 0 \Rightarrow q^+ = q^- \Rightarrow$ as $k^+ \gg k^- \Rightarrow$ drop $2k^-q^+$

dropping $k^2,\ \underline{k}\cdot\underline{q} \ll Q^2$ get

$$(k+q)^2 \approx 2k^+q^- - Q^2$$

$$\Rightarrow \delta\left((k+q)^2\right) \approx \delta\left(2k^+q^- - Q^2\right) = \delta\left(\frac{k^+}{p^+} 2p^+q^- - Q^2\right)$$

as $p\cdot q \approx p^+q^- \Rightarrow$ and $x_{Bj} = \dfrac{Q^2}{2p\cdot q}$

$$\Rightarrow \delta\left((k+q)^2\right) \approx \frac{x_{Bj}}{Q^2} \delta\left(x_{Bj} - \frac{k^+}{p^+}\right)$$

$$\Rightarrow \quad x_{Bj} = \frac{k^+}{p^+}$$ Feynman x = Bjorken x

physical meaning: light cone momentum fraction of the struck quark!

$$\gamma \cdot (k+q) = \gamma^+ (k^- + q^-) + \gamma^-(k^+ + q^+) - \underline{\gamma} \cdot (\underline{k} + \underline{q})$$

after d^4k : $\gamma^+ \to p^+$ $\quad \gamma^- \to p^-$ $\quad \underline{\gamma} \to \underline{p} = 0$

$\Rightarrow$ as $p^+ >> p^-$ keep γ^+ only, $k^- + q^- \simeq \dfrac{Q^2}{x \cdot 2p_+}$

$$'' \frac{(\underline{k}+\underline{q})^2}{2(k^+ + q^+)} \simeq \frac{\underline{q}^2}{2q^+} = \frac{Q^2}{2xp^+}$$

$$W_{\mu\nu} = \frac{1}{4 m_p p_+} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k) \left[\gamma_\mu \gamma^+ \gamma_\nu\right]_{ba} \cdot \delta\left(x - \frac{k^+}{p^+}\right)$$

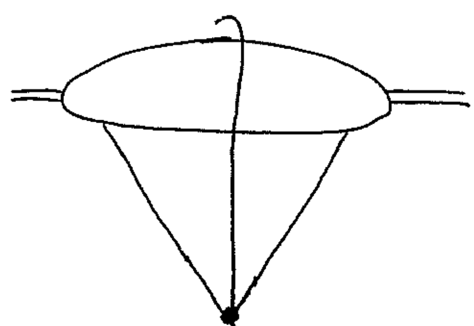
(see $p^\mu q^\nu$ decomp.)

symmetrize, as $W_{\mu\nu}$ is symmetric

Concentrate on $W_{ij} \sim \frac{1}{2}\left[\gamma_i \gamma^+ \gamma_j + \gamma_j \gamma^+ \gamma_i\right] =$

$= -\frac{1}{2} \gamma^+ \{\gamma_i, \gamma_j\} = -g_{ij} \gamma^+$ $\quad$ (we used $W_{ij} = W_{ji}$)

DIS now looks like



$\gamma^+ \delta\left(x - \frac{k^+}{p^+}\right)$.

(Mueller vertex)

We have $W_{ij} \propto g_{ij}$ from diagram calculations.

On the other hand, since $\underline{p} = 0$

$$W_{ij} = -W_1\left(g_{ij} - \frac{q_i q_j}{q^2}\right) + \frac{W_2}{m_p^2} q_i q_j \left(\frac{p \cdot q}{q^2}\right)^2 =$$

$$= -W_1 g_{ij} + \frac{q_i q_j}{q^2}\left[W_1 + \frac{W_2}{m_p^2} \frac{(p\cdot q)^2}{q^2}\right] \propto g_{ij}$$

$$\Rightarrow \quad W_1 + \frac{W_2}{m_p^2}\frac{(p\cdot q)^2}{q^2} = 0$$

as $\nu = \frac{p\cdot q}{m_p}$ and $x = \frac{Q^2}{2p\cdot q} = -\frac{q^2}{2p\cdot q}$

we write $\boxed{\nu W_2 = 2 m x W_1}$ Callan-Gross Relation '69

follows from spin-$\frac{1}{2}$ nature of quarks!

(would be different for particles with different spin); equivalently: $\boxed{F_2(x,Q^2) = 2x F_1(x,Q^2)}$

Exercise: show that Callan-Gross relation leads to $\frac{d\sigma}{d^3k'} \sim \left[1 + \left(1 - \frac{\nu}{\varepsilon}\right)^2\right] W_1$

CG relation leads to

$$\nu W_2 = 2m_p x W_1 = \cancel{2}\cancel{m_p} x \cdot \frac{1}{\cancel{2}\cancel{m_p} 2p_+} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k) \cdot$$

$$\cdot (\gamma^+)_{ba}\, \delta\left(x - \frac{k^+}{p^+}\right) \Rightarrow$$ <u>defining</u> <u>quark distribution function</u>:

$$\boxed{q^f(x) \equiv \frac{1}{2p^+}\int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k)\,(\gamma^+)_{ba}\,\delta\left(x - \frac{k^+}{p^+}\right)}$$

we get $\boxed{\underset{(x,Q^2)}{\nu W_2} = \sum_f e_f^2\, x\, q^f(x)}$ $= F_2$

no Q^2-dependence, only x-dependent

<u>Bjorken scaling</u> (see attached)

Bjorken scaling was first measured at SLAC in 1968: it killed string models and brought back field theories.

$$\boxed{\begin{aligned} F_2(x) &= \sum_f e_f^2 \, x \, q_f(x) \\ F_1 &= \frac{F_2}{2x} = \frac{1}{2} \sum_f e_f^2 \, q_f(x) \end{aligned}}$$

F_1 = counts # of quarks in the proton with the longitudinal momentum fraction $= x$ (weighed by $\frac{1}{2} e_f^2$)

F_2 = gives the average x carried by quarks (weighed by e_f^2) $\otimes$ # of quarks at x.

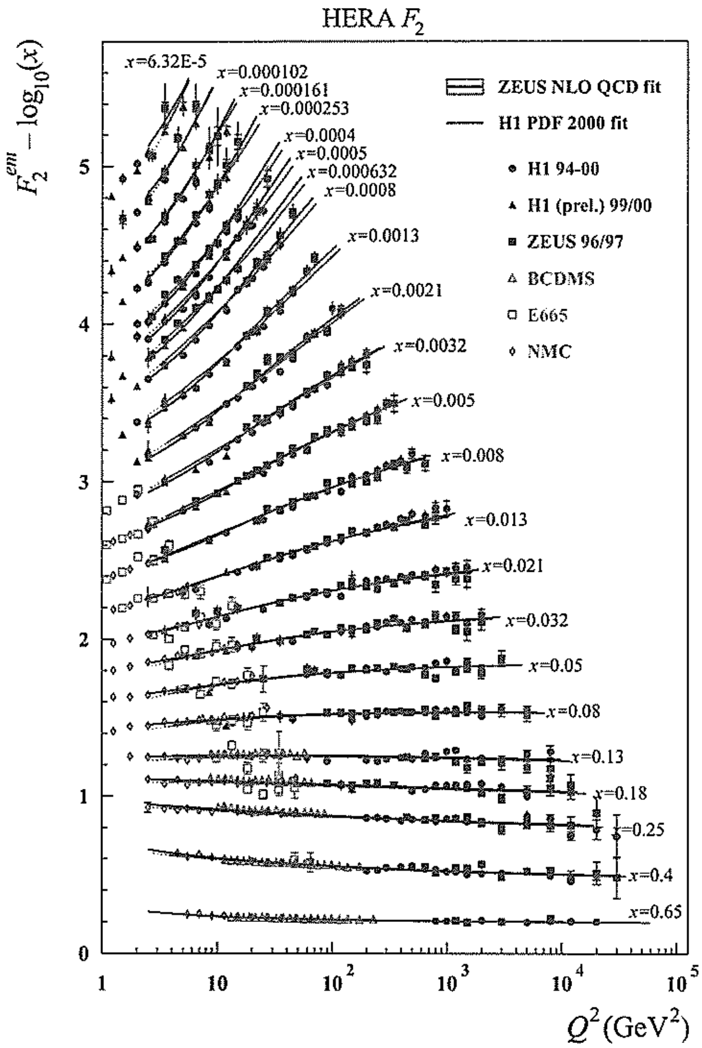
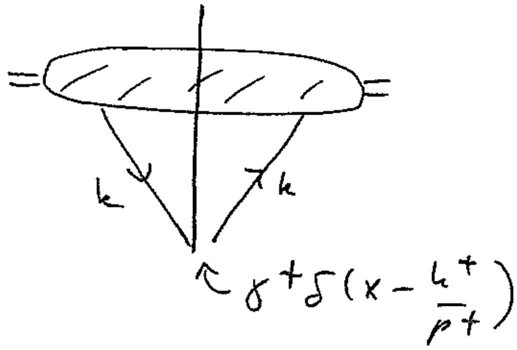


Fig. 2.7. Compilation of the world F_2 data for DIS on a proton. The proton F_2 structure function is plotted as a function of Q^2 for a range of values of x, as indicated next to the data. It can be seen that, except for very small x, F_2 is independent of Q^2, a manifestation of Bjorken scaling. (We thank Kunihiro Nagano for providing us with this figure.) A color version is available online at www.cambridge.org/9780521112574.

In Fig. 2.7 we show a summary of the world knowledge of the proton F_2 structure function. This structure function is plotted as a function of Q^2 for many different fixed values of Bjorken-x. One can clearly see that, when x is not too small, F_2 is independent of Q^2. This is the experimental manifestation of Bjorken scaling. We see that the theory we

$q^f(x) = \frac{1}{2p^+}$ 

$\Rightarrow$ often $p^f(x)$ is denoted $q(x)$.

$>$ $q^f(x, Q^2)$ counts # of quarks with light cone momentum x and transverse momentum $k_T \lesssim Q$.

parton distribution function $\left(q^f \sim a^+ a \right)$

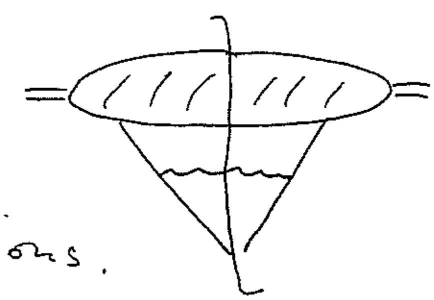
$\Rightarrow$ for a free quark $A^f_{ab}(p,k)(\gamma^+)_{ba} = \delta^4(p-k)\cdot(2\pi)^4$.

$\underbrace{\bar{u}_b(p)(\gamma^+)_{ba} u_a(p)}_{= 2p^+} = 2p^+ (2\pi)^4 \delta^4(p-k)$ $\Rightarrow$ $\boxed{q^f_{quark}(x) = \delta(x-1)}$ plug in.

one quark at $x=1$

'eskin, ch. 17.5
terman ch 14
'K, Levin ch. 2.4

QCD Improved Parton Model: DGLAP equation

How about corrections like  ?

These are important corrections.

However, let us first discard the negligible diagrams like