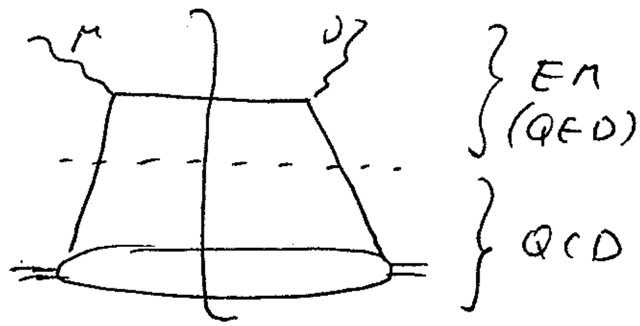
Particle Production in High Energy Hadronic Collisions.

Collinear Factorization

When we considered DIS above, we have factorized EM & QCD parts of the diagram:

We got

$$F_2(x, Q^2) = \sum_f e_f^2 x q^f(x)$$

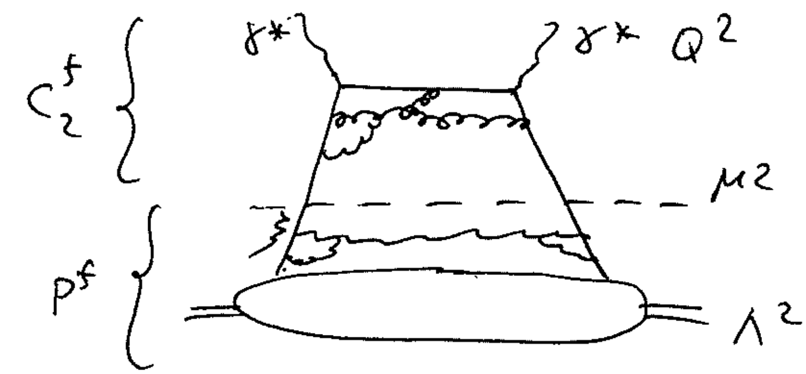


$\Rightarrow$ this is an example of <u>collinear factorization</u>

In general one writes:

$$F_2(x, Q^2) = \sum_{f, \bar{f}, \text{gluons}} \int_0^1 d\xi \; C_2^f\left(\frac{x}{\xi}, Q^2, \mu^2\right) \, p^f(\xi, \mu^2) + \underbrace{O\left(\frac{m^2}{Q^2}\right)}_{\text{higher twists}}$$

C_2^f ~ coefficient ftn (may contain QCD corrections at higher orders)



$p^f = \{q^f, q^{\bar{f}}, G\}$; ξ – momentum fraction of the parton in p^f.

μ^2 is called factorization scale.

<u>Note:</u> C_2^f is <u>perturbatively calculable</u>, p^f is not (though one has DGLAP for p^f)

$\Lambda^2 \lesssim \mu^2 \lesssim Q^2$ => but F_2 does not depend on μ^2, it is arbitrary => $\mu^2 \frac{d}{d\mu^2} F_2(x, Q^2) = 0$

Write $F_2 = C_2^f \otimes p^f$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = 0 = \left(\mu^2 \frac{d}{d\mu^2} C_2^f\right) \otimes p^f + C_2^f \otimes \mu^2 \frac{d}{d\mu^2} p^f$$

=> what happens (separation of variables, only C_2 depends on Q^2, only p^f depends on Λ^2):

$$\mu^2 \frac{d}{d\mu^2} P_f = \gamma(\alpha_s) \otimes P_f$$

~ DGLAP evolution

γ ~ splitting function.

$$\mu^2 \frac{d}{d\mu^2} C_2^f = -\gamma(\alpha_s) \otimes C_2^f$$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = -\gamma \otimes C_2^f \otimes p^f + C_2^f \otimes \gamma \otimes p^f = 0.$$

as desired.

=> can "place" corrections into PDF or coefficient function

Collinear factorization in DIS is a theorem which can be proven $\Rightarrow$ must be right! (at large-Q^2 only!)

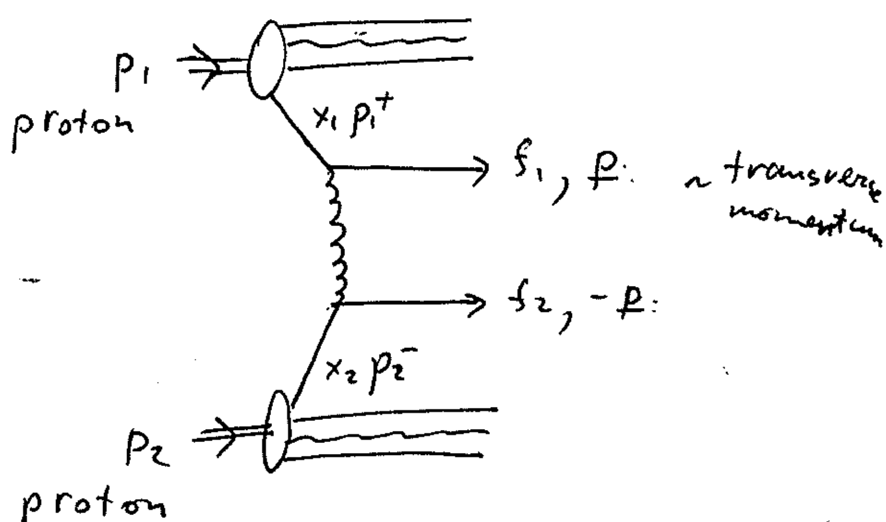
~ at LO have $C_2^f = \delta\left(\frac{x}{\xi} - 1\right) e_f^2$, f = quarks only

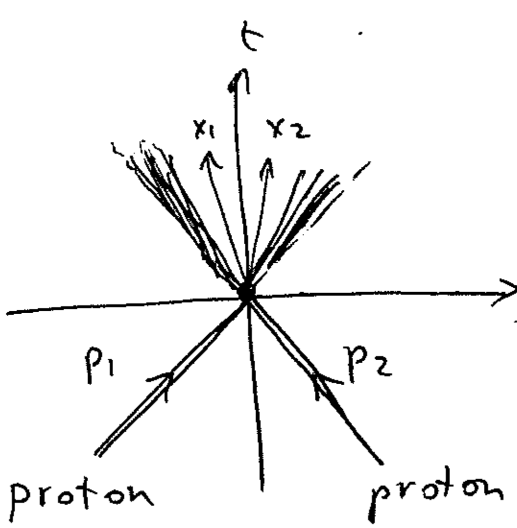
$$\Rightarrow F_2(x, Q^2) = \sum_f \underbrace{\int_0^1 d\xi \, \delta\left(\frac{x}{\xi} - 1\right)}_{x} e_f^2 \, q^f(\xi)$$

$$= \sum_f e_f^2 \, x q^f(x) \quad \text{as expected!}$$

Jet Production in Hadronic Collisions.

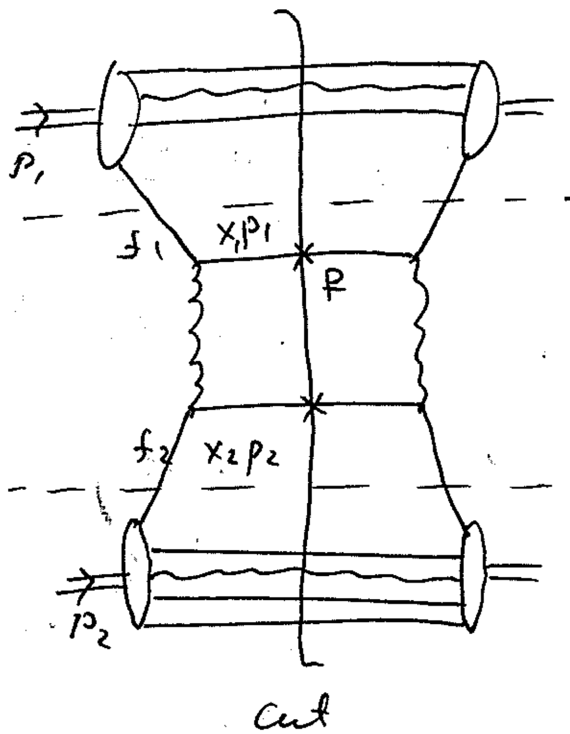
Collinear factorization also applies to hadron-hadron collisions. Consider quark production:





~ collision happens very fast on proton's time scales $\Rightarrow$ factorization.

Square the diagram:



$\Big\}$ $p^{f_1}(x_1, \mu^2)$ ~ parton distribution

$\Big\}$ $\mathcal{E}_p \dfrac{d\hat{\sigma}_{f_1 f_2 \to f_1' f_2'}}{d^3 p}$ ~ "hard" cross section

$\Big\}$ $p^{f_2}(x_2, \mu^2)$ ~ parton distribution

The collinear factorization formula then reads:

$$\mathcal{E}_p \frac{d\sigma}{d^3 p} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \; p^{f_i}(x_1, \mu^2) \cdot \mathcal{E}_p \frac{d\hat{\sigma}_{f_i f_j \to f_i' f_j'}}{d^3 p} \cdot p^{f_j}(x_2, \mu^2)$$

Usually put $\mu^2 = p_T^2$ for large p_T jets (or hadrons)

After the collision quarks (gluons) that are produced get dressed by further emissions. But the flow of energy is not likely to be modified much by those. (Still people construct other IR-safe observables insensitive to late-time emissions: (— + ... + ...))

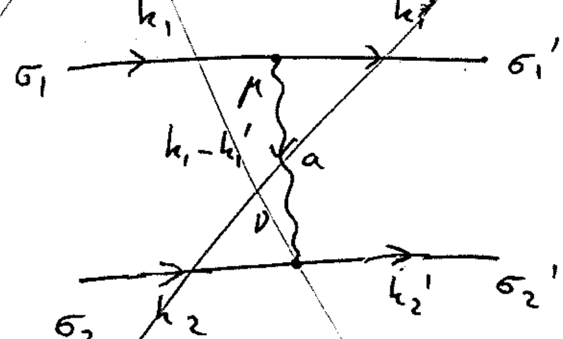
Example | Quark jet production (coming from quarks). Replace $p^{f_i} \to q^f$ $\Rightarrow$ write

$$\varepsilon_p \frac{d\sigma}{d^3p} = \sum_{f_1,f_2} \int_0^1 dx_1\, dx_2\; q^{f_1}(x_1,p_T^2)\; \varepsilon_p \frac{d\hat{\sigma}_{f_1 f_2 \to f_1' f_2'}}{d^3p}\; q^{f_2}(x_2,p_T^2)$$

q^f ~ to be found from DGLAP (PDF data)

We can calculate the hard cross section:

$$d\hat{\sigma} = \frac{1}{2E_1\; 2E_2 \cdot \underset{\underset{|\vec{v}_1-\vec{v}_2|}{\shortparallel}}{2} \cdot} \; \frac{d^3k_1'}{(2\pi)^3 2E_1'} \cdot$$



$$\cdot \frac{d^3k_2'}{(2\pi)^3 2E_2'} (2\pi)^4 \delta^{(4)}(k_1+k_2-k_1'-k_2') \cdot$$

$$\cdot |M|^2 \left[\delta^3(\vec{k}_1'-\vec{p}) + \delta^3(\vec{k}_2'-\vec{p})\right] \cdot d^3p$$

measured jet can be either quark!

$$\Rightarrow \varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \,?$$

$\delta^{(3)}$ kills d^3k_2' kills d^3k_1'

$$\frac{d^3k_1'}{(2\pi)^3 2E_1'} \frac{d^3k_2'}{(2\pi)^3 2E_2'} (2\pi)^4 \delta^{(4)}(k_1+k_2-k_1'-k_2')\; \delta^3(\vec{k}_1'-\vec{p}) =$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4E_p E_2'} \delta(E_1+E_2-E_p-E_2')$$