Particle Production in High Energy

Hadronic Collisions.

Collinear Factorization

When we considered DIS above, we have factorized EM & QCD parts of the diagram:

\[ F_2(x, Q^2) = \sum_{\pm} e_{\pm}^2 x_{\pm} f_{\pm}(x) \]

\[ \text{(EN)} \quad \text{(QED)} \quad \{ \text{QCD} \} \]

\[ \Rightarrow \text{this is an example of collinear factorization} \]

In general one writes:

\[ F_2(x, Q^2) = \sum_{\frac{1}{2}, \frac{3}{2}, \text{gluons}} \int d\frac{\sigma}{\mu^2} C_2^f \left( \frac{\zeta}{\bar{s}}, \frac{Q^2}{\zeta^2}, \frac{m^2}{Q^2} \right) p_f^s \left( \frac{\Delta^2}{\zeta^2} \right) + o \left( \frac{m^2}{Q^2} \right) \]

\[ C_2^f \text{ coefficient ftw} \]

\[ \text{(may contain QCD corrections at higher orders)} \]

\[ p_f^s = \{ q_f^s, s_f^s, q_s^s \}; \frac{\Delta}{s} \text{- momentum fraction of the parton in } p_f^s. \]
$m^2$ is called factorization scale.

Note: $C_2$ is perturbatively calculable, $p^f$ is not (though one has DGLAP for $p^f$)

$\Lambda^2 \lesssim m^2 \lesssim Q^2 \Rightarrow$ but $F_2$ does not depend on $m^2$, it is arbitrary $\Rightarrow m^2 \frac{d}{dm^2} F_2(x, Q^2) = 0$

Write $F_2 = C_2^f \otimes p^f$

$\Rightarrow m^2 \frac{d}{dm^2} F_2 = 0 = \left( m^2 \frac{d}{dm^2} C_2^f \right) \otimes p^f + C_2^f \otimes m^2 \frac{d}{dm^2} p^f$

$\Rightarrow$ what happens (separation of variables, $C_2^f$ depends on $Q^2$, only $p^f$ depends on $\Lambda^2$):

\[ m^2 \frac{d}{dm^2} p^f = \Delta (x) \otimes p^f \sim \text{DGLAP evolution} \]

\[ m^2 \frac{d}{dm^2} C_2^f = -\Delta (x) \otimes C_2^f \]

$\Rightarrow m^2 \frac{d}{dm^2} F_2 = -\Delta \otimes C_2^f \otimes p^f + C_2^f \otimes \delta \otimes p^f = 0$

as desired.

$\Rightarrow$ can "place" corrections into PDF or coefficient function.
Collinear factorization in DIS is a theorem which can be proven -> must be right! (at large - Q^2 only!)

At LO have \( C_2^f = 8 \left( \frac{x}{Q^2} - 1 \right) e_f^2 \), \( f = \text{quark only} \)

\[
\Rightarrow F_2(x, Q^2) = \sum_f \int_0^1 \left( \frac{x}{Q^2} - 1 \right) e_f^2 g_f^x \left( \frac{x}{Q^2} \right)
\]

\[
= \sum_f e_f^2 x g_f^x \text{ as expected!}
\]

Jet Production in Hadronic Collisions.

Collinear factorization also applies to hadron-hadron collisions. Consider quark production:

\[ p_1 \rightarrow x_1 p_1^+ \]
\[ x_1 p_1^+ \rightarrow s_1, p_1 \]
\[ s_1, p_1 \rightarrow s_2, -p_1 \]
\[ x_2 p_2^- \]
\[ p_2 \rightarrow \text{proton} \]

A collision happens very fast on proton's time scales -> factorization.
Square the diagram:

\[ p f_1(x_1, m^2) \sim \text{parton distribution} \]

\[ \frac{d\sigma_{f_1 f_2 \rightarrow f_1' f_2'}}{d^3 p} \sim \text{"hard" cross section} \]

\[ p f_2(x_2, m^2) \sim \text{parton distribution} \]

The collinear factorization formula then reads:

\[ \frac{d\sigma}{d^3 p} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \ p f_i(x_1, m^2) \cdot \frac{d\sigma_{f_i f_j \rightarrow f_i' f_j'}}{d^3 p} \cdot p f_j(x_2, m^2) \]

Usually put \( m^2 = \rho^2 \) for large \( \rho \) jets (hadrons)\).

After the collision, quarks (gluons) that are produced get dressed by further emissions. But the flow of energy is not likely to be modified much by those. (Still people construct other IR-safe observables insensitive to late-time emissions: (\( \cdots \))).
We can calculate the hard cross section from DISAP (PDF data) 

\[ \frac{d^3 \sigma_{gg \rightarrow q\bar{q}}}{d^3 p_1 d^3 p_2} = \int \frac{d^3 \sigma_{gg \rightarrow 2 \ell \nu \bar{\nu}}}{d^3 p_1 d^3 p_2} \]

\[ \frac{d^3 \sigma_{gg \rightarrow 2 \ell \nu \bar{\nu}}}{d^3 p_1 d^3 p_2} = \frac{1}{2} \frac{1}{2} \sum_{\ell = e, \mu, \tau} \frac{1}{m_\ell^2 - m_\ell^2} \]

\[ \frac{d^3 \sigma_{gg \rightarrow 2 \ell \nu \bar{\nu}}}{d^3 p_1 d^3 p_2} = \frac{1}{2} \frac{1}{2} \sum_{\ell = e, \mu, \tau} \frac{1}{m_\ell^2 - m_\ell^2} \]

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