The Higgs Mechanism (U(1) model)

\[ L = (D_\mu \phi)^* (D^\mu \phi) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \mu^2 \phi^* \phi - \lambda (\phi \phi^*)^2 \]

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad A_\mu \sim \text{real Abelian vector field} \]

\[ \phi \sim \text{scalar complex field} \]

\[ \sigma = \frac{\mu}{\sqrt{2} \lambda} \]

\[ V \]

\[ \text{Im } \phi \]

\[ \text{Re } \phi \]

\[ \text{Wrote } \phi(x) = \frac{f(x) + v}{\sqrt{2}} e^{i \theta(x)} \]

\[ B_\mu = A_\mu - \frac{1}{g} \partial_\mu \theta(x) \]

\[ L = \frac{1}{2} \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^2 - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \frac{1}{2} g^2 B_\mu B^\mu \]

\[ + \frac{1}{2} g^2 B_\mu B^\mu (2 \rho \delta + \rho^2) - 2 v \rho \delta - \frac{1}{2} \rho^2 \]

\[ m_\phi \approx \mu \sqrt{2} \quad m_B = g \sigma \]

no NG boson, as \( \theta(x) \) got "eaten up" by \( B_\mu \)

\( B_\mu \) has a mass \( \Rightarrow \) Higgs phenomenon.
SSB of gauge symmetry — no Goldstone bosons

but get massive vector fields!

(e.g. Meissner effect in superconductivity when photon gets a "mass" and is screened in superconductor — P.W. Anderson, ’58)

=> in particle physics this is known as the Higgs phenomenon. (Higgs, 1964).

**SU(2) \times U(1) Electroweak Theory.**

**History:** Pauli postulated neutrinos to explain \(\beta\)-decay (1930).

Fermi (’34): to explain \(\beta\)-decay \(n \rightarrow p e^-\bar{\nu}\)
suggested an interaction term

\[
L_F = - \frac{G_F}{\sqrt{2}} [\bar{p} \gamma_\mu n] [\bar{\epsilon} \gamma^\mu \nu] + \text{h.c.}
\]

with \(G_F = \frac{10^{-5}}{m_p^2}\), \(\Rightarrow\) but as \([G_F] \approx \frac{1}{\Lambda^2} \Rightarrow\) not renormalizable vector

\(\Rightarrow\) theory may have \((W, Z)\) bosons \(\Rightarrow\) Glauber, Salam proposed a gauge theory (’61, ’64)
problem with massive gauge fields:

the propagator is: $i \frac{g^2 \pi^2}{m^2} \Rightarrow$ also non-renormalizable, as $\Rightarrow$ constant as $k^2 \Rightarrow \infty \Rightarrow$

loops badly diverge...

Weinberg (1967) suggested using SSB to cure the problem

1983 $W, Z$ bosons discovered at CERN

Glashow-Weinberg-Salam model (1979 Nobel Prize in Physics)

define fermion fields of leptons:

$e, \mu, \tau, \nu_e, \nu_{\mu}, \nu_{\tau}$


define left & right handed ones

left-handed

$\ell_L = (\nu_{\ell})_L$, $\mu_L = (\nu_{\mu})_L$, $\tau_L = (\nu_{\tau})_L$.

and in right-handed isospin singlets:

$\ell_R = e_R$, $\nu_{\ell_R} = \mu_R$, $\nu_{\tau_R} = \epsilon_R$

write the Lagrangian for the 3 generations (aka families) of leptons:

...
\[ L_{\text{free}} = \bar{R}e + i \delta \cdot \delta R e + \bar{L} e + i \delta \cdot \delta L e + (\mu \& \tau \text{- terms}) \]

\[ \Rightarrow \text{quantum } \#\text{'s: } I \text{ weak } \Rightarrow \text{sospin } \Rightarrow \]

\[ \Rightarrow \text{doublets have } I = \frac{1}{2}, \text{ singlet has } I = 0. \]

\[ \Rightarrow \text{neutrinos have zero electric charge: } Q_{\text{electric}} = 0 \]

\[ \Rightarrow \text{if we want to have } (\text{Gell-Mann-Nishijima-type}) \]

\[ Q = I_3 + \frac{Y}{2} \]

\[ \Rightarrow \text{define weak hypercharge } Y : \text{neutrinos have } Q = 0, I_3 = +\frac{1}{2} \Rightarrow Y = -1 \Rightarrow \text{all doublets } \bar{L}_e, \bar{L}_\mu, \bar{L}_\tau \text{ have } Y = -1. \]

(check: electron has \( Q = -1 \Rightarrow -1 = -\frac{1}{2} -\frac{1}{2}, \text{ OK} \))

in the singlet: electron \( Q = -1 = I_3 + \frac{Y}{2} \Rightarrow Y = -2 \)

\[ \Rightarrow \text{iso-singlets have weak hypercharge } (Y = -2). \]

\( R_e, R_\mu, R_\tau \)

\[ \Rightarrow \text{back to } L_{\text{free}} : \text{it clearly has the following global symmetries:} \]

\[ U(1): \bar{L}_e \rightarrow e^{-i \delta y} \bar{L}_e, \bar{R}_e \rightarrow e^{-2 i \delta y} \bar{R}_e \]

\[ SU(2): \bar{L}_e \rightarrow e^{i \frac{\tau_2}{2}} \bar{L}_e, \bar{\tau} \sim \text{Pauli matrices} \]
Gauge $U(1)$ symmetry first: introduce an abelian vector field $B_\mu(x)$ with field strength $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ & coupling to leptons of $g'/2$: \( y = -2 \Rightarrow (\pm, -\frac{g'}{2} y B_\mu) \)

\[
L = \bar{\psi} \gamma^\mu \left( \partial_\mu + 2i \left( \frac{g'}{2} \right) B_\mu \right) \psi + \bar{\psi} e^{i \gamma^5} \psi.
\]

\[
\left( \partial_\mu + i \left( \frac{g'}{2} \right) B_\mu \right) \bar{\psi} e^{-i \gamma^5} \psi - \frac{i}{4} f_{\mu\nu} f^{\mu\nu} + (m, \bar{e}).
\]

Now let us gauge the $SU(2)$ symmetry: introduce a gauge field $W_\mu = (W_1^\mu, W_2^\mu, W_3^\mu)$ with the field strength:

\[
F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig [W_\mu, W_\nu]
\]

\[
W_\mu = \tilde{W}_\mu - \frac{g}{2} \tilde{W}_\mu \tilde{W}_\mu
\]

\[
L = \bar{\psi} \gamma^\mu \left( \partial_\mu + ig' B_\mu \right) \psi + \bar{\psi} \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} B_\mu - i g \frac{\tilde{g}}{2} \cdot \tilde{W}_\mu \right) \psi - \frac{i}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot \tilde{F}_{\mu\nu} + (m, \bar{e}).
\]

Now we have a Lagrangian for the leptons & 4 gauge fields $(B_\mu, \tilde{W}_\mu)$, but so far everything is massless ($\Rightarrow$ bad).
\( \rightarrow \) to give particles (especially \( \bar{W} \)'s) a mass need \$SL_3\$ mechanism

\( \rightarrow \) so for the Lagrangian is \( SU(2) \otimes U(1) \) invariant

\( \rightarrow \) to break this symmetry introduce Higgs field: a Weak isospin doublet

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}
\Rightarrow Q = +1
\Rightarrow Q = I_3 + \frac{Y}{2}, \quad I_3 = \pm \frac{1}{2}
\Rightarrow \text{weak hypercharge}
\Rightarrow Y = 1.
\]

\[
\phi^+ = (\phi^-, \phi^{0+})
\]

\( \Rightarrow \) add Higgs field to the Lagrangian:

\[
L_{\text{Higgs}} = \left( \partial \phi - i \frac{g'}{2} B \phi - i g \frac{\tau}{2} \phi \right)^2 \left( \partial \phi^* - i \frac{g'}{2} B^* \phi^* - i g \frac{\tau}{2} \phi^* \right)^2
\]

\[
+ \mu^2 \phi^+ \phi - \lambda (\phi^+ \phi)^2
\]

Note that Higgs has \( Y = +1 \) \( \Rightarrow \partial \phi - i \frac{g'}{2} Y B = \partial \phi - i \frac{g'}{2} B \)

\( \Rightarrow \) Higgs also couples to fermions (Yukawa coupling)

\[
L_{\text{Higgs + leptons}} = -G_e \left[ \bar{L}_e \phi \bar{R}_e + \bar{R}_e \phi^+ L_e \right] + (\mu, \xi \text{-terms})
\]
\[ L = \bar{\nu}_e (\bar{\nu}_e^c L) = \bar{\nu}_e \phi^+ e_R + \bar{\nu}_e^c \phi^0 e_R \] (matrices in isospin space).

\[ \Rightarrow \text{the full Lagrangian is:} \]
\[ L = \bar{\nu}_e^c i \gamma^\mu (\partial_\mu + ig' B_\mu) e_R + \bar{\nu}_e i \gamma^\mu (\partial_\mu + ig Z_\mu W_\mu^-) e_R + (\mu, \tau \text{-terms}) - \frac{i}{4} f_{\mu \nu \rho} f^{\mu \nu \rho} - \frac{i}{4} \tilde{F}_{\mu \nu} \tilde{F}^{\mu \nu} + \left[ \left( \partial_\mu - ig' B_\mu - ig \frac{\tau_3}{2} W_\mu^- \right) \phi \right]^\dagger \left[ \left( \partial_\mu - ig' B_\mu - ig \frac{\tau_3}{2} W_\mu^- \right) \phi \right] + m^2 \phi^+ \phi - \lambda (\phi^+ \phi)^2 - G_\nu \left[ \bar{\nu}_e^c \phi^+ e_R + \bar{\nu}_e \phi^0 L e \right] \]

\[ SU(2)_L \otimes U(1)_Y \text{ electroweak theory.} \]
\[ \Rightarrow \text{use the Higgs field to break the symmetry:} \]
\[ SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM} \]

\[ \Rightarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \] has electric charge +1 \Rightarrow \text{it in vacuum have } \langle 0 | \phi | 0 \rangle \neq 0 \text{ (SUSY)}

\[ \Rightarrow \text{no charge symmetry} \Rightarrow \text{no electric charge conservation} \Rightarrow \text{don't want this} \]

\[ \Rightarrow \text{to conserve the electric charge require the vacuum of Higgs field to be at} \]
\[ \langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ \psi / \sqrt{2} \end{pmatrix}. \]
\[ v = \frac{\theta}{\sqrt{2}} \]  
\[ \Rightarrow \text{write} \quad \phi(x) = e^{-i \frac{\theta}{2} \cdot \hat{\theta}(x)} \left( \begin{array}{c} 0 \\ \frac{v + \gamma(x)}{\sqrt{2}} \end{array} \right) \]

with \( \hat{\theta}, \gamma \) real fields.

Just like in the Abelian \( U(1) \) case can absorb \( \hat{\theta} \) field into \( \hat{W}_\mu \) by performing gauge rotation:

\[ \Phi(x) = e^{-i \frac{\theta}{2} \cdot \hat{\theta}(x)} \Rightarrow \Phi \rightarrow \Phi' = S \Phi, \quad \hat{L}_e \rightarrow \hat{L}_e' = S \hat{L}_e \]

and \( \hat{W}_\mu \rightarrow \hat{W}_\mu' = S \hat{W}_\mu S^{-1} - \frac{i}{g} \left( \partial_\mu S \right) S^{-1} \left( \hat{\theta} \text{ would be Goldstone boson} \right) \)

\[ \Rightarrow \text{can drop primes to write (keep electrons only)} \]

\[ L = \bar{e}_e i \gamma^\mu \left( \partial_{\mu} + i g' \hat{B}_\mu \right) e_e + \bar{e}_e i \gamma^\mu \left( \partial_{\mu} + i \frac{g' q'}{2} \hat{B}_\mu - i g \frac{\hat{\theta}}{2} \cdot \hat{W}_\mu \right) e_e \]

\[ - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} F_{\mu \nu} \cdot \hat{F}^{\mu \nu} + \frac{i}{2} \left[ \left( \partial_{\mu} - i \frac{g' q'}{2} \hat{B}_\mu - i g \frac{\hat{\theta}}{2} \cdot \hat{W}_\mu \right) (0) \right] \hat{W}_{\mu} \]

\[ \left[ \left( \partial^\mu - i \frac{g' q'}{2} \hat{B}^\mu - i g \frac{\hat{\theta}}{2} \cdot \hat{W}^\mu \right) (0_{\nu+_{\theta}}) \right] + \frac{\mu^2}{2} (v + y)^2 - \frac{\lambda}{4} (v + y)^4 \]

\[ - G e \left[ \frac{1}{\sqrt{2}} \left( \bar{v} e \cdot \bar{e}_e \right) (0_{\nu+_{\theta}}) e_e + h.c. \right] \]

\[ = \text{the potential} \]

Start with \( \gamma \)-particle: linear terms in \( \gamma \) cancel as usual, as we are expanding around a minimum in \( \gamma \).