Last time we finished deriving the Electroweak Lagrangian:

\[ \mathcal{L}_{\text{EW}} = \bar{e} i \gamma ^{\mu} e + \bar{\nu}_e c \sigma ^{\mu \nu} \nu_e - \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma ^{\mu} (\gamma ^{\nu} + i \gamma ^{5} \gamma ^{\nu}) \gamma ^{\mu} \nu_e - \frac{i}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

\[ + \frac{1}{2} \bar{\nu}_e m_e \bar{\nu}_e - m_{\nu}^2 \bar{\nu}_e \nu_e - \frac{1}{4} \bar{\nu}_e \gamma ^{5} \gamma ^{\mu} \gamma ^{\nu} \gamma ^{5} \nu_e + \frac{g^2}{4} (\gamma ^{\mu} + \gamma ^{5}) \bar{\nu}_e \gamma ^{\mu} \nu_e \]

\[ + \frac{g^2}{8 \cos^2 \theta_W} (\gamma ^{\mu} + \gamma ^{5}) \bar{Z} \gamma ^{\mu} Z + \frac{g}{2 \cos \theta_W} \left[ 2 m^2 \sin^2 \theta_W \bar{e} \gamma ^{\mu} \gamma ^{5} \sigma ^{\mu \nu} \gamma ^{5} \nu_e \right. \]

\[ + \left( 2 \sin^2 \theta_W - 1 \right) \bar{e} \gamma ^{\mu} \gamma ^{5} \sigma ^{\mu \nu} \gamma ^{5} \nu_e \right] - e \bar{e} \gamma ^{\mu} A_c \gamma ^{5} \nu_e + \frac{g}{2 \cos \theta_W} \bar{\nu}_e \gamma ^{5} \gamma ^{\mu} \nu_e \]

\[ + \frac{g}{\sqrt{2}} \left[ \bar{\nu}_e \gamma ^{\mu} W^+ \gamma ^{5} \nu_e + \bar{\nu}_e \gamma ^{\mu} W^- \gamma ^{5} \nu_e \right] + (\lambda, \phi - \text{terms}) \]

\[ M_W = 80.4 \text{ GeV}, \quad M_Z = 91.2 \text{ GeV} \]

\[ \frac{M_W}{M_Z} = \cos \theta_W, \quad \sin^2 \theta_W = 0.22 \]

\[ \frac{g^2}{4 \pi} = \frac{1}{30} \quad \text{weak coupling} \]

\[ \frac{g_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} \rightarrow g_F \approx 10^{-5} \text{GeV}^{-2} \]
Let’s derive this from the Electroweak Lagrangian we wrote. At the lowest order the process is given by the diagram:

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{e} \gamma \cdot W e_{L} + \bar{e}_{L} \gamma \cdot W^{+} \nu_{e} \right] \]

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{\mu} \gamma \cdot W e_{L} + \bar{e}_{L} \gamma \cdot W^{+} \nu_{\mu} \right] \]

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{\tau} \gamma \cdot W e_{L} + \bar{e}_{L} \gamma \cdot W^{+} \nu_{\tau} \right] \]

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{e} \gamma \cdot W (1-\delta_{5}) e + \bar{e} \gamma \cdot W^{+} (1-\delta_{5}) \nu_{e} \right] \]

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{\mu} \gamma \cdot W (1-\delta_{5}) \nu_{\mu} + \bar{e} \gamma \cdot W^{+} (1-\delta_{5}) \nu_{e} \right] \]

\[ \frac{g}{\sqrt{2}} \left[ \bar{\nu}_{\tau} \gamma \cdot W (1-\delta_{5}) \nu_{\tau} + \bar{e} \gamma \cdot W^{+} (1-\delta_{5}) \nu_{e} \right] \]

\[ \Rightarrow \text{the diagram is} \]

\[ \left( \frac{i g}{2 \sqrt{2}} \right)^{2} \left[ \bar{U}_{\nu_{\mu}}(p_{2}) \gamma_{\alpha} (1-\delta_{5}) U_{\mu}(p_{1}) \right] \]

\[ \left[ \bar{\nu}_{e}(h_{2}) \gamma_{\beta} (1-\delta_{5}) \nu_{e}(h_{1}) \right] \]

\[ \Rightarrow \text{looks just like the term in Fermi theory} \]

\[ \Rightarrow \text{equate the prefactors:} \]
\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2}
\]

\[\Rightarrow G_F = \frac{g^2}{4\pi} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{M_w^2} \approx 8.5 \times 10^{-5} \text{ GeV}^{-2} \approx 10^{-5} \text{ GeV}^{-2}\]

as advertised.

\[\Rightarrow \text{What about Higgs and related parameters?}\]

\[M_w = \frac{g \, v^2}{2} \Rightarrow v = \frac{2}{g} M_w \Rightarrow \text{as } g \approx 0.63\]

\[\Rightarrow \text{get } v \approx 289 \text{ GeV} \quad (v \approx 246 \text{ GeV PDG value})\]

The Higgs mass is \(M_H = M_{\sqrt{2}} = v \sqrt{2} \lambda\)

\[M_{\text{Higgs}} = 125.09 \pm 0.21 \pm 0.11 \text{ GeV} \quad \text{stat.} \quad \text{syst.}\]

\[\Rightarrow \lambda \approx \frac{1}{2} \left(\frac{M_H}{v}\right)^2 \approx 0.09 \quad \text{fairly small, } \lambda \approx 0.1\]

about right at the scale of \(\sim 1 \text{ TeV}\).
Quarks in the Electroweak Theory.

Quarks also form left-handed doublets under weak isospin:

\[
\begin{align*}
L_u &= \left( \begin{array}{c}
U \\
d^1
\end{array} \right)_L \\
L_c &= \left( \begin{array}{c}
c \\
s^1
\end{array} \right)_L \\
L_t &= \left( \begin{array}{c}
t \\
b^1
\end{array} \right)_L
\end{align*}
\]

\[
\begin{align*}
R_u &= u_R \\
R_d &= d_R \\
R_c &= c_R \\
R_s &= s_R \\
R_t &= t_R \\
R_b &= b_R
\end{align*}
\]
$L_u = \begin{pmatrix} u \\ d' \\ d'' \end{pmatrix}$ 

$doublet \Rightarrow I_3 = \frac{+1}{2} \Rightarrow Q = I_3 + \frac{Y}{2}$

$\Rightarrow Y = 2(Q - I_3) \Rightarrow \text{for } u \text{ have } Q = \frac{+2}{3}, I_3 = \frac{+1}{2} \Rightarrow$ 

$\Rightarrow Y = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}$; for $d'$ have $Q = -\frac{1}{3}, I_3 = -\frac{1}{2} \Rightarrow$ 

$\Rightarrow Y = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{3} \Rightarrow \boxed{Y = \frac{1}{3}}$ for the doublet!

$S\text{inglets: } R_u = uR_e \text{ has } Q = \frac{+2}{3}, I_3 = 0 \Rightarrow \boxed{Y = \frac{y}{3}}$

$R_d = Ud \text{ has } Q = -\frac{1}{3}, I_3 = 0 \Rightarrow \boxed{Y = -\frac{2}{3}}$

(Same for other quark generations/families)

$\Rightarrow$ We have defined the quark weak eigenstates $d', s', b'$ by:

$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$\Rightarrow$ Weak eigenstates CKM quarks in QCD matrix (mass eigenstates)

$1963 \Rightarrow$ CKM = Cabibbo–Kobayashi–Maskawa matrix

$1973 \Rightarrow$ Nobel Prize 2008
CKM matrix is unitary: $V^+V = VV^+ = 1$.

(logic: our mass matrix for quarks is diagonal, but there is no reason for a EW interaction one to be diagonal too.)

Let's write down the Lagrangian:

$L_{\text{quarks + gauge}} = \bar{L}_u \, i \gamma^\mu \left( \partial_\mu - i \frac{g'}{2} \gamma \beta_3 - i g \frac{\tilde{\omega}_\mu}{2} \right) L_u$

$+ \bar{R}_u \, i \gamma^\mu \left( \partial_\mu - i \frac{g'}{2} \gamma \beta_3 \right) R_u + \bar{R}_d \, i \gamma^\mu \left( \partial_\mu - i \frac{g'}{2} \gamma \beta_3 \gamma^5 \right) R_d$

$+ \text{other 2 generations}$,

$\Rightarrow$

$L_{\text{quarks + gauge}} = \bar{L}_u \, i \gamma^\mu \left( \partial_\mu - i \frac{g'}{6} \beta_3 - i g \frac{\tilde{\omega}_\mu}{2} \right) L_u$

$+ \bar{R}_u \, i \gamma^\mu \left( \partial_\mu - i \frac{2}{3} g' \beta_3 \right) R_u + \bar{R}_d \, i \gamma^\mu \left( \partial_\mu + i \frac{1}{3} g' \beta_3 \gamma^5 \right) R_d$

$+ \text{2 more generations}$.

The need to couple quarks to Higgs: (don't have to, but it would be nice)

$\phi = \begin{pmatrix} \phi^\dagger \cr \phi \end{pmatrix}$

We may write a term like $\bar{L}_u \phi R_u$ and $\bar{L}_d \phi R_d$.

However the VEV is $\langle 0 | \phi | 10 \rangle = \begin{pmatrix} \phi \cr \phi^\dagger \end{pmatrix}$ $\Rightarrow$
near the Higgs VEV get

\[ \mathcal{L}_\mu \phi R_\mu = (\bar{u}_L \cdot d_L^c) (\sigma_i \phi) u_R = \bar{d}_L \cdot U_R \frac{\sigma_i}{\sqrt{2}} \sim \text{no mass} \]

for \( U = \begin{pmatrix} \gamma & 0 \\ 0 & 1 \end{pmatrix} \) \( \gamma \neq 1 \), \( \gamma \neq -i \frac{1}{2} \) not \( U(1)_Y \) invariant too...

in like neutrinos, \( \nu \) would not get a mass...

(same for \( c, t \) quarks).

\( \Rightarrow \) to give quarks mass define

\[ \overline{\phi}(x) = i \tau^2 \phi^* \]

for the VEV: \( \langle 0 | \overline{\phi} | 0 \rangle = i \tau^2 \begin{pmatrix} 0 \\ \frac{\gamma}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{\gamma}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \gamma \frac{\gamma}{\sqrt{2}} \\ 0 \end{pmatrix} \)

have the VEV \( \neq 0 \) on top now

Under \( SU(2)_L \) gauge transform: \( \phi \rightarrow e^{i \frac{\tau_1}{2}} \phi \)

\( \Rightarrow \overline{\phi} \rightarrow i \tau^2 \left( e^{i \frac{\tau_1}{2}} \phi \right)^* = i \tau^2 e^{-i \frac{\tau_2 \cdot \tau_2^*}{2}} \phi^* = (\tau^2)^2 = 1 \)

\( = i \tau^2 e^{-i \frac{\tau_2 \cdot \tau_2^*}{2}} \overline{\phi} \)

\( \overline{\phi} = \tau^2 i \tau^2 \overline{\phi} \)

this is true because: \( \tau^2 \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \tau^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \)

\( \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau^1 \)

Similarly \( \tau^2 \begin{pmatrix} \tau^2 & \tau^2 \\ -\tau^2 & \tau^2 \end{pmatrix} \tau^2 = \tau^2 \) (obvious) and

\( \tau^2 \begin{pmatrix} \tau^2 & \tau^2 \\ -\tau^2 & \tau^2 \end{pmatrix} \tau^2 = \tau^3 \Rightarrow \text{eqn is true} \)

(\( \tau^2 \begin{pmatrix} \tau^2 & \tau^2 \\ -\tau^2 & \tau^2 \end{pmatrix} \tau^2 = \overline{\tau^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \overline{\tau^2} = \tau^2 \))

Sandwich \( (\tau^2 \text{'s in}) \)
\[ \Rightarrow \text{under } SU(2)_L \text{ have } \Phi \rightarrow e^{-\frac{\alpha}{2}} \Phi \]

\[ \Rightarrow \text{transforms just like } \phi ! \]

\[ \Rightarrow \text{can write } \tilde{L}_u \Phi R_u \sim SU(2)_L \text{ invariant!} \]

\[ \text{hence VEV: } \tilde{L}_u \Phi R_u = (\tilde{u}_L \tilde{d}_L') \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) U_R = \frac{v}{\sqrt{2}} \tilde{u}_L U_R \]

\[ \angle(\Phi^{10}) = (\frac{\nu}{\sqrt{2}}) \]

\[ \Rightarrow \text{may give } u \text{-quark mass!} \]

Terms like \[ \tilde{L}_u \phi R_d = (\tilde{u}_L \tilde{d}_L') \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) d_R = \frac{v}{\sqrt{2}} \tilde{d}_L d_R \]

can give \( d \)-quark mass (and \( s, b \) quarks too).

\[ \Rightarrow \text{also need to check weak hypercharge:} \]

\[ \phi \text{ has } Y = +1 \Rightarrow \Phi \text{ has } Y = -1 \Rightarrow \tilde{L}_u \Phi R_u \Rightarrow \text{let } Y = 0 \]

\[ \alpha = Y = 0 \]

\[ Y = -\frac{1}{3}, \quad Y = -\frac{1}{3} \]

\[ \tilde{L}_u \Phi R_d \Rightarrow \text{let } Y = 0 \quad \text{two both work!} \]

\[ Y = -\frac{1}{3}, \quad Y = +1 \]

To write quarks + Higgs couplings let's limit ourselves to 2 generations: \( L_u, L_d, R_u, R_d, R_c, R_s \).

First write all possible terms:

\[ L \text{ quarks+Higgs } = -G_1 \left[ \tilde{L}_u \Phi R_u + \tilde{R}_u \Phi^+ L_u \right] - G_2 \left[ \tilde{L}_u \Phi R_d + \tilde{R}_d \Phi^+ L_u \right] - G_3 \left[ \tilde{L}_u \Phi R_s + \tilde{R}_s \Phi^+ L_u \right] - G_4 \left[ \tilde{L}_c \Phi R_c + \tilde{R}_c \Phi^+ L_c \right] \]
\[
-G_5 \left[ L_c \phi R_d + \bar{R}_d \phi^+ L_c \right] - G_6 \left[ L_c \phi R_\bar{s} + \bar{R}_s \phi^+ L_c \right] \\
-G_7 \left[ L_u \bar{\phi} R_c + \bar{R}_c \bar{\phi}^+ L_u \right] - G_8 \left[ L_c \bar{\phi} R_u + \bar{R}_u \bar{\phi}^+ L_c \right] \tag{162}
\]

Plug in \( \langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix} \), \( \langle 01 | \phi | 10 \rangle = \begin{pmatrix} u/\sqrt{2} \\ 0 \end{pmatrix} \):

\[ 2 \text{ generations quark + Higgs} = - \frac{v}{\sqrt{2}} \left\{ G_1 \bar{u} u + G_2 (\bar{d}_L d_R + \bar{d}_R d'_L) + G_3 (\bar{d}'_L s_R + \bar{s}_R d'_L) + G_4 \bar{c} c + G_5 (\bar{s}_L d_R + \bar{d}_R s'_L) + G_6 (\bar{s}'_L s_R + \bar{s}_R s'_L) + G_7 (\bar{u}_L c_R + \bar{c}_R u_L) + G_8 (\bar{c}_L u_R + \bar{u}_R c_L) \right\} \]

\[ \Rightarrow \text{first of all we see} \quad m_u = \frac{G_1 v}{\sqrt{2}} \quad m_c = \frac{G_4 v}{\sqrt{2}} \]

\[ \Rightarrow \text{can't have} \quad u \to c \quad \text{& vice versa} \Rightarrow G_7 = G_8 = 0 \]

\[ \Rightarrow \text{left with} \quad d, s, \bar{s} \text{ quarks} \quad \text{for those write:} \]

\[
\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}
\]

\( \theta_c \sim \text{Cabibbo angle} \), in CKM matrix \( V_{ud} = \cos \theta_c \sim v_s \)

\[ V_{us} = \sin \theta_c = -v_c \]

\( \theta_c \approx 13^\circ \quad \text{small mixing} \)

\[ d' = d \cos \theta_c + s \sin \theta_c \]

\[ s' = -d \sin \theta_c + s \cos \theta_c \]
\[ \text{part \ L \ quark + \ Higgs} = - \frac{v}{\sqrt{2}} \left\{ G_2 \left[ \bar{d} \bar{d} \cos \theta_c + (\bar{s}_L \bar{d}_R + \bar{d}_R \bar{s}_L) \right] \sin \theta_c \right\} + G_3 \left[ \bar{s} \bar{s} \sin \theta_c + (\bar{d}_L \bar{s}_R + \bar{s}_R \bar{d}_L) \cos \theta_c \right] + G_6 \left[ -\bar{d} \bar{d} \sin \theta_c + (\bar{s}_L \bar{d}_R + \bar{d}_R \bar{s}_L) \cos \theta_c \right] + \]

\[ + G_6 \left[ \bar{s} \bar{s} \cos \theta_c - (\bar{d}_L \bar{s}_R + \bar{s}_R \bar{d}_L) \sin \theta_c \right] \right\} \]

\[ = - \bar{d} \bar{d} \frac{v}{\sqrt{2}} \left[ G_2 \cos \theta_c - G_5 \sin \theta_c \right] - \bar{s} \bar{s} \frac{v}{\sqrt{2}} \left[ G_3 \sin \theta_c + G_6 \cos \theta_c \right] - \frac{v}{\sqrt{2}} \left( \bar{s}_L \bar{d}_R + \bar{d}_R \bar{s}_L \right) \left[ G_2 \sin \theta_c + G_5 \cos \theta_c \right] \]

\[ - \frac{v}{\sqrt{2}} \left( \bar{d}_L \bar{s}_R + \bar{s}_R \bar{d}_L \right) \left[ G_3 \cos \theta_c - G_6 \sin \theta_c \right] = 0 \]

\( \Rightarrow \) don't want \( d \leftrightarrow s \) \( \Rightarrow \) \( G_5 = - G_2 \tan \theta_c \)

\( G_6 = G_3 \cot \theta_c \)

\[ \Rightarrow m_d = \frac{v}{\sqrt{2}} \left[ G_2 \cos \theta_c - G_5 \sin \theta_c \right] = \frac{v}{\sqrt{2}} \frac{G_2}{\cos \theta_c} = m_d \]

\[ m_s = \frac{v}{\sqrt{2}} \left[ G_3 \sin \theta_c + G_6 \cos \theta_c \right] = \frac{v}{\sqrt{2}} \frac{G_3}{\sin \theta_c} = m_s \]

\( \Rightarrow \) instead of unknown \( m_d, m_d, m_s, m_c \) have constants \( G_1, G_2, G_3, G_4 \) also unknown...

\[ \text{L \ quark+Higgs} = - \sum m_c \bar{q}_c g^f q^f \phi \phi(x) \]
CKM matrix (approximate values)

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix} \approx
\begin{pmatrix}
0.974 & 0.225 & 0.004 \\
0.225 & 0.973 & 0.043 \\
0.009 & 0.041 & 0.999
\end{pmatrix}
\]

"almost" diagonal.

Why do we need $d'$, $s'$, $b'$? Look at $Q$:

\[
\mathcal{L}^{e+e-} \supset \bar{u}_L d_L \rightarrow W^+ u_L \nu_L \rightarrow W^+ \nu_L \rightarrow W^+ d_L^{+}\frac{g}{2} W^- u_L \rightarrow W^+ \nu_L d_L^{+} + g \bar{d}_L^{'} \nu_L W^+ u_L
\]

Experimentally, one has the following decays:

\[
\begin{align*}
\bar{K}^- & \rightarrow \mu^- \bar{\nu}_\mu \\
K^- & \rightarrow \mu^- \bar{\nu}_\mu
\end{align*}
\]