Last time | Quarks in Electroweak Theory (cont'd)

$$\boxed{\begin{aligned}\mathcal{L}_{quarks+gauge} &= \bar{L}_u\, i\gamma^\mu\left(\partial_\mu - i\frac{g'}{6}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u \\ &+ \bar{R}_u\, i\gamma^\mu\left(\partial_\mu - i\frac{2}{3}g'B_\mu\right)R_u + \bar{R}_d\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{3}B_\mu\right)R_d \\ &+ (c,t\text{ - generations})\end{aligned}}$$

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}, \quad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V\cdot\begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad V = 3\times 3 \text{ CKM matrix}$$

$$\boxed{\mathcal{L}_{quarks+Higgs} = -\sum_f m_f\, \bar{q}^f q^f\, \frac{v+\eta(x)}{v}}$$

$$\Rightarrow \mathcal{L}^{d,s\ \text{part}}_{\text{quark + Higgs}} = -\frac{v}{\sqrt{2}} \Big\{ G_2 \Big[\underline{\bar{d} d \cos\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L) \cdot \sin\theta_c \Big] + G_3 \Big[\bar{s} s \sin\theta_c + (\bar{d}_L s_R + \bar{s}_R d_L) \cos\theta_c \Big] +$$

$$+ G_5 \Big[\underline{-\bar{d} d \sin\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L) \cos\theta_c \Big] +$$

$$+ G_6 \Big[\bar{s} s \cos\theta_c - (\bar{d}_L s_R + \bar{s}_R d_L) \sin\theta_c \Big] \Big\} =$$

$$= -\bar{d} d \frac{v}{\sqrt{2}} \Big[G_2 \cos\theta_c - G_5 \sin\theta_c \Big] - \bar{s} s \frac{v}{\sqrt{2}} \Big[G_3 \sin\theta_c + G_6 \cos\theta_c \Big] - \frac{v}{\sqrt{2}} (\bar{s}_L d_R + \bar{d}_R s_L) \underbrace{\Big[G_2 \sin\theta_c + G_5 \cos\theta_c \Big]}_{=0}$$

$$- \frac{v}{\sqrt{2}} (\bar{d}_L s_R + \bar{s}_R d_L) \underbrace{\Big[G_3 \cos\theta_c - G_6 \sin\theta_c \Big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2 \tan\theta_c$

$G_6 = G_3 \cot\theta_c$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}} \Big[G_2 \cos\theta_c - G_5 \sin\theta_c \Big] = \boxed{\frac{v}{\sqrt{2}} \frac{G_2}{\cos\theta_c} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}} \Big[G_3 \sin\theta_c + G_6 \cos\theta_c \Big] = \boxed{\frac{v}{\sqrt{2}} \frac{G_3}{\sin\theta_c} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

$\mathcal{L}_{\text{quark+Higgs}} = -\sum m_f \bar{q}^f q^f \, \underline{v + h(x)}$ ~ the net quarks + Higgs Lagrangian

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

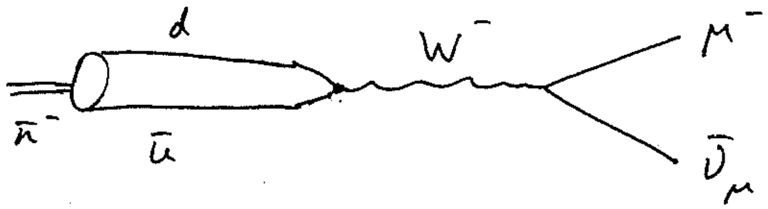
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

$$g(\bar{u}_L\ \bar{d}'_L)\, i\gamma^\mu \underbrace{\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \Rightarrow \text{has}$$

$$g\,\bar{u}_L\,\gamma\cdot W_\mu\, d'_L + g\,\bar{d}'_L\,\gamma\cdot W^+_\mu\, u_L$$

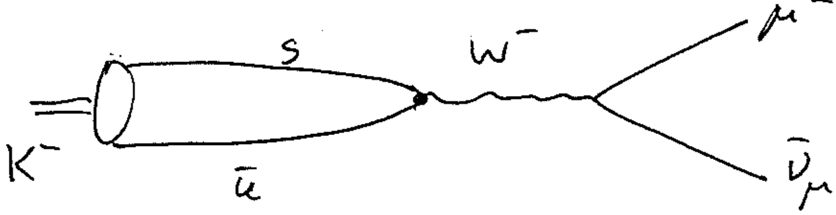
Experimentally one has the following leptonic decays:



$\pi^- \to \mu^- \bar{\nu}_\mu$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Cabibbo angle



$K^- \to \mu^- \bar{\nu}_\mu$

$\Rightarrow$ if $d' = d$ $\Rightarrow$ then $K^- \to \mu^- \bar{\nu}_\mu$ process would have been prohibited $\Rightarrow$ but it exists!

$\Rightarrow$ in 1963 Cabibbo postulated his mixing

$\Rightarrow$ as $d' = d\cos\theta_c + s\sin\theta_c$ $\Rightarrow$ get $s\bar{u}$ coupling!

$\theta_c \simeq 13^\circ$

$$\left.\begin{aligned} &\Rightarrow M_{\pi^- \to \mu^- \bar{\nu}_\mu} \propto \cos\theta_c \\ &M_{K^- \to \mu^- \bar{\nu}_\mu} \propto \sin\theta_c \end{aligned}\right\} \Rightarrow \frac{\Gamma_{\pi^- \to \mu^- \bar{\nu}_\mu}}{\Gamma_{K^- \to \mu^- \bar{\nu}_\mu}} \approx \frac{\cos^2\theta_c}{\sin^2\theta_c} =$$

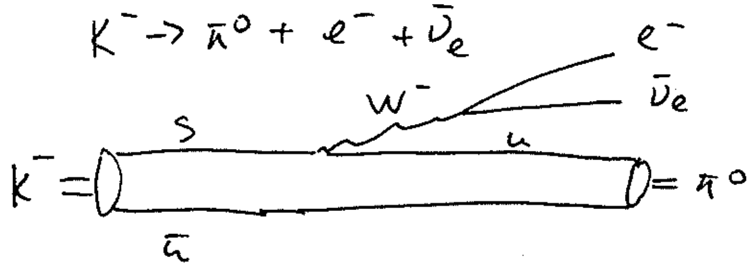
$= \cot^2\theta_c \simeq 18.8$ (experiment $\simeq 13.2$)

$\pi^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-favored

$K^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-surpressed.

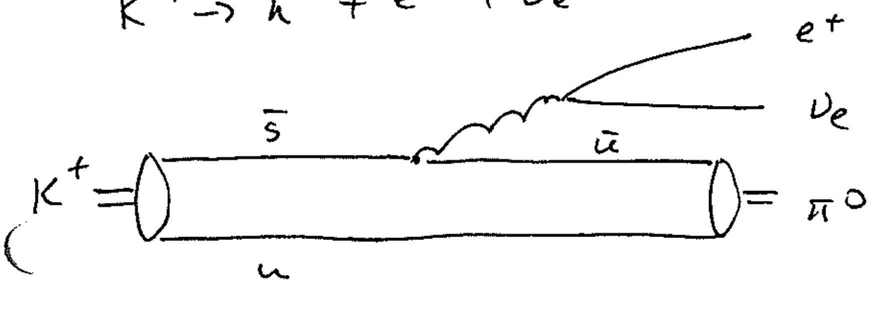
Other relevant processes:

$K^- \to \pi^0 + e^- + \bar{\nu}_e$



$\Rightarrow M_{K^- \to \pi^0 + e^- + \bar{\nu}_e} \propto \sin\theta_c \sim V_{us}$

$K^+ \to \pi^0 + e^+ + \nu_e$



$\Rightarrow M_{K^+ \to \pi^0 + e^+ + \nu_e} \propto \sin\theta_c \sim V_{us}$ too.

<u>Semi-leptonic decays.</u>

cf. $K^+ \to \mu^+ + \nu_\mu$ ~ leptonic decay
(all leptons in final state)

$K^- \to \pi^0 \pi^-$ hadronic (non-leptonic) decay.

<u>Interactions of W's and Z's with Quarks</u>

$$\mathcal{L}_{\text{quarks}+W,Z} = \bar{L}_u i\gamma^\mu \left(\partial_\mu - i\frac{g'}{6}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u +$$

$$+ \bar{R}_u i\gamma^\mu\left(\partial_\mu - i\frac{2}{3}g'B_\mu\right)R_u + \bar{R}_d i\gamma^\mu\left(\partial_\mu + i\frac{1}{3}g'B_\mu\right)R_d + (c,t)$$

$$= \bar{L}_u i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{L_u}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u + \bar{R}_u i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{R_u}B_\mu\right)$$

$$\cdot R_u + \bar{R}_d\, i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{R_d}B_\mu\right)R_d + (c,t).$$

Define $\quad U_L = \begin{pmatrix} u \\ c \\ t\end{pmatrix}_L$, $\quad D_L = \begin{pmatrix} d \\ s \\ b\end{pmatrix}_L$, $\quad U_R, D_R$ ~ similarly

with $\psi = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$, $\quad M = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & V\end{pmatrix}$, V is a 3×3 CKM Matrix,

M is 6×6, $M^\dagger M = \mathbb{1}$

$$\Rightarrow \boxed{\mathcal{L} = \bar{\psi} M^\dagger i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}\overset{=1/3}{Y} B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)M\psi + \sum_f \bar{R}_f i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_f B_\mu\right)R_f}$$

(more compact notation)

Write
$$\begin{cases} B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{cases}$$

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L} = \bar{\Psi}\, i\gamma^\mu \Big[\partial_\mu - i\frac{g'}{6}\left(A_\mu \cos\theta_W - Z_\mu \sin\theta_W\right) - i g \frac{\tau^3}{2} \cdot \\ \cdot \left(A_\mu \sin\theta_W + Z_\mu \cos\theta_W\right) - i\frac{g}{\sqrt{2}}\, m^+ \left(\tau^+ W_\mu + \tau^- W_\mu^+\right) m \Big] \Psi \\ + \sum_f \bar{R}_f\, i\gamma^\mu \left(\partial_\mu - i\frac{g'}{2} Y_f \left(A_\mu \cos\theta_W - Z_\mu \sin\theta_W\right)\right) R_f . \end{aligned}}$$

We have used: $m^+ \tau^3 m = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} =$

$$= \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -V^+V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tau^3 .$$

<u>We defined</u>: $\tau^\pm = \dfrac{\tau_1 \pm i\tau_2}{2} \Rightarrow \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\Rightarrow \frac{1}{2}\left(\tau^1 W_\mu^1 + \tau^2 W_\mu^2\right) = \left| \begin{array}{l} W_\mu^1 = \dfrac{W_\mu + W_\mu^+}{\sqrt{2}} \\ W_\mu^2 = (-i)\dfrac{W_\mu^+ - W_\mu}{\sqrt{2}} \end{array} \right. =$$

$$= \frac{1}{2}\frac{1}{\sqrt{2}}\left[\tau^1 \left(W_\mu + W_\mu^+\right) - i\tau^2\left(W_\mu^+ - W_\mu\right)\right] = \frac{1}{\sqrt{2}}\Big[W_\mu \frac{\tau^1 + i\tau^2}{2} +$$

$$+ W_\mu^+ \frac{\tau^1 - i\tau^2}{2}\Big] = \frac{1}{\sqrt{2}}\left(\tau^+ W_\mu + \tau^- W_\mu^+\right) \quad \text{as desired.}$$

(i) Charged current (coupling of $W^{\mp}$ bosons)

$$m^+ \tau^+ m = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix}; \quad m^+ \tau^- m = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ V^+ & 0 \end{pmatrix}$$ $\Rightarrow$ the charged current part of the Lagrangian is:

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}} \bar{\psi} \gamma^\mu \left[\begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix} W_\mu + \begin{pmatrix} 0 & 0 \\ V^+ & 0 \end{pmatrix} W_\mu^+ \right] \psi =$$

$$= \frac{g}{\sqrt{2}} (\bar{U}_L \; \bar{D}_L) \gamma^\mu \begin{pmatrix} 0 & V W_\mu \\ V^+ W_\mu^+ & 0 \end{pmatrix} \begin{pmatrix} U_L \\ D_L \end{pmatrix} = \frac{g}{\sqrt{2}} \Big[\bar{U}_L \gamma^\mu V W_\mu D_L$$

$$+ \bar{D}_L \gamma^\mu V^+ W_\mu^+ U_L \Big] = \boxed{\frac{g}{2\sqrt{2}} \Big\{ \bar{u} \gamma \cdot W (1-\gamma_5) [V_{ud} \cdot d + V_{us} s + V_{ub} b] + \bar{c} \gamma \cdot W (1-\gamma_5) [V_{cd} d + V_{cs} s + V_{cb} b] + \bar{t} \gamma \cdot W (1-\gamma_5) [V_{td} d + V_{ts} s + V_{tb} b] + h.c. \Big\} = \mathcal{L}_{cc}}$$

$\Rightarrow$ we dropped L subscripts $\Rightarrow$ got $(1-\gamma_5)$'s

$\Rightarrow$ spelled out $V D_L$ $\Rightarrow$ got CKM matrix elements.

(ii) Neutral current (coupling of Z bosons, photons)

a) Photons $\Rightarrow$ A_μ terms

$$\mathcal{L}^{\text{photons}} = \frac{g'}{6}\cos\theta_W\, \bar{\Psi}\,\gamma\cdot A\,\Psi + \frac{g}{2}\sin\theta_W\, \bar{\Psi}\,\gamma\cdot A\,\tau^3\Psi$$

$$+ \sum_f \frac{g'}{2} Y_f \cos\theta_W\, \bar{R}_f\,\gamma\cdot A\, R_f$$

$\Rightarrow$ remember $e = g'\cos\theta_W = g\sin\theta_W$

$$\mathcal{L}^{\text{photons}} = \bar{\Psi}\,\gamma\cdot A\left(\frac{e}{6} + \frac{e}{2}\tau^3\right)\Psi + \sum_f \frac{e}{2} Y_f\, \bar{R}_f\,\gamma\cdot A\, R_f$$

$$\Rightarrow e\left(\frac{1}{6} + \frac{\tau^3}{2}\right) = e\left(\frac{Y}{2} + \frac{\tau^3}{2}\right) = e\left(\frac{Y}{2} + I_3\right) = Q_{LHQ} = e_f$$

$\Rightarrow$ Gell-Mann-Nishijima formula

(check: $\frac{Y}{2} + \frac{\tau^3}{2} = \frac{1}{6} + \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{pmatrix}$ ← charge of u, c, t; ← charge of d, s, b)

$$\frac{e}{2}\cdot Y_f = e\cdot\frac{1}{2}\cdot\begin{cases} 4/3 & \text{for } u, c, t\\ -2/3 & \text{for } d, s, b\end{cases} = e\cdot\begin{cases} 2/3 & \text{for } u, c, t\\ -1/3 & \text{for } d, s, b\end{cases}$$

$\Rightarrow$ get $Q_{RHQ} = e_f$ again

$$\boxed{e_f = e\cdot\begin{cases} 2/3 & \text{for } u, c, t\\ -1/3 & \text{for } d, s, b\end{cases}}$$

$$\Rightarrow \boxed{\mathcal{L}^{\text{photons}} = \sum_f e_f\, \bar{q}_f\,\gamma\cdot A\, q_f}$$

Regular QED term as expected!

b) Z-bosons: $\mathcal{L}^Z = \bar{\Psi}\gamma^\mu\left[-\frac{g'}{6}\sin\theta_W + g\frac{\tau^3}{2}\cdot\right.$

$\left.\cdot\cos\theta_W\right] Z_\mu \Psi - \sum_f \bar{R}_f \gamma^\mu Z_\mu \frac{g'}{2} Y_f \sin\theta_W R_f =$ $\left| \begin{array}{l} g'\sin\theta_W = \\ = g\frac{\sin^2\theta_W}{\cos\theta_W}\end{array}\right.$

$$= (\bar{U}_L\ \bar{D}_L)\,\gamma\cdot Z \begin{pmatrix} \frac{g}{2}\cos\theta_W - \frac{g}{6}\frac{\sin^2\theta_W}{\cos\theta_W} & 0 \\ 0 & -\frac{g}{2}\cos\theta_W - \frac{g}{6}\frac{\sin^2\theta_W}{\cos\theta_W}\end{pmatrix}\begin{pmatrix}U_L\\ D_L\end{pmatrix}$$

$$-\frac{g\sin^2\theta_W}{2\cos\theta_W}\cdot\sum_f \bar{R}_f\gamma\cdot Z\, Y_f R_f = \frac{g}{2\cos\theta_W}\Big[\bar{u}_L\gamma\cdot Z u_L\cdot$$

$$\cdot\left(\cos^2\theta_W - \frac{1}{3}\sin^2\theta_W\right) - \bar{D}_L\gamma\cdot Z D_L\left(\cos^2\theta_W + \frac{1}{3}\sin^2\theta_W\right) -$$

$$-\frac{g\sin^2\theta_W}{2\cos\theta_W}\left(\bar{u}_R\gamma\cdot Z u_R\cdot\frac{4}{3} + \bar{D}_R\gamma\cdot Z D_R\left(-\frac{2}{3}\right)\right)\Big]$$

$$\Rightarrow \boxed{\mathcal{L}^Z = \frac{g}{4\cos\theta_W}\Big\{\bar{u}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{4}{3}\sin^2\theta_W\right]\cdot u - \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{2}{3}\sin^2\theta_W\right] D}$$

Putting photons & Z-bosons together get

$$\boxed{\mathcal{L}_{n.c.} = \frac{g}{4\cos\theta_W}\Big\{\bar{u}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{4}{3}\sin^2\theta_W\right)-(1+\gamma_5)\frac{4}{3}\sin^2\theta_W\right]u - \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{2}{3}\sin^2\theta_W\right)-(1+\gamma_5)\frac{2}{3}\sin^2\theta_W\right]D + \sum_f e_f\bar{q}_f\gamma\cdot A\, q_f}$$

(neutral current $\mathcal{L}$)