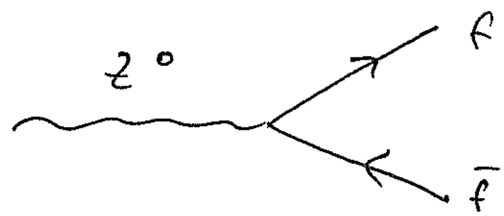
Last time

Decay of the Z-boson



after some algebra we obtained

$$\boxed{\Gamma_Z = \frac{g^2 M_Z}{192\pi \cos^2\theta_W}\left\{ 2N_\nu + \left[1+(1-4\sin^2\theta_W)^2\right]N_e + 3\left[1+\left(1-\frac{8}{3}\sin^2\theta_W\right)^2\right]N_u + 3\left[1+\left(1-\frac{4}{3}\sin^2\theta_W\right)^2\right]N_d \right\}}$$

$N_u = 2$ (u, c) ~ up type quarks, top is too heavy

$N_d = 3$ (d, s, b)

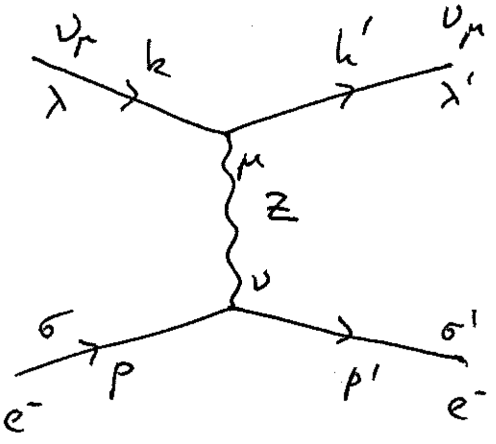
$N_e = 3$ (e, μ, τ)

N_ν ~ can measure indirectly

LEP: $N_\nu = 2.984 \pm 0.008$ ~ 3 neutrino generations!

Elastic electron-neutrino scattering.

Consider $\nu_\mu + e \to \nu_\mu + e$



We know that

$$\mathcal{L}_Z = \frac{g}{4\cos\theta_W}\Big\{ \bar{\nu}_e \gamma\cdot Z(1-\gamma_5)\nu_e$$

$$+ 2\sin^2\theta_W\, \bar{e}\gamma\cdot Z(1+\gamma_5)e$$

$$+ (2\sin^2\theta_W - 1)\,\bar{e}\gamma\cdot Z(1-\gamma_5)e\Big\} + (\mu,\tau)$$

$$\equiv \frac{g}{2\cos\theta_W}\Big\{ g_L^{\nu_e}\,\bar{\nu}_e\gamma\cdot Z(1-\gamma_5)\nu_e + g_R^e\,\bar{e}\gamma\cdot Z(1+\gamma_5)e$$

$$+ g_L^e\,\bar{e}\gamma\cdot Z(1-\gamma_5)e\Big\} + \dots$$

$\uparrow$ Definition

Scattering amplitude:

$$iM = \left(\frac{ig}{2\cos\theta_W}\right)^2 g_L^{\nu}\bar{u}_{\lambda'}(k')\gamma_\mu(1-\gamma_5)u_\lambda(k)\,\bar{u}_{\sigma'}(p')\left[g_L^e\gamma_\nu(1-\gamma_5)\right.$$

$$\left. + g_R^e\gamma_\nu(1+\gamma_5)\right]u_\sigma(p)\quad \underbrace{\frac{-i\left[g^{\mu\nu} - \frac{(k-k')^\mu(k-k')^\nu}{M_Z^2}\right]}{(k-k')^2 - M_Z^2 + i\varepsilon}}_{\simeq \frac{i}{M_Z^2}g^{\mu\nu} \text{ at low energy} \ll M_Z^2}$$

$$\Rightarrow\quad M \simeq \frac{-g^2}{4M_Z^2\cos^2\theta_W}\, g_L^{\nu}\bar{u}_{\lambda'}(k')\gamma_\mu(1-\gamma_5)u_\lambda(k)\,\bar{u}_{\sigma'}(p')\left[g_L^e\gamma^\mu(1-\gamma_5)\right.$$

$$\left. + g_R^e\gamma^\mu(1+\gamma_5)\right]u_\sigma(p) \quad\Rightarrow$$

$$\Rightarrow \sum_{\lambda,\lambda',\sigma,\sigma'} |M|^2 \simeq \frac{g^4}{16 M_Z^4 \cos^4\theta_W} L^-_{\mu\nu}(k,k') (g_L^{\nu_\mu})^2 \cdot \Big[(g_L^e)^2 L^{-\mu\nu}(p,p')$$

$$+ (g_R^e)^2 L^{+\mu\nu}(p,p') \Big] = \frac{g^4}{16 M_Z^4 \cos^4\theta_W} (g_L^{\nu_\mu})^2 \Big[(g_L^e)^2 \cdot$$

$$\cdot L^-_{\mu\nu}(k,k') L^{-\mu\nu}(p,p') + (g_R^e)^2 L^-_{\mu\nu}(k,k') L^{+\mu\nu}(p,p') \Big],$$

where we have used the following result and definition: [Use $\mathrm{tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$, $\mathrm{tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\varepsilon^{\mu\nu\rho\sigma}$]

$$\sum_{\lambda,\lambda'} \bar{u}_{\lambda'}(k') \gamma_\mu (1\mp\gamma_5) u_\lambda(k) \cdot u^+_\lambda(k) (1\mp\overset{\gamma_5}{\overset{\shortparallel}{\gamma_5^+}}) \gamma_\nu^+ \gamma^0 u_{\lambda'}(k')$$

$$= \mathrm{tr}[\gamma_\mu(1\mp\gamma_5) \not{k} \gamma^0 (1\mp\gamma_5)\gamma_\nu^+ \gamma^0 \not{k}'] = 2 \cdot$$

$$\cdot \mathrm{tr}[\gamma_\mu(1\mp\gamma_5)\not{k} \underbrace{\gamma^0\gamma_\nu^+\gamma^0}_{\gamma_\nu} \not{k}'] = 2\,\mathrm{tr}[\gamma_\mu(1\mp\gamma_5) \cdot$$

$$\cdot \not{k}\gamma_\nu\not{k}'] = 8[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i\varepsilon^\alpha{}_\nu{}^\beta{}_\mu k_\alpha k'_\beta]$$

$$= 8[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \pm i \underbrace{\varepsilon^{\alpha\nu\beta\mu}}_{=\varepsilon_{\mu\nu\alpha\beta}} k_\alpha k'_\beta] \equiv L^\mp_{\mu\nu}(k,k')$$

Now, $L^-_{\mu\nu}(k,k') L^{\mp\mu\nu}(p,p') = 64[k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu}$

$$+ i\varepsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta][p^\mu p'^\nu + p^\nu p'^\mu - p\cdot p' g^{\mu\nu} \pm i\varepsilon^{\mu\nu\rho\sigma} p_\rho p'_\sigma]$$

$$= 64[2p\cdot k\, p'\cdot k' + 2p\cdot k'\, p'\cdot k \cancel{- 4p\cdot p' k\cdot k'} + \cancel{\overset{\sim g_{\mu\nu}g^{\mu\nu}}{4\, p\cdot p' k\cdot k'}}$$

$$\mp \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} k^\alpha k'^\beta p_\rho p'_\sigma] = \Big| \text{as } \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = -2\delta_\alpha{}^\rho\delta_\beta{}^\sigma + 2\delta_\alpha{}^\sigma\delta_\beta{}^\rho$$

$$= 128\left[p\cdot k\, p'\cdot k' + p\cdot k' p'\cdot k \mp \left(-p\cdot k\, p'\cdot k' + p\cdot k' p'\cdot k\right)\right]$$

$$= 128 \cdot \begin{cases} 2\, p\cdot k\, p'\cdot k' & \text{"}-\text{"} \\ 2\, p\cdot k' p'\cdot k & \text{"}+\text{"} \end{cases}$$

$$\underbrace{\frac{1}{2}\sum_{\lambda,\lambda',\sigma,\sigma'} |M|^2}_{\langle |M|^2 \rangle\ \parallel} = \frac{1}{2}\,\frac{g^4}{16\cdot M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2 128\cdot 2$$

averaging over electron polarizations

$$\cdot\left[\left(g_L^e\right)^2 p\cdot k\, p'\cdot k' + \left(g_R^e\right)^2 p\cdot k' p'\cdot k\right]$$

$$= \frac{8 g^4}{M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2\left[\left(g_L^e\right)^2 \underbrace{p\cdot k\, p'\cdot k'}_{(p\cdot k)^2} + \left(g_R^e\right)^2 \underbrace{p\cdot k' p'\cdot k}_{(p\cdot k')^2}\right]$$

$$p+k = p'+k' \Rightarrow p\cdot k = p'\cdot k' \Rightarrow$$

$$p-k' = p'-k \Rightarrow p\cdot k' = p'\cdot k$$

The cross section is

$$d\sigma = \frac{1}{2E_p\, 2E_K\, |\vec{v}_p - \vec{v}_k|}\,\frac{d^3p'}{(2\pi)^3 2E_{p'}}\,\frac{d^3k'}{(2\pi)^3 2E_{k'}}\,\langle |M|^2\rangle (k')$$

$$\cdot (2\pi)^4\,\delta^4(p'+k'-p-k) =$$

work in the lab frame = rest frame of electron $\Rightarrow \vec{v}_p = 0,\ E_p = m_e$

$|\vec{v}_k| = 1$ as $m_\nu = 0. \Rightarrow p\cdot k = m_e E_k$

$p\cdot k' = m_e E_{k'}$

$$= \frac{1}{2m_e\, 2E_K}\,\frac{d^3k'}{(2\pi)^3 2E_k}\,\frac{1}{2E_{p'}}\,2\pi\,\delta(\underbrace{E_{p'}}_{\sqrt{(\vec{k}-\vec{k}')^2+m_e^2}} + E_{k'} - \underbrace{E_p}_{m_e} - E_K)\,.$$

$$\cdot\,\frac{8 g^4 m_e^2}{M_Z^4\cos^4\theta_W}\left(g_L^{\nu_\mu}\right)^2\left[\left(g_L^e\right)^2 E_k^2 + \left(g_R^e\right)^2 E_{k'}^2\right]$$

Now, we can integrate over angles:

$$\frac{d^3k'}{(2\pi)^3\, 2E_{k'}} \frac{1}{2E_{p'}} \cdot 2\pi\, \delta\Big(\sqrt{(\vec{k}-\vec{k}')^2+m_e^2} + \underbrace{k'}_{=|\vec{k}'|} - m_e - \underbrace{k}_{=|\vec{k}|}\Big) =$$

$$= \frac{k'^2\, dk' \cdot 2\pi \cdot d\cos\theta}{(2\pi)^3\, 2k'\, 2E_{p'}}\, \delta\Big(\sqrt{k^2+k'^2-2kk'\cos\theta+m_e^2} + k' - m_e - k\Big)$$

$$= \frac{\cancel{k'}\, dk'}{8\pi\, \cancel{E_{p'}}} \cdot \frac{1}{\frac{1}{\cancel{2E_{p'}}}\, \cancel{2}k\cancel{k'}} = \frac{dk'}{8\pi k} = \frac{dE_{p'}}{8\pi k}$$ ← can measure recoil electron

since $E_{p'} + k' = m_e + k \Rightarrow |dk'| = |dE_{p'}|$

$$\Rightarrow \frac{d\sigma}{dE_{p'}} = \frac{1}{\cancel{8}\pi k} \frac{1}{4m_e E_k} \frac{\cancel{8}g^4 m_e^2}{M_Z^4 \cos^4\theta_W} (g_L^{\nu_\mu})^2 \Big[(g_L^e)^2 E_k^2 + (g_R^e)^2 \cdot E_{k'}^2 \Big]$$

$$\frac{d\sigma}{dE_{p'}} = \frac{g^4 m_e}{4\pi M_Z^4 \cos^4\theta_W} (g_L^{\nu_\mu})^2 \Big[(g_L^e)^2 + (g_R^e)^2 \Big(\frac{E_{k'}}{E_k}\Big)^2 \Big]$$

Here $E_{k'} = m_e + E_k - E_{p'}$.

$\Rightarrow$ can measure $|g_L^e|$ and $|g_R^e|$ (given $g_L^{\nu_\mu}$), to test SM predictions

What if we do not make this low-energy approximation? Need to keep the whole Z-boson propagator. However:

$$\bar{u}_{\lambda'}(k')(\not{k}-\not{k}')(1-\gamma_5)u_\lambda(k) = \left| \begin{array}{l} (\not{p}-m)u(p)=0 \\ \bar{u}(p)(\not{p}-m)=0 \end{array} \right\} \text{Dirac eq'n}$$

$$= \bar{u}_{\lambda'}(k')(\not{k}-m)(1-\gamma_5)u_\lambda(k) = \bar{u}_{\lambda'}(k')\Big[(1+\gamma_5)\underset{m}{\not{k}}$$

$$-m(1-\gamma_5)\Big]u_\lambda(k) = 2m\,\bar{u}_{\lambda'}(k')\gamma_5 u_\lambda(k) = 0$$

as $m_\nu = 0$ $\Rightarrow$ can drop $\dfrac{(k-k')^\mu (k-k')^\nu}{M_Z^2}$ term in the Z propagator for massless ν's.

$\Rightarrow$ the only relativistic/high energy correction is $\dfrac{1}{M_Z^2} \to \dfrac{1}{M_Z^2 - t}$

where $t = (k-k')^2 = (p-p')^2 = 2m_e^2 - 2p\cdot p' =$

$= \Big|\begin{array}{l}\text{rest frame} \\ \text{of the } e^-\end{array} = 2m_e^2 - 2m_e E_{p'} = 2m_e(m_e - E_{p'})$

$$\Rightarrow \boxed{\frac{d\sigma}{dE_{p'}} = \frac{g^4 m_e}{4\pi\left[M_Z^2 - 2m_e(m_e - E_{p'})\right]^2 \cdot \cos^4\theta_W} \cdot \left(g_L^{\nu_\mu}\right)^2 \cdot \left[(g_L^e)^2 + (g_R^e)^2\left(\frac{E_{k'}}{E_k}\right)^2\right]}$$

exact result

When is this correction important?

Need $M_Z^2 \sim 2 m_e E_{p'} \Rightarrow E_{p'} \sim \frac{M_Z^2}{m_e}$

$\Rightarrow E_{p'} \sim 2 \cdot 10^5 M_Z \simeq 1.7 \cdot 10^7 \, GeV \approx$

$\simeq 17 \, PeV \Rightarrow$ need $E_{p'} \simeq 17 \, PeV$,

very energetic e^- $\Rightarrow$ need very energetic

ν's, possible in cosmic rays.