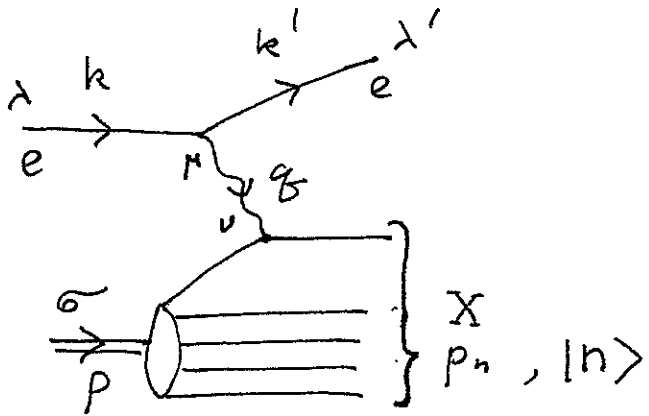


Parton Model and Deep Inelastic Scattering (80)

DIS



$$e(k) + \text{proton}(p) \rightarrow e(k') + X$$

Rest frame of the proton: $p = (m_p, 0, 0, 0)$

$$k = (\epsilon, 0, 0, k) \approx (\epsilon, 0, 0, \epsilon) \quad \begin{matrix} \text{energy} \sim \text{many GeV} \\ \text{neglect } m_e \end{matrix}$$

$$k' = (\epsilon', \epsilon' \sin \theta, 0, \epsilon' \cos \theta)$$

Define:

$$\begin{aligned} \rightarrow Q^2 &\equiv -q^2 = -(k-k')^2 = 2k \cdot k' = 2\epsilon\epsilon'(1-\cos\theta) = \\ &= 4\epsilon\epsilon' \sin^2 \frac{\theta}{2} \end{aligned}$$

$$v \equiv \frac{p \cdot q}{m_p} = \epsilon - \epsilon' \quad \begin{matrix} \leftarrow \text{only in } p\text{'s rest frame} \\ \leftarrow \text{energy lost by } e^- \end{matrix}$$

$$\rightarrow x \equiv \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2m_p v} \quad \text{Bjorken } x \text{ variable}$$

$$\hat{s} \equiv (p+q)^2 \Rightarrow x = \frac{Q^2}{Q^2 + \hat{s}} \quad \left\| \begin{aligned} x &= \frac{Q^2}{2p \cdot q} = \frac{-q^2}{(p+q)^2 - p^2 - q^2} \\ &= \frac{Q^2}{\hat{s} - m_p^2 + Q^2} \approx \frac{Q^2}{\hat{s} + Q^2} \end{aligned} \right.$$

Q^2 and x are important / independent!

$$\left(v = \frac{Q^2}{2xm} \right)$$

Interaction amplitude:

$$T_{\sigma, \lambda, \lambda'}(n) = +ie \bar{u}_{\lambda'}(k') \gamma_{\mu} u_{\lambda}(k) \frac{-ig^{\mu\nu}}{q^2}$$

$$ie \langle n | j_{\nu}(0) | p, \sigma \rangle$$

where
$$j_{\nu}(x) = \sum_f e_f \bar{q}_f(x) \gamma_{\nu} q_f(x)$$

with $e_f = +2/3, -1/3, \dots$ (quark flavors)
electric charges

j_{ν} is EM current

Let's calculate the ^{total} cross-section ($|\vec{v}_1 - \vec{v}_2| = 1$):

$$d\sigma = \frac{1}{4} \sum_{\sigma, \lambda, \lambda'} \sum_n |T_{\sigma, \lambda, \lambda'}(n)|^2 (2\pi)^4 \delta^4(q + p - p_n)$$

spin averaging

$$\frac{1}{4} \frac{d^3k'}{2m} \frac{2\varepsilon}{2\varepsilon'} \frac{1}{(2\pi)^3} = \frac{e^4}{Q^4}$$

incoming particles

$$\frac{1}{2} \sum_{\lambda, \lambda'} [\bar{u}_{\lambda'}(k') \gamma_{\mu} u_{\lambda}(k)]^* [\bar{u}_{\lambda'}(k') \gamma_{\nu} u_{\lambda}(k)]$$

$$\frac{1}{2} \sum_{\sigma, n} \langle n | j^{\mu}(0) | p, \sigma \rangle^* \langle n | j^{\nu}(0) | p, \sigma \rangle$$

$$(2\pi)^4 \delta(q + p - p_n) \frac{d^3k'}{2m} \frac{2\varepsilon}{2\varepsilon'} \frac{1}{(2\pi)^3} \quad \text{--- } 4\pi m_p W^{\mu\nu}$$

Therefore

(82)

$$\frac{d\sigma}{d^3k'} = \frac{\alpha_{EM}^2}{Q^4 \epsilon \cdot \epsilon'} l_{\mu\nu} W^{\mu\nu}$$

(rest frame of proton)

$$l_{\mu\nu} = \frac{1}{2} \sum_{\lambda, \lambda'} \left[\bar{u}_{\lambda'\alpha}(k') (\gamma_\mu)_{\alpha\beta} u_{\lambda\beta}(k) \right]^* \bar{u}_{\lambda'\alpha'}(k')$$

$$(\gamma_\nu)_{\alpha'\beta'} u_{\lambda\beta'}(k) \stackrel{\text{see next page}}{=} \frac{1}{2} \sum_{\lambda, \lambda'} \bar{u}_\lambda(k) \gamma_\nu u_{\lambda'}(k')$$

$$\bar{u}_{\lambda'}(k') \gamma_\nu u_\lambda(k) = \frac{1}{2} \text{Tr} [\gamma_\mu \gamma \cdot k' \gamma_\nu \gamma \cdot k]$$

as $\sum_{\lambda'} u_{\lambda'}(k') \bar{u}_{\lambda'}(k') = \gamma \cdot k' + m_e \approx \gamma \cdot k'$

$$\text{Using } \text{Tr} [\gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\nu] = 4 [g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\mu} g_{\beta\nu}]$$

we get

$$l_{\mu\nu} = 2 (k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k')$$

$$W_{\mu\nu} = \frac{1}{4\pi m} \frac{1}{2} \sum_{\sigma, n} \langle n | j^\mu(0) | p, \sigma \rangle^* \langle n | j^\nu(0) | p, \sigma \rangle$$

$$\cdot (2\pi)^4 \delta^4(q + p - p_n) = \frac{1}{4\pi m} \frac{1}{2} \sum_{\sigma, n} \int d^4x e^{iq \cdot x}$$

$$\langle p, \sigma | j^\mu(x) | n \rangle \langle n | j^\nu(0) | p, \sigma \rangle$$

$$\stackrel{''}{=} e^{ip \cdot x} j^\mu(0) e^{-ip \cdot x} \quad (\text{Heisenberg picture})$$

$$[\bar{u}_{\lambda'}(k') \gamma^M u_{\lambda}(k)]^* = [u_{\lambda'}^{\dagger} \gamma^0 \gamma^M u_{\lambda}]^{\dagger} = \quad (82')$$

↑
as this is a scalar

$$= u_{\lambda}^{\dagger} \underbrace{(\gamma^0)^2}_{\mathbb{1}} \gamma^{+M} \gamma^{\dagger 0} u_{\lambda'} = \bar{u}_{\lambda}(k) \gamma^0 \gamma^{+M} \gamma^{\dagger 0} u_{\lambda'}$$

$$\text{now, } \gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Rightarrow \gamma^{\dagger 0} = \gamma^0 \Rightarrow \gamma^0 \gamma^{+M} \gamma^{\dagger 0} = \gamma^0 \gamma^{+M} \gamma^0$$

Let's find $\gamma^0 \gamma^{+M} \gamma^0$:

$$M=0 \Rightarrow \gamma^0 \gamma^{\dagger 0} \gamma^0 = \gamma^0$$

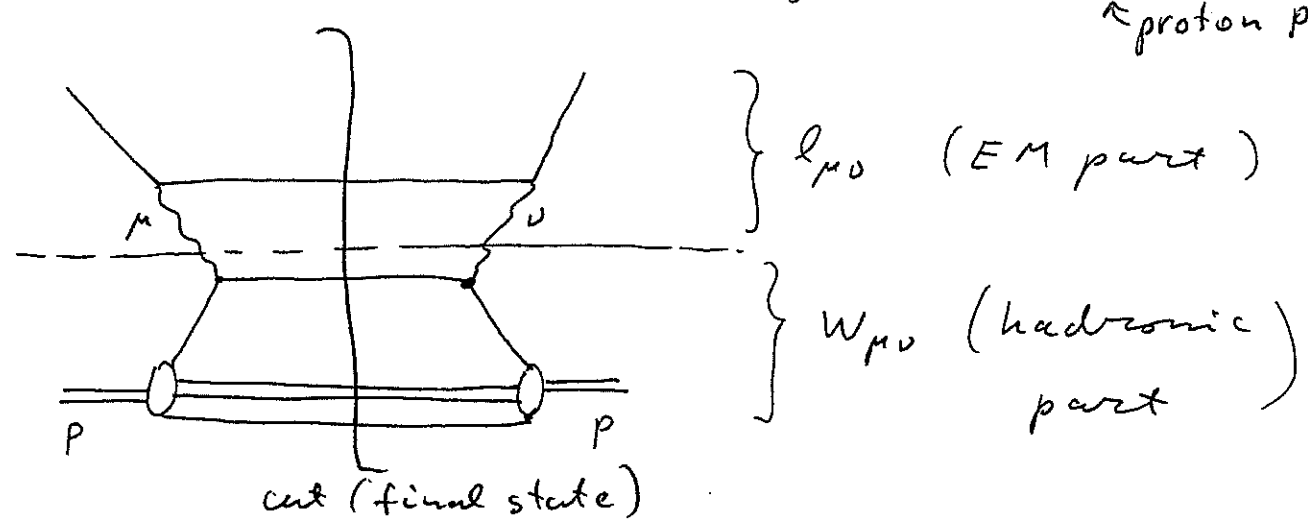
$$M=i \Rightarrow \gamma^0 \gamma^{\dagger i} \gamma^0 = -\gamma^0 \gamma^i \gamma^0 = \gamma^i (\gamma^0) \gamma^0 = \gamma^i$$

$$\Rightarrow \boxed{\gamma^0 \gamma^{+M} \gamma^0 = \gamma^M}$$

\Rightarrow get $\bar{u}_{\lambda}(k) \gamma^M u_{\lambda'}(k')$ as desired.

$$W_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x e^{iq \cdot x} \langle p | j_\mu(x) j_\nu(0) | p \rangle$$

(averaged over σ)
 \uparrow proton polarization



$$W_{\mu\nu}(p, q) = A(x, Q^2) p_\mu p_\nu + B(x, Q^2) g_\mu g_\nu + C(x, Q^2) g_{\mu\nu} + D(x, Q^2) (p_\mu g_\nu + p_\nu g_\mu) + E(x, Q^2) (p_\mu g_\nu - p_\nu g_\mu) + F(x, Q^2) \epsilon_{\mu\nu\sigma\tau} p^\sigma q^\tau$$

$F = 0$ in $\gamma^* p, \gamma^* A$ (F comes from γ_5 's, appears in ν DIS).

(i) $q_\mu W^{\mu\nu} = 0$ (current conservation)
 $q_\mu \rightarrow \partial_\mu \Rightarrow \partial_\mu j^\mu(x) = 0$

$$A p_\nu (p \cdot q) + B q_\nu \cdot q^2 + C q_\nu + D (p \cdot q q_\nu + q^2 p_\nu) + E (p \cdot q q_\nu - q^2 p_\nu) = 0$$

(ii) $q_\nu W^{\mu\nu} = 0 = A p_\mu (p \cdot q) + B q^2 g_\mu^\nu + D (p \cdot q q_\mu + q^2 p_\mu) + E (p_\mu q^2 - p \cdot q q_\mu) = 0$

