

Last time

DIS

$$\frac{d\sigma}{d^3k'} = \frac{4dEm^2}{Q^4} \left[2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

$$= \frac{4dEm^2}{Q^4} \left[\frac{2}{m_p} F_1(x, Q^2) \sin^2 \frac{\theta}{2} + \frac{2m_p x}{Q^2} F_2(x, Q^2) \cos^2 \frac{\theta}{2} \right]$$

W_1, W_2 = structure functions

F_1, F_2 = — (— , $F_1 = m_p W_1$

$F_2 = \nu W_2$

The Parton Model (cont'd)

Infinite Momentum Frame:

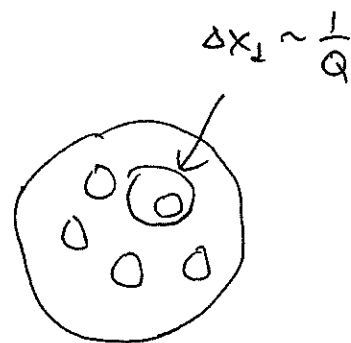
$$p^\mu = \left(p + \frac{m^2}{2p}, 0, 0, p \right)$$

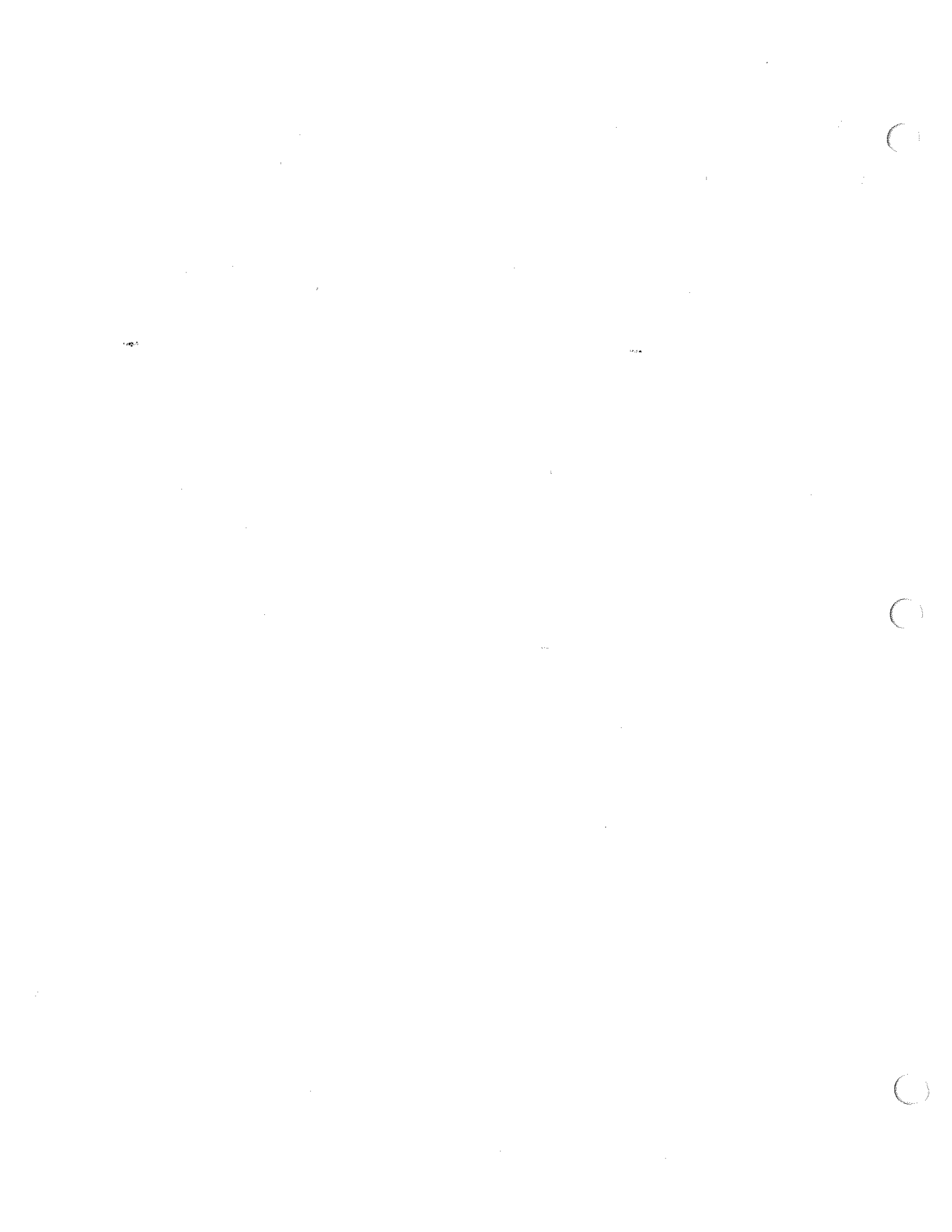
$$q^\mu = \left(q^0, \vec{q}_\perp, 0 \right)$$

$p \gg Q$, p is very large

$$q^0 = \frac{Q^2}{2xp} \ll q_\perp \Rightarrow Q^2 \simeq -q_\perp^2 \Rightarrow$$

$\frac{1}{Q} = \Delta x_\perp \sim$ distance probed by γ^* in DIS \rightarrow



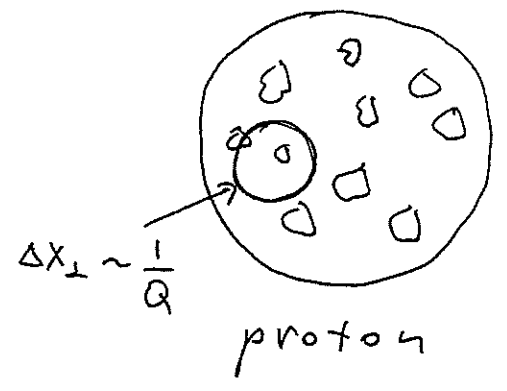


$Q^2 = q^2 \Rightarrow$ photon acts like a microscope

in transverse plane:

$$\Delta x_{\perp} \cdot q_{\perp} \sim 1 \quad (\hbar = 1)$$

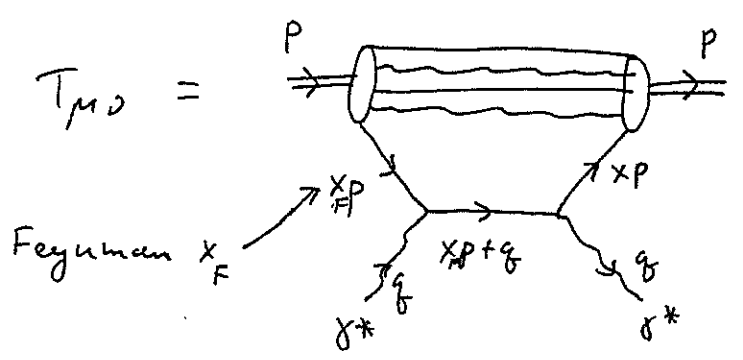
$$\Delta x_{\perp} \sim \frac{1}{q_{\perp}} \sim \frac{1}{Q}$$



large $Q \sim$ resolve just 1 quark

Define
$$T_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x e^{iq \cdot x} \frac{1}{2} \sum_{\sigma} \langle p, \sigma | T j_{\mu}(x) j_{\nu}(0) | p, \sigma \rangle$$

$W_{\mu\nu} = 2 \text{Im} (i T_{\mu\nu})$ (optical theorem)



"Forward Amplitude"

(Def) Feynman-x: the fraction of proton's longitudinal momentum carried by struck quarks

typical interaction time in proton's rest frame

is $\frac{1}{\Lambda_{QCD}} \Rightarrow$ boost to get $\frac{P}{m} \frac{1}{\Lambda} \equiv \tau_{\Lambda}$

int. time of DIS is $\tau_{DIS} \approx \frac{1}{q^0}$, where $q^0 \approx \frac{Q^2}{2xP}$ is struck quark's velocity: $\tau_{DIS} \approx \frac{2xP}{Q^2}$

time-ordered product: (denoted T) (88)

$$T j_\mu(x) j_\nu(y) \equiv \theta(x^0 - y^0) j_\mu(x) j_\nu(y) + \theta(y^0 - x^0) j_\nu(y) j_\mu(x)$$

note: currents do not commute with each other in general \Rightarrow not a trivial object.

$$2 \text{Im}(i T_{\mu\nu}) = 2 \text{Im} \left[i \cdot \frac{1}{4\pi m_p} \int d^4x e^{i q \cdot x} \langle p | \theta(x^0) j_\mu(x) j_\nu(0) + \theta(x^0) j_\nu(0) j_\mu(x) | p \rangle \right]$$

$$= 2 \cdot \frac{1}{4\pi m_p} \sum_n \text{Re} \left\{ \int d^4x e^{i q \cdot x + i p \cdot x - i p_n \cdot x} \theta(x^0) \langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle + \int d^4x e^{i q \cdot x + i p_n \cdot x - i p \cdot x} \theta(-x^0) \langle p | j_\nu(0) | n \rangle \langle n | j_\mu(0) | p \rangle \right\}$$

$$= 2 \frac{1}{4\pi m_p} \sum_n (2\pi)^3 \delta(\vec{q} + \vec{p} - \vec{p}_n) \left(\frac{-1}{i(q^0 + p^0 - p_n^0 + i\epsilon)} \langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle + (2\pi)^3 \delta(\vec{q} + \vec{p}_n - \vec{p}) \cdot \text{Re} \left(\frac{1}{i(q^0 + p_n^0 - p^0 - i\epsilon)} \langle p | j_\nu(0) | n \rangle \langle n | j_\mu(0) | p \rangle \right) \right)$$

not physical \Rightarrow drop (after including $\delta(q^0 + p_n^0 - p^0)$)

$$= 2 \frac{1}{4\pi m_p} \sum_n (2\pi)^3 \delta(\vec{q} + \vec{p} - \vec{p}_n) (-) \left(\text{Im} \frac{1}{q^0 + p^0 - p_n^0 + i\epsilon} \right) \cdot \langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle$$

$$\langle n | j_\nu(0) | p \rangle = \int dx \text{Im} \frac{1}{x + i\epsilon} = -\pi \delta(x) = \frac{1}{4\pi m_p} \sum_n (2\pi)^4 \delta^4(q + p - p_n)$$

$$\langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle = W_{\mu\nu} \text{ as desired.}$$

(can prove that $\langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle$ is real)

consider $\langle p | j_\mu(0) | n \rangle$. This is a 4-vector (88')
which depends on two 4-vectors, p_μ and p_n^μ .

We write

$$\langle p | j_\mu(0) | n \rangle = \alpha e^{i\varphi_1} p_\mu + \beta e^{i\varphi_2} p_{n\mu}, \quad (*)$$

where α, β, φ_1 , and φ_2 are all real. (and unknown)

Next consider $\langle p | j_\mu(x) | n \rangle$. Current conservation,

$$\partial_\mu j^\mu(x) = 0, \text{ implies that } \partial^\mu \langle p | j_\mu(x) | n \rangle = 0$$

$$\Rightarrow \partial^\mu \langle p | e^{i\hat{p}\cdot x} j_\mu(0) e^{-i\hat{p}\cdot x} | n \rangle = 0$$

$$\Rightarrow \left(\partial^\mu e^{i(p-p_n)\cdot x} \right) \langle p | j_\mu(0) | n \rangle = 0$$

$$\Rightarrow (p-p_n)^\mu \langle p | j_\mu(0) | n \rangle = 0$$

Now, plug in formula (*):

$$(p-p_n)^\mu \left[\alpha e^{i\varphi_1} p_\mu + \beta e^{i\varphi_2} p_{n\mu} \right] = 0$$

$$\underbrace{\alpha p \cdot (p-p_n)}_{\text{real}} e^{i\varphi_1} = - \underbrace{\beta p_n \cdot (p-p_n)}_{\text{real}} e^{i\varphi_2}$$

\Rightarrow for these two complex numbers to be equal, one has to have $\varphi_1 = \varphi_2 + \pi n$, $n \in \text{integers}$

$$\Rightarrow e^{i\varphi_1} = \pm e^{i\varphi_2}$$

\uparrow due to the πn term possibility

back to (*)

$$\Rightarrow \langle p | j_\mu(0) | n \rangle = (\alpha p_\mu \pm \beta p_{n\mu}) e^{i\varphi_1}$$

$$\Rightarrow \langle p | j_\mu(0) | n \rangle \langle n | j_\nu(0) | p \rangle =$$

$$= (\alpha p_\mu \pm \beta p_{n\mu}) (\alpha p_\nu \pm \beta p_{n\nu}) = \text{real \#}.$$

if x is small (≤ 1) and Q is large



interaction is "instantaneous"

as $\frac{2xp}{Q^2} \ll \frac{p}{m} \frac{1}{\Lambda} \Rightarrow$ as $2xm\Lambda \ll 2m\Lambda \ll Q^2$

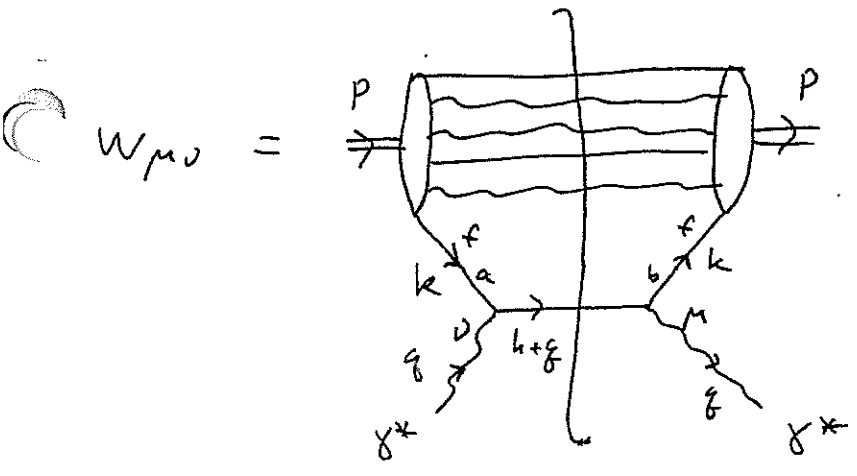
Define light cone variables:

for vector V^M one has $V^+ = \frac{1}{\sqrt{2}} (V^0 + V^3)$

$\underline{V} = (V^1, V^2)$ $V^- = \frac{1}{\sqrt{2}} (V^0 - V^3)$

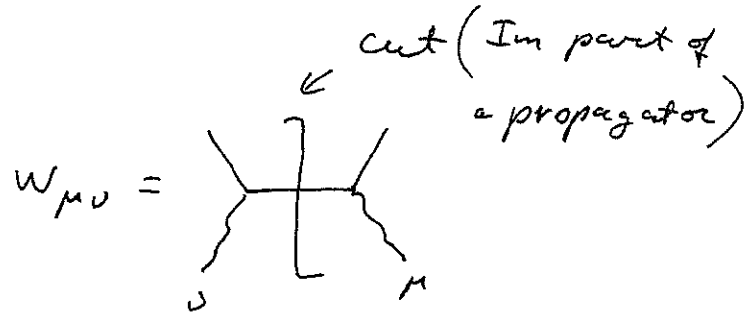
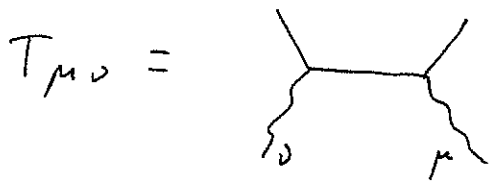
(2d + transverse vector)

$V_1 \cdot V_2 = V_{1\mu} V_2^\mu = V_{1+} V_{2-} + V_{1-} V_{2+} - \underline{V}_1 \cdot \underline{V}_2$



$A_{ab}^f(p, k)$
 Dirac indices
 e.g.
 $p^+ = \frac{p}{\sqrt{2}}$
 $p^- = \frac{m^2}{\sqrt{2}p}$
 $p^+ \gg p^-$

as $W_{\mu\nu} = 2 \text{Im} (i T_{\mu\nu})$



$\frac{k}{i} \frac{1}{k^2 - m^2 + i\epsilon} \Rightarrow \frac{k}{2\pi} \int \frac{1}{\delta^{(+)}(k^2 - m^2)}$

as $2 \text{Im} \frac{-1}{k^2 - m^2 + i\epsilon} = 2\pi \delta^{(+)}(k^2 - m^2)$

We write

(90)

$$W_{\mu\nu} = \frac{1}{\frac{4\pi m_p}{2}} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A_{ab}^f(p, k) [\delta_\mu \delta_0(k+q) \delta_\nu]_{ba}$$

\cdot $(2\pi) \delta((k+q)^2)$ where A_{ab}^f is the rest of the diagram (see p.89).

Start calculating assuming that

$$Q^2 \gg k^2, \quad \underline{k} \cdot \underline{q}, \quad h^+ \gg h^- \quad (\text{IMF})$$

$$\therefore (k+q)^2 = k^2 + 2h^+ q^- + 2h^- q^+ - \underline{k} \cdot \underline{q} - Q^2$$

$$q_3 = 0 \Rightarrow q^+ = q^- \Rightarrow \text{as } h^+ \gg h^- \Rightarrow \text{drop } 2h^- q^+$$

dropping $k^2, \underline{k} \cdot \underline{q} \ll Q^2$ get

$$(k+q)^2 \approx 2h^+ q^- - Q^2$$

$$\Rightarrow \delta((k+q)^2) \approx \delta(2h^+ q^- - Q^2) = \delta\left(\frac{h^+}{p^+} 2p^+ q^- - Q^2\right)$$

$$\text{as } p \cdot q \approx p^+ q^- \Rightarrow \text{and } x_{Bj} = \frac{Q^2}{2p \cdot q}$$

$$\Rightarrow \delta((k+q)^2) \approx \frac{x_{Bj}}{Q^2} \delta\left(x_{Bj} - \frac{h^+}{p^+}\right)$$

$$\Rightarrow \left. x_{Bj} = \frac{k^+}{p^+} \right\} \text{ Feynman } x = \text{ Bjorken } x$$

physical meaning: light cone momentum fraction of the struck quark!