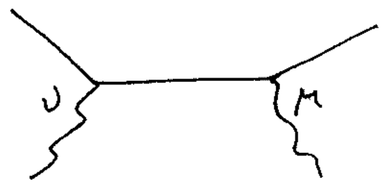
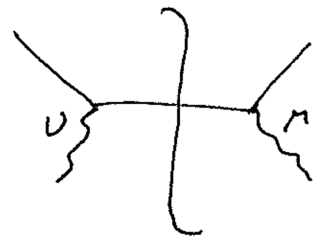
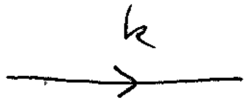
Last time | The Parton Model (cont'd)

We discussed the optical theorem in terms of its manifestation in DIS: $\boxed{W_{\mu\nu} = 2\,\mathrm{Im}(i\,T_{\mu\nu})}$

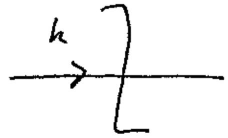
$T_{\mu\nu} =$ 

$W_{\mu\nu} =$ 

e.g. scalar propagator:



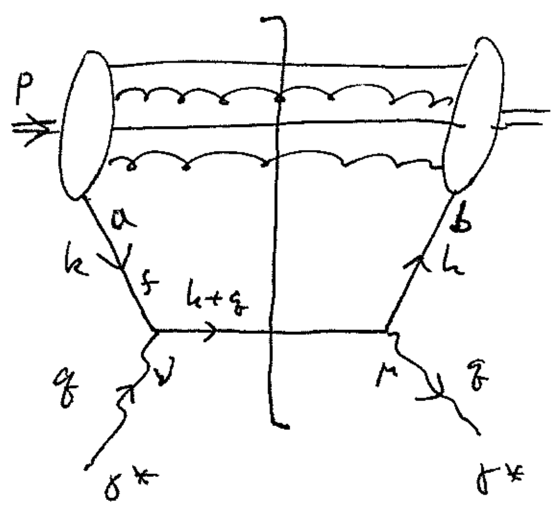
$$\frac{i}{k^2 - m^2 + i\varepsilon}$$

$\Rightarrow$ 

$$2\pi\,\delta^{(+)}(k^2 - m^2)$$
$$\parallel$$
$$2\pi\,\theta(k^0)\,\delta(k^2 - m^2)$$

↑ positive energy.

Indeed, $$2\,\mathrm{Im}\left(i\,\frac{i}{k^2 - m^2 + i\varepsilon}\right) = 2\pi\,\delta(k^2 - m^2)$$

$W_{\mu\nu} =$ 

$\Big\}\ A^f_{ab}(p,k)$ — ab: Dirac indices

We got

$$W_{\mu\nu} = \frac{1}{2m_p} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k) \left[\gamma_\mu (\not{k} + \not{q}) \gamma_\nu \right]_{ba} \cdot$$

$$\cdot \, \delta\left((k+q)^2 \right)$$

Using $Q^2 \gg k^2, \underline{k} \cdot q$ and $k^+ \gg k^-$ we got

$$\delta\left((k+q)^2 \right) \simeq \frac{x}{Q^2} \, \delta\left(x - \frac{k^+}{p^+} \right)$$

$\Rightarrow$ Bjorken x is the light-cone momentum fraction of the struck quark!

$$\gamma\cdot(k+q) = \gamma^+(k^-+q^-) + \gamma^-(k^++q^+) - \underline{\gamma}\cdot(\underline{k}+\underline{q})$$

after d^4k : $\gamma^+ \to p^+$ $\quad \gamma^- \to p^-$ $\quad \underline{\gamma} \to \underline{p} = 0$

$\Rightarrow$ as $p^+ \gg p^-$ keep γ^+ only, $k^-+q^- \simeq \dfrac{Q^2}{x\cdot 2p_+}$

$$'' \ \frac{(\underline{k}+\underline{q})^2}{2(k^++q^+)} \simeq \frac{\underline{q}^2}{2k^+} = \frac{Q^2}{2xp^+}$$

$$W_{\mu\nu} = \frac{1}{4m_p p_+} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A^f_{ab}(p,k) \left[\gamma_\mu \gamma^+ \gamma_\nu\right]_{ba} \cdot \delta\left(x - \frac{k^+}{p^+}\right)$$

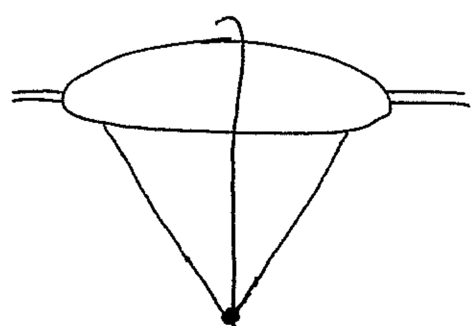
(see $p^\mu q^\nu$ decomp.)

leading $W_{\mu\nu}$ component $\searrow$ $\quad\quad$ $\swarrow$ symmetrize, as $W_{\mu\nu}$ is symmetric

Concentrate on $W_{ij} \sim \frac{1}{2}\left[\gamma_i \gamma^+ \gamma_j + \gamma_j \gamma^+ \gamma_i\right] =$

$= -\frac{1}{2}\gamma^+ \{\gamma_i, \gamma_j\} = -g_{ij}\gamma^+$ $\quad$ (we used $W_{ij} = W_{ji}$)

DIS now looks like



We have $W_{ij} \propto g_{ij}$ from diagram calculations.

On the other hand, since $\underline{p} = 0$

$$W_{ij} = -W_1\left(g_{ij} - \frac{q_i q_j}{q^2}\right) + \frac{W_2}{m_p^2} q_i q_j \left(\frac{p\cdot q}{q^2}\right)^2 =$$

$$= -W_1 g_{ij} + \frac{q_i q_j}{q^2}\left[W_1 + \frac{W_2}{m_p^2}\frac{(p\cdot q)^2}{q^2}\right] \propto g_{ij}$$

$$\Rightarrow \quad W_1 + \frac{W_2}{m_p^2} \frac{(p\cdot q)^2}{q^2} = 0$$

as $\nu = \frac{p\cdot q}{m_p}$ and $x = \frac{Q^2}{2p\cdot q} = -\frac{q^2}{2p\cdot q}$

we write $\boxed{\nu W_2 = 2 m x W_1}$ Callan-Gross Relation '69

follows from spin-$\frac{1}{2}$ nature of quarks!

(would be different for particles with different spin); equivalently: $\boxed{F_2(x,Q^2) = 2x F_1(x,Q^2)}$

Exercise: show that Callan-Gross relation leads to $\frac{d\sigma}{d^3k'} \sim \left[1 + \left(1 - \frac{\nu}{\varepsilon}\right)^2\right] W_1$

CG relation leads to

$$\nu W_2 = 2m_p x W_1 = \not{2}\not{m} x \cdot \frac{1}{\not{2}\not{m}2p_+} \sum_f e_f^2 \int \frac{d^4k}{(2\pi)^4} A_{ab}^f(p,k) \cdot$$

$$\cdot(\gamma^+)_{ba}\, \delta\left(x - \frac{k^+}{p^+}\right) \quad \Rightarrow \quad \underline{\text{defining}}\ \underline{\text{quark distribution function}}:$$

$$\boxed{q^f(x) \equiv \frac{1}{2p_+} \int \frac{d^4k}{(2\pi)^4} A_{ab}^f(p,k)\, (\gamma^+)_{ba}\, \delta\left(x - \frac{k^+}{p^+}\right)}$$

we get $\boxed{\nu W_2 = \sum_f e_f^2\, x\, q^f(x)}$ with $\nu W_2 = F_2(x,Q^2)$

no Q^2-dependence, only x-dependent

<u>Bjorken scaling</u> (see attached)

Bjorken scaling was first measured at SLAC in 1968: it killed string models and brought back field theories.

$$F_2(x) = \sum_f e_f^2 \, x \, q_f(x)$$

$$F_1 = \frac{F_2}{2x} = \frac{1}{2} \sum_f e_f^2 \, q_f(x)$$

F_1 = counts # of quarks in the proton with the longitudinal momentum fraction $= x$ (weighed by $\frac{1}{2} e_f^2$)

F_2 = gives the average x carried by quarks (weighed by e_f^2) $\otimes$ # of quarks at x.

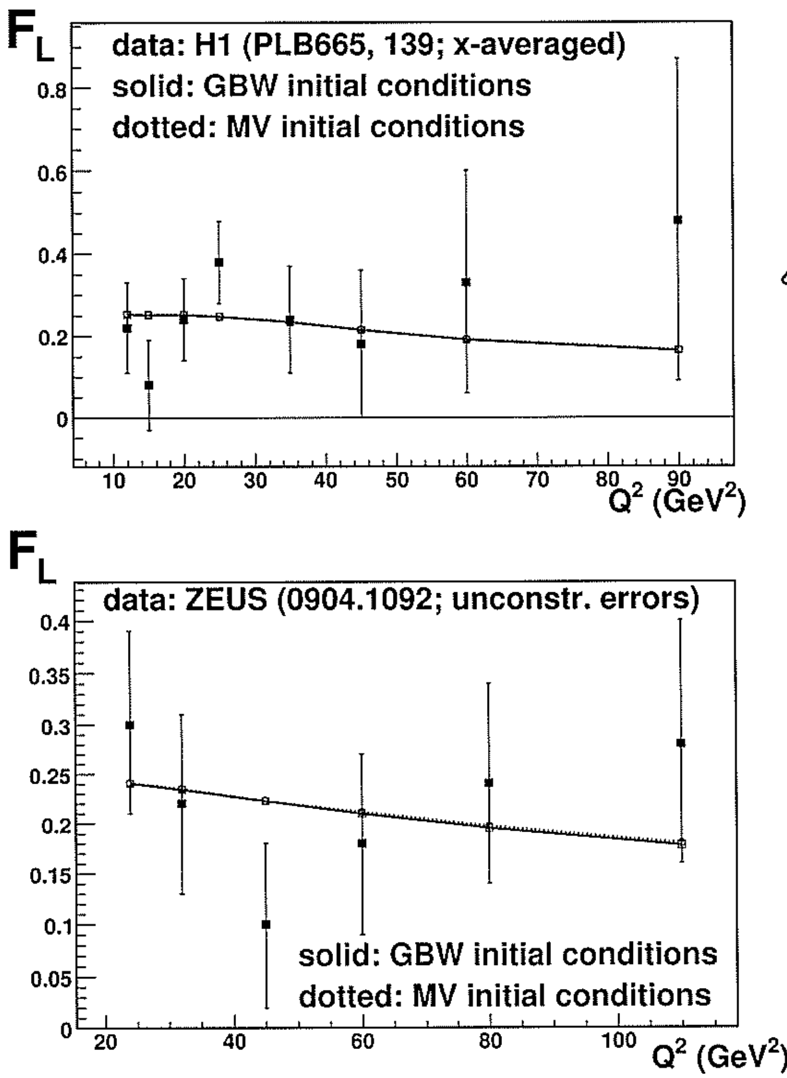
~ Jerome Friedman, Henry Kendall and Richard Taylor, Nobel Prize in Physics 1990 for the DIS experiments which led to establishing quark model (hence, for Bjorken scaling)

What about Bjorken?

(92'')

$F_L \equiv F_2 - 2x F_1$, F_L is zero in the naive Parton model

arXiv: 0902.1112 [hep-ph]



F_L is non-zero, but small (e.g. compared to F_2)

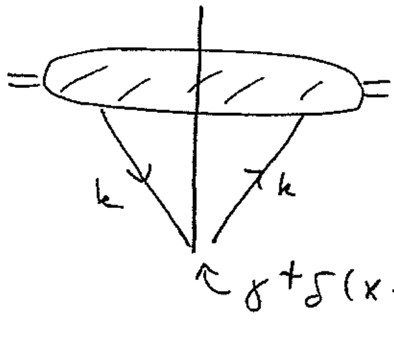
⇓

Callan-Gross relationship works!

Figure 3: Comparison between experimental data from the H1 [17] (upper plot) and ZEUS [18] (lower plot) Collaborations and the predictions of our model for $F_L(x, Q^2)$. Red solid lines and open squares correspond to GBW i.c., and blue dotted lines and open circles to MV i.c. The theoretical results have been computed at the same $\langle x \rangle$ as the experimental data, and then joined by straight lines. The error bars correspond to statistical and systematic errors added in quadrature for those data coming from [17], while they correspond to the error quoted for the unconstrained fit for those data coming from [18].

to allow a discrimination of the different UV behaviors of the two employed i.c.

Second, the fits using GBW i.c. and obtained by letting γ vary as a free pa-

$q^f(x) = \frac{1}{2p^+}$ 

$\Rightarrow$ often $p^f(x)$ is denoted $q(x)$.

> $q^f(x, Q^2)$ counts # of quarks with light cone momentum x and transverse momentum $k_T \leq Q$.

parton distribution function $(q^f \sim a^+ a)$

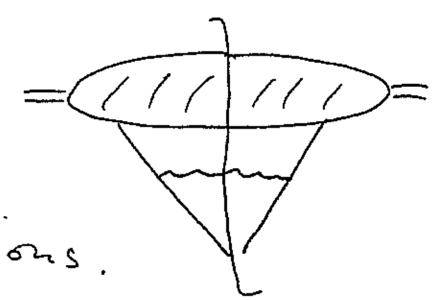
$\Rightarrow$ for a free quark $A^f_{ab}(p,k)(\gamma^+)_{ba} = \delta^4(p-k)\cdot(2\pi)^4$.

$\underbrace{\bar{u}_b(p)(\gamma^+)_{ba} u_a(p)}_{= 2p^+} = 2p^+ (2\pi)^4 \delta^4(p-k)$ $\Rightarrow$ (plug in.) $\boxed{q^f_{quark}(x) = \delta(x-1)}$

one quark at $x=1$

Peskin, ch. 17.5
Sterman ch 14
(K, Levin ch. 2.4

QCD Improved Parton Model: DGLAP equation

How about corrections like  ?

These are important corrections.

However, let us first discard the negligible diagrams like