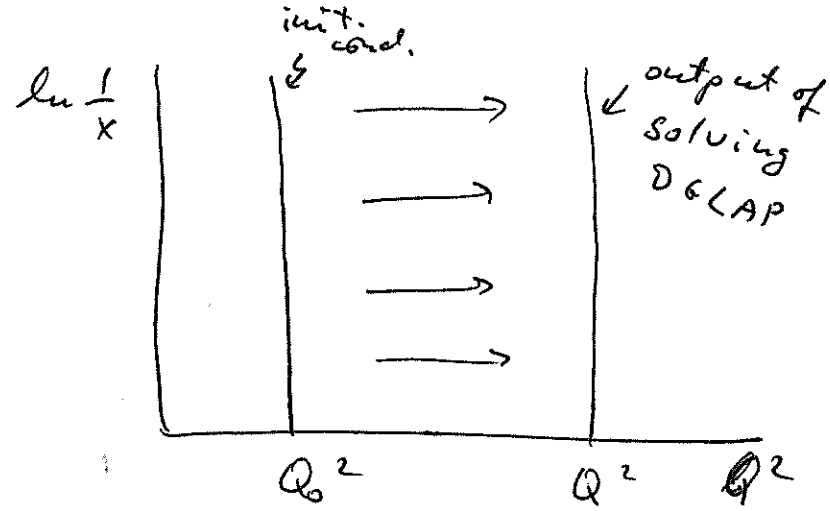
Last time | talked about DGLAP equation

~ usually one sets initial conditions at some $Q^2 = Q_0^2$ & "evolves" the PDFs with DGLAP to $Q^2 > Q_0^2$:



DGLAP at small x

Gluons dominate at small x ⇒ drop quarks

$$\frac{\partial}{\partial \ln Q^2} G(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dx_1}{x_1} P_{GG}\left(\frac{x}{x_1}\right) G(x_1, Q^2)$$

Def. moments: $G_n(Q^2) = \int_0^1 dx \cdot x^{n-1} \cdot G(x, Q^2)$

anomalous dimension: $\gamma_{GG}^{(n)} = \int_0^1 dz \cdot z^{n-1} P_{GG}(z)$

$P_{GG}(z) \simeq \frac{2N_c}{z}$ at small-x

In moments space, the solution is easy:

$$G_n(Q^2) = e^{\int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \frac{\alpha_s(Q'^2)}{2\pi} \gamma_{GG}^{(n)}} G_n(Q_0^2)$$

such that

$$\boxed{G(x,Q^2) = \int \frac{dn}{2\pi i}\, x^{-n}\, e^{\frac{\gamma_{GG}^{(n)}}{2\pi\beta_2} \ln\left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\right)} G_n(Q_0^2)}$$

with $\gamma_{GG}^{(n)} = \frac{2N_c}{n-1}$

Performing saddle-point approximation we got

$$x\,G(x,Q^2) \sim e^{2\sqrt{\frac{N_c}{\pi\beta_2} \ln\frac{1}{x} \ln\left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\right)}}$$

$\Rightarrow$ xG grows at small x, but slower than a power of x

$\Rightarrow$ the data shows that $xG \sim \left(\frac{1}{x}\right)^{\lambda}$, with $\lambda \simeq 0.3$ (a number). DGLAP evolution cannot give us this result $\Rightarrow$

$\Rightarrow$ another evolution equation which resums logarithms of x (powers of $\alpha_s \ln\frac{1}{x}$) is needed to elucidate the small-x behaviour.

This equation exists and is known as the BFKL equation (Balitsky, Fadin, Kuraev, Lipatov, '77)

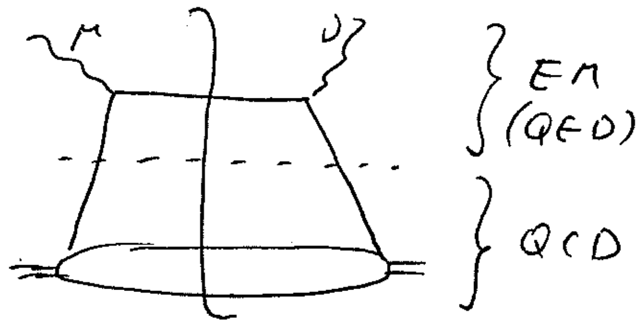
Particle Production in High Energy Hadronic Collisions.

Collinear Factorization

When we considered DIS above, we have factorized EM & QCD parts of the diagram:

We got

$$F_2(x, Q^2) = \sum_f e_f^2 x q^f(x)$$



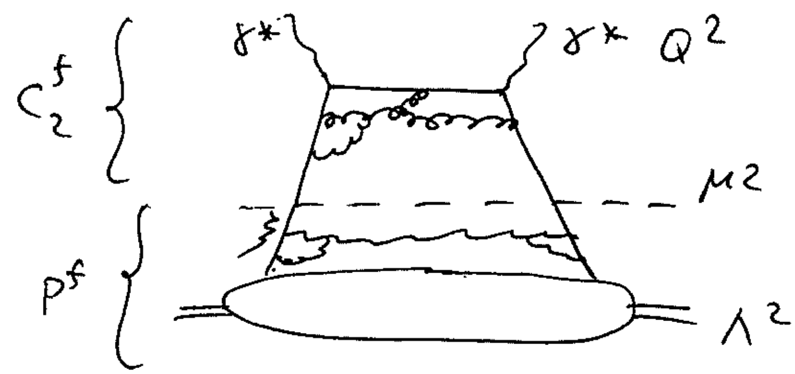
$\Rightarrow$ this is an example of collinear factorization

In general one writes:

$$F_2(x, Q^2) = \sum_{f, \bar{f}, \text{gluons}} \int_0^1 d\xi \; C_2^f\left(\frac{x}{\xi}, Q^2, \mu^2\right) P^f(\xi, \mu^2) + O\left(\frac{m^2}{Q^2}\right)$$

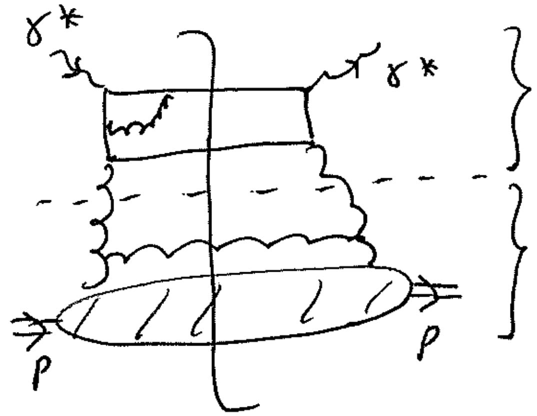
higher twists

C_2^f ~ coefficient ftn (may contain QCD corrections at higher orders)



$p^f = \{q^f, q^{\bar{f}}, G\}$; ξ – momentum fraction of the parton in p^f.

May also have



C_2^G

$G \sim$ gluon distribution

μ^2 is called factorization scale.

<u>Note:</u> C_2^f is <u>perturbatively calculable</u>, p^f is not (though one has DGLAP for p^f)

$\Lambda^2 \lesssim \mu^2 \lesssim Q^2 \Rightarrow$ but F_2 does not depend on μ^2, it is arbitrary $\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2(x, Q^2) = 0$

Write $\quad F_2 = C_2^f \otimes p^f$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = 0 = \left(\mu^2 \frac{d}{d\mu^2} C_2^f\right) \otimes p^f + C_2^f \otimes \mu^2 \frac{d}{d\mu^2} p^f$$

$\Rightarrow$ what happens (separation of variables, only C_2 depends on Q^2, only p^f depends on Λ^2):

$$\mu^2 \frac{d}{d\mu^2} P_f = \gamma(\alpha_s) \otimes P_f$$

~ DGLAP evolution

γ ~ splitting function.

$$\mu^2 \frac{d}{d\mu^2} C_2^f = -\gamma(\alpha_s) \otimes C_2^f$$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = -\gamma \otimes C_2^f \otimes p^f + C_2^f \otimes \gamma \otimes p^f = 0.$$

as desired.

$\Rightarrow$ can "place" corrections into PDF or coefficient function

Collinear factorization in DIS is a <u>theorem</u> which can be <u>proven</u> $\Rightarrow$ must be right! (at large-Q^2 only!)

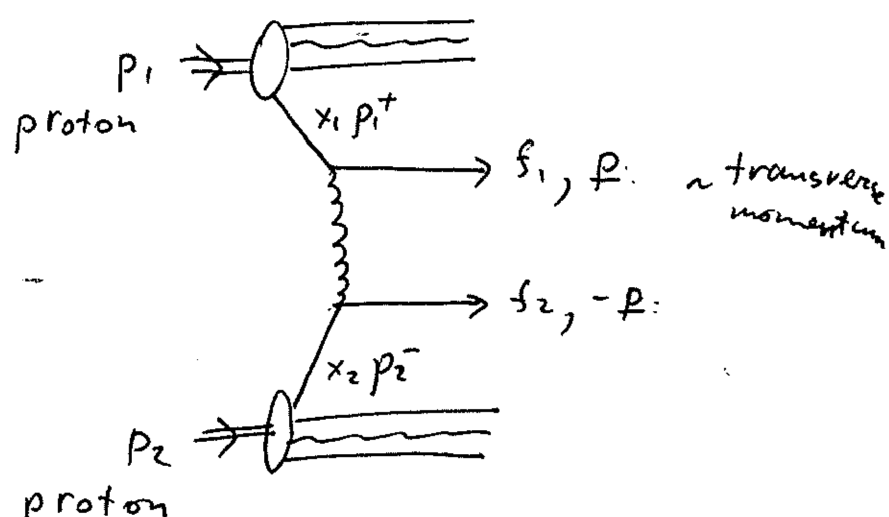
~ at LO have $C_2^f = \delta\left(\frac{x}{\xi} - 1\right) e_f^2$, f = quarks only

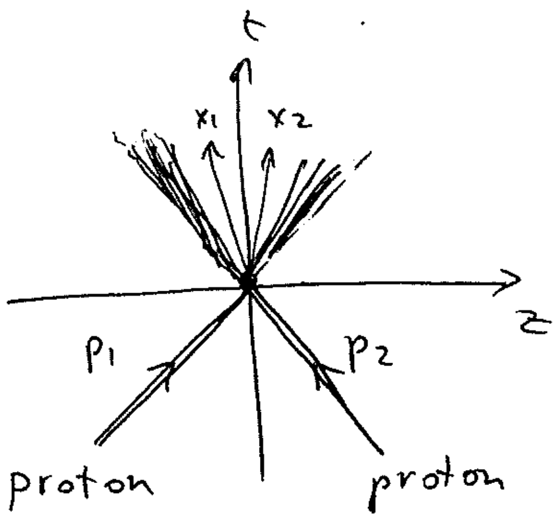
$$\Rightarrow F_2(x, Q^2) = \sum_f \underbrace{\int_0^1 d\xi \, \delta\left(\frac{x}{\xi} - 1\right)}_{x} e_f^2 \, q^f(\xi)$$

$$= \sum_f e_f^2 \, x \, q^f(x)$$ as expected!

Jet Production in Hadronic Collisions.

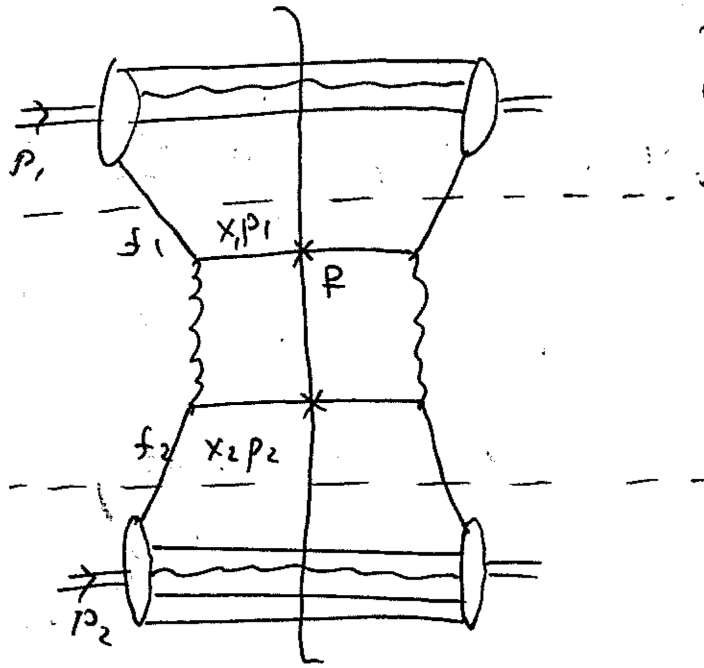
Collinear factorization also applies to hadron-hadron collisions. Consider quark production:





~ collision happens very fast on proton's time scales $\Rightarrow$ factorization.

Square the diagram:



cut

$\} \; p^{f_1}(x_1, \mu^2) \sim$ parton distribution

$\} \; \mathcal{E}_p \dfrac{d\hat{\sigma}_{f_1 f_2 \to f_1' f_2'}}{d^3 p} \sim$ "hard" cross section

$\} \; p^{f_2}(x_2, \mu^2) \sim$ parton distribution

The collinear factorization formula then reads:

$$\boxed{\mathcal{E}_p \frac{d\sigma}{d^3 p} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \; p^{f_i}(x_1, \mu^2) \cdot \mathcal{E}_p \frac{d\hat{\sigma}_{f_i f_j \to f_i' f_j'}}{d^3 p} \cdot p^{f_j}(x_2, \mu^2)}$$

Usually put $\mu^2 = p_T^2$ for large p_T jets (or hadrons)

After the collision quarks (gluons) that are produced get dressed by further emissions. But the flow of energy is not likely to be modified much by those. (Still people construct other IR-safe observables insensitive to late-time emissions: (+ + ...)).

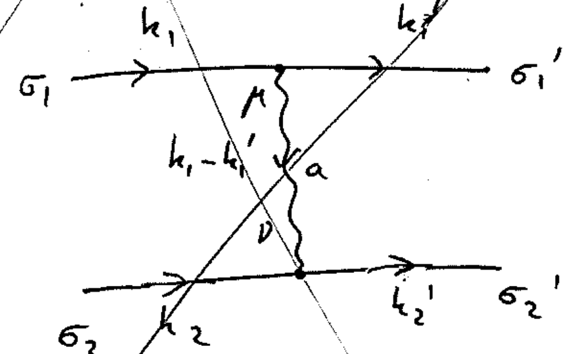
Example Quark jet production (coming from quarks). Replace $p^{f_i} \to q^f$ $\Rightarrow$ write

$$\mathcal{E}_p \frac{d\sigma}{d^3p} = \sum_{f_1,f_2} \int_0^1 dx_1\, dx_2\; q^{f_1}(x_1, p_T^2)\; \mathcal{E}_p \frac{d\hat{\sigma}_{f_1 f_2 \to f_1' f_2'}}{d^3p}\; q^{f_2}(x_2, p_T^2)$$

q^f ~ to be found from DGLAP (PDF data)

We can calculate the hard cross section:

$$d\hat{\sigma} = \frac{1}{2\mathcal{E}_1\, 2\mathcal{E}_2 \cdot \underset{|\vec{v}_1 - \vec{v}_2|}{\overset{2}{''}} \cdot} \frac{d^3k_1'}{(2\pi)^3 2\mathcal{E}_1'} \cdot$$



$$\cdot \frac{d^3k_2'}{(2\pi)^3 2\mathcal{E}_2'} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_1' - k_2') \cdot$$

$$\cdot |M|^2 \left[\delta^3(\vec{k}_1' - \vec{p}) + \delta^3(\vec{k}_2' - \vec{p}) \right] \cdot d^3p$$

measured jet can be either quark!

$$\Rightarrow \mathcal{E}_p \frac{d\hat{\sigma}}{d^3p} = ?$$

$\delta^{(3)}$ kills d^3k_2'

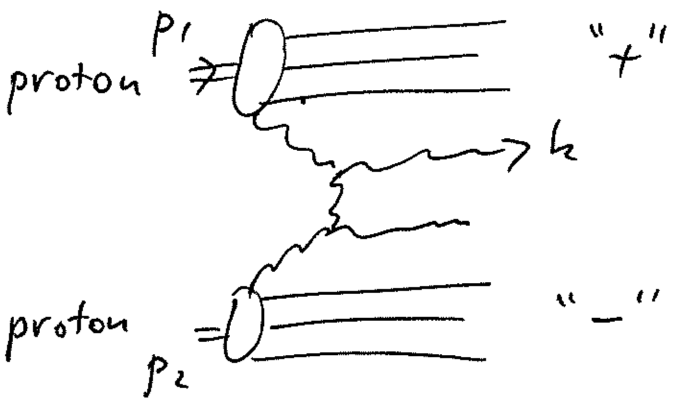
kills d^3k_1'

$$\frac{d^3k_1'}{(2\pi)^3 2\mathcal{E}_1'} \frac{d^3k_2'}{(2\pi)^3 2\mathcal{E}_2'} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_1' - k_2')\; \delta^3(\vec{k}_1' - \vec{p}) =$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4\mathcal{E}_p \mathcal{E}_2'} \delta(E_1 + \mathcal{E}_2 - \mathcal{E}_p - \mathcal{E}_2')$$

Rapidity in p+p collisions

Def. Rapidity variable $\boxed{\eta \equiv \frac{1}{2} \ln \frac{k^+}{k^-}}$ for an outgoing particle with momentum 4-vector k^μ.



at a collider, one proton comes in with

$$p_1^\mu = \left(p_1^+ , \frac{m^2}{2p_1^+} , \underline{0} \right) ,$$

another one has

$$p_2^\mu = \left(\frac{m^2}{2p_2^-} , p_2^- , \underline{0} \right) .$$

p_1^+ , p_2^- are very large, m = proton mass

$$s = (p_1 + p_2)^2 \simeq 2 p_1 \cdot p_2 = 2 p_1^+ p_2^- .$$

Proton rapidities: $y_1 = \frac{1}{2} \ln \frac{p_1^+}{m^2/2p_1^+} = \ln \frac{p_1^+ \sqrt{2}}{m}$.

$$y_2 = \frac{1}{2} \ln \frac{m^2/2p_2^-}{p_2^-} = - \ln \frac{p_2^- \sqrt{2}}{m} .$$

y_1 is large and positive, y_2 is large and <0.

The net rapidity interval between the protons is $y_1 - y_2 = \ln \frac{2p_1^+ p_2^-}{m^2} = \ln \frac{s}{m^2}$ $\Rightarrow$ the higher is the energy, the wider is the interval.

$$\boxed{y_1 - y_2 = \ln \frac{s}{m^2}}$$

Rapidity interval between our particle k and the proton p_1 is $y_1 - \eta = \ln \frac{p_1^+ \sqrt{2}}{m} - \frac{1}{2} \ln \frac{k^+}{k^-} =$

$=$ | as $k^- = \frac{k^2}{2k^+}$ for massless particle | $= \ln \frac{p_1^+ \sqrt{2}}{m} - \ln \frac{k^+ \sqrt{2}}{k_\perp} = \ln \frac{p_1^+}{k^+} + \ln \frac{k_\perp}{m}$

for $k_\perp \simeq m$ get $y_1 - \eta = \ln \frac{p_1^+}{k^+} = \ln \frac{1}{x_1}$, where $x_1 = \frac{k^+}{p_1^+}$ is the Bjorken-x in proton #1: $\boxed{y_1 - \eta \approx \ln \frac{1}{x_1}}$

Similarly, $\boxed{\eta - y_2 \simeq \ln \frac{1}{x_2}}$ with $x_2 = \frac{k^-}{p_2^-}$ ~ Bjorken-x in proton #2

We thus have