Quarks in the Electroweak Theory.

Quarks also form left-handed doublets under weak isospin:

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_o$$

(A2)

$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$ a doublet $\Rightarrow I_3 = \frac{\pm 1}{2} \Rightarrow Q = I_3 + \frac{Y}{2}$

$\Rightarrow Y = 2(Q - I_3) \Rightarrow$ for u have $Q = +\frac{2}{3}$, $I_3 = +\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}$; for d' have $Q = -\frac{1}{3}$, $I_3 = -\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{3} \Rightarrow Y = \frac{1}{3}$ for the doublet!

Singlets: $R_u = u_R$ has $Q = +\frac{2}{3}$, $I_3 = 0 \Rightarrow Y = \frac{4}{3}$

$R_d = u_d$ has $Q = -1/3$, $I_3 = 0 \Rightarrow Y = -2/3$

(same for other quark generations/families)

$\Rightarrow$ We have *defined* the quark *weak* eigenstates d', s', b' by:

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{weak eigenstates}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{quarks in QCD (mass eigenstates)}}$$

$$\text{CKM} = \underbrace{\overset{1963}{\text{Cabibbo}}}_{\text{? No prize?}}-\underbrace{\overset{1973}{\text{Kobayashi-Maskawa}}}_{\text{Nobel Prize '08}} \text{ matrix}$$

CKM matrix is unitary: $V^{+}V = VV^{+} = \mathbb{1}$.

(Logic: our mass matrix for quarks is diagonal, but there is no reason for EW interaction one to be diagonal too.)

Let's write down the Lagrangian:

$$\mathcal{L}_{\text{quarks + gauge}} = \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2}\underbrace{Y}_{=1/3} B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big)L_u$$

$$+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2}\underbrace{Y}_{=\frac{4}{3}} B_\mu\Big)R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2}B_\mu \underbrace{Y}_{=-2/3}\Big)R_d$$

+ other 2 generations.

$$\Rightarrow \boxed{\mathcal{L}_{\text{quarks + gauge}} = \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{6}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big)L_u + \bar{R}_u \, i\gamma^\mu\Big(\partial_\mu - i\frac{2}{3}g' B_\mu\Big)R_u + \bar{R}_d \, i\gamma^\mu\Big(\partial_\mu + i\frac{1}{3}g' B_\mu\Big)R_d + \text{2 more generations.}}$$

Need to couple quarks to the Higgs: (don't have to, but it would be nice $\Uparrow$ for $SU(2)_L \times U(1)_Y$)

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}.$$

If we write a term like $\bar{L}_u \phi R_u$ and $\bar{L}_u \phi R_d$.

However the VEV is $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Rightarrow$

$\Rightarrow$ near the Higgs VEV get

$$\bar{L}_u \phi R_u = (\bar{u}_L \ \bar{d}_L') \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} u_R = \bar{d}_L' u_R \frac{v}{\sqrt{2}}$$ ~ no mass for u ...

$Y=-1/3$, $Y=+1$, $Y=+4/3$ $\Rightarrow$ not $U(1)_Y$ invariant too...

~ like neutrinos, u would not get a mass... ?

(same for c, t quarks).

$\Rightarrow$ to give quarks mass define $\boxed{\tilde{\phi}(x) \equiv i\tau^2 \phi^*}$

for the VEV: $$\langle 0|\tilde{\phi}|0\rangle = i\underbrace{\tau^2}_{= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}} \cdot \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} =$$

$$= \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}.$$ ~ have the VEV $\neq 0$ on top now

Under $SU(2)_L$ gauge transform: $\phi \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}} \phi$

$$\Rightarrow \tilde{\phi} \to i\tau^2 \left(e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}} \phi\right)^* = i\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \phi^* =$$

$(\tau^2)^2 = \mathbb{1}$

$$= \underbrace{\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \tau_2}_{e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}} \tilde{\phi}$$

this is true because: $\tau^2(-\tau^{1*})\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

$$\cdot \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau^1$$

Similarly $\tau^2(-\tau^{2*})\tau^2 = \tau^2$ (obvious) and

$\tau^2(-\tau^{3*})\tau^3 = \tau^3$ $\Rightarrow$ eqn is true $\left(\begin{matrix} \tau^2(-\vec{\tau}^*)\tau^2 = \vec{\tau}, (\tau^2)^2 = \mathbb{1} \\ \Rightarrow \text{sandwich } (\tau^2)\text{'s in} \end{matrix}\right)$

$\Rightarrow$ under $SU(2)_L$ have $\tilde{\phi} \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}} \tilde{\phi}$

$\Rightarrow$ transforms just like ϕ !

$\Rightarrow$ can write $\bar{L}_u \tilde{\phi} R_u$ ~ $SU(2)_L$ invariant!

near VEV: $\bar{L}_u \tilde{\phi} R_u = (\bar{u}_L \; \bar{d}'_L) \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} u_R = \frac{v}{\sqrt{2}} \bar{u}_L u_R$

$\langle 0 | \tilde{\phi} | 0 \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\Rightarrow$ may give u-quark mass!

terms like $\bar{L}_u \phi R_d = (\bar{u}_L \; \bar{d}'_L) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} d_R = \frac{v}{\sqrt{2}} \bar{d}'_L d_R$

can give d-quark mass (and s, b quarks too).

$\Rightarrow$ also need to check weak hypercharge:

ϕ has $Y = +1$ $\Rightarrow$ $\tilde{\phi}$ has $Y = -1$ $\Rightarrow$ $\underbrace{\bar{L}_u}_{Y=-1/3} \underbrace{\tilde{\phi}}_{Y=-1} \underbrace{R_u}_{Y=4/3}$ $\Rightarrow$ net $Y = 0$

$\underbrace{\bar{L}_u}_{Y=-1/3} \underbrace{\phi}_{Y=+1} R_d$ $\Rightarrow Y = -2/3$ $\Rightarrow$ net $Y = 0$ ~ both work!

To write quarks + Higgs couplings let's limit ourselves to 2 generations: $L_u, L_c, R_u, R_d, R_c, R_s$.

First write all possible terms:

$$\mathcal{L}_{\text{quarks-Higgs}} = -G_1 [\bar{L}_u \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^+ L_u] - G_2 [\bar{L}_u \phi R_d + \bar{R}_d \phi^+ L_u] - G_3 [\bar{L}_u \phi R_s + \bar{R}_s \phi^+ L_u] - G_4 [\bar{L}_c \tilde{\phi} R_c + \bar{R}_c \tilde{\phi}^+ L_c]$$

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$$- G_5 \left[\bar{L}_c \phi R_d + \bar{R}_d \phi^+ L_c \right] - G_6 \left[\bar{L}_c \phi R_s + \bar{R}_s \phi^+ L_c \right]$$

$$- G_7 \left[\bar{L}_u \tilde{\phi} R_c + \bar{R}_c \tilde{\phi}^+ L_u \right] - G_8 \left[\bar{L}_c \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^+ L_c \right].$$

Plug in $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$:

$$\mathcal{L}^{2\text{ generations}}_{\text{quark - Higgs}} = -\frac{v}{\sqrt{2}} \Big\{ G_1 \bar{u} u + G_2 (\bar{d}'_L d_R + \bar{d}_R d'_L) + G_3 (\bar{d}'_L s_R +$$

$$+ \bar{s}_R d'_L) + G_4 \bar{c} c + G_5 (\bar{s}'_L d_R + \bar{d}_R s'_L) + G_6 (\bar{s}'_L s_R + \bar{s}_R s'_L)$$

$$+ G_7 (\bar{u}_L c_R + \bar{c}_R u_L) + C_8 (\bar{c}_L u_R + \bar{u}_R c_L) \Big\}$$

$\Rightarrow$ first of all we see $m_u = G_1 \frac{v}{\sqrt{2}}$ $m_c = \frac{G_4 v}{\sqrt{2}}$.

$\Rightarrow$ can't have $u \to c$ & vice versa $\Rightarrow \boxed{G_7 = G_8 = 0}$.

$\Rightarrow$ Left with d, s quarks : for those write:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$\theta_c \sim$ Cabibbo angle , in CKM matrix $V_{ud} \simeq \cos\theta_c \simeq V_{cs}$,

$V_{us} \simeq \sin\theta_c \simeq -V_{cd}$

$\theta_c \simeq 13°$ small mixing.

$\Rightarrow d' = d\cos\theta_c + s\sin\theta_c$

$s' = -d\sin\theta_c + s\cos\theta_c$

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$$\Rightarrow \mathcal{L}^{d,s \text{ part}}_{\text{quark-Higgs}} = -\frac{v}{\sqrt{2}} \Big\{ G_2 \Big[\underline{\bar{d} d \cos\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L) \cdot$$

$$\sin\theta_c \Big] + G_3 \Big[\underline{\bar{s} s \sin\theta_c} + (\bar{d}_L s_R + \bar{s}_R d_L) \cos\theta_c \Big] +$$

$$+ G_5 \Big[\underline{-\bar{d} d \sin\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L) \cos\theta_c \Big] +$$

$$+ G_6 \Big[\underline{\bar{s} s \cos\theta_c} - (\bar{d}_L s_R + \bar{s}_R d_L) \sin\theta_c \Big] \Big\} =$$

$$= -\bar{d} d \frac{v}{\sqrt{2}} \Big[G_2 \cos\theta_c - G_5 \sin\theta_c \Big] - \bar{s} s \frac{v}{\sqrt{2}} \Big[G_3 \sin\theta_c +$$

$$+ G_6 \cos\theta_c \Big] - \frac{v}{\sqrt{2}} (\bar{s}_L d_R + \bar{d}_R s_L) \underbrace{\Big[G_2 \sin\theta_c + G_5 \cos\theta_c \Big]}_{=0}$$

$$- \frac{v}{\sqrt{2}} (\bar{d}_L s_R + \bar{s}_R d_L) \underbrace{\Big[G_3 \cos\theta_c - G_6 \sin\theta_c \Big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2 \tan\theta_c$

$G_6 = G_3 \cot\theta_c$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}} \Big[G_2 \cos\theta_c - G_5 \sin\theta_c \Big] = \boxed{\frac{v}{\sqrt{2}} \frac{G_2}{\cos\theta_c} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}} \Big[G_3 \sin\theta_c + G_6 \cos\theta_c \Big] = \boxed{\frac{v}{\sqrt{2}} \frac{G_3}{\sin\theta_c} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.226 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

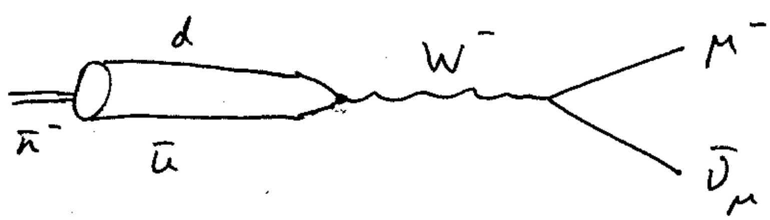
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

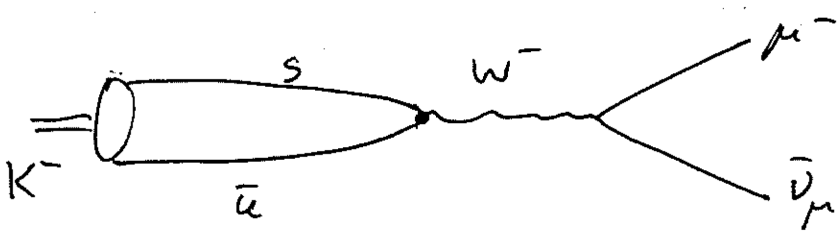
$$g(\bar u_L \ \bar d'_L)\, i\gamma^\mu \underbrace{\frac{\vec\tau}{2}\cdot \vec W_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix}$$ => has

$$g\, \bar u_L\, \gamma\cdot W_\mu\, d'_L + g\, \bar d'_L\, \gamma\cdot W^+_\mu\, u_L$$

Experimentally one has the following decays:



$\pi^- \to \mu^- \bar\nu_\mu$



$K^- \to \mu^- \bar\nu_\mu$